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# **Intermittent turbulence, eddy mixing in stably stratified atmospheric boundary layers and in upper troposphere and lower stratosphere**

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## OUTLINE

- ▶ **Urban Boundary Layer**
- ▶ **Turbulence intermittency**  
in stably stratified boundary layers (SBL)
- ▶ **Features of eddy mixing in the SBL**  
and in the free atmosphere

**Study motivation**: The turbulence in stably stratified planetary boundary layers has scientifically intriguing nature and practical significance (e.g., pollutant transport). Indeed, its dynamics includes such phenomena as occurrence of Kelvin-Helmholtz instability (K-H), gravity waves, low-level jets (LLJ).

# RANS approach for turbulence modeling

- Reynolds stresses,  $\tau_{ij} \equiv \langle u_i u_j \rangle$

$$\frac{D}{Dt} \tau_{ij} + D_{ij} = P_{ij} + \beta_i h_j + \beta_j h_i - \Pi_{ij} - \varepsilon_{ij}$$

- Heat fluxes,  $h_i \equiv \langle u_i \theta \rangle$

$$\frac{Dh_i}{Dt} + D_i^h = -h_j \frac{\partial U_i}{\partial x_j} - \tau_{ij} \frac{\partial \Theta}{\partial x_j} + \beta g_i \langle \theta^2 \rangle - \Pi_{i\theta}$$

$$\Pi_{i\theta} \equiv \left\langle \theta \frac{\partial p}{\partial x_i} \right\rangle$$

## Modification of pressure-scalar correlation $\Pi_{i\theta} = \langle \theta \frac{\partial p}{\partial x_i} \rangle$ in the stably stratified turbulence

The relaxation linear model employed for the slow term:  $\Pi_{i\theta} \cong \overline{u'_i \theta'} \cdot \tau_{p\theta}^{-1}$

and in 'standard' the second order closure models usually assume, that

$$\tau_{p\theta} \propto \tau = E / \varepsilon$$

But, such closure may not necessarily apply to stably stratified flows!

Indeed, the time scale  $\tau_{p\theta}$  must include a buoyancy frequency  $N$  (effect of internal waves on turbulent transport) :

$$(*) \quad \tau_{p\theta} = \tau / [1 + \alpha \tau^2 N^2]$$

The physical reason behind (\*) is that in stably stratified flows, eddies work against gravity and lose the TKE, which converted to potential energy.

# Three-parameter theory of stratified turbulence: eddy diffusivities of momentum and heat

$$\left( \langle uw \rangle, \langle vw \rangle \right) = -K_m \left( \frac{\partial U}{\partial z}, \frac{\partial V}{\partial z} \right)$$

$$\langle w\theta \rangle = -K_h \frac{\partial \Theta}{\partial z} + \gamma_c$$

$$K_m = \frac{E^2}{\varepsilon} S_M$$

$$K_h = \frac{E^2}{\varepsilon} S_H$$

# Three-parameter theory of stratified turbulence : closing

**Turbulent kinetic energy**  $E = (1/2)\langle u_i u_i \rangle$

$$\frac{DE}{Dt} + \frac{1}{2} D_{ii} = -\tau_{ij} \frac{\partial U_i}{\partial x_j} + \beta_i h_i - \varepsilon,$$

**TKE dissipation,  $\varepsilon$**

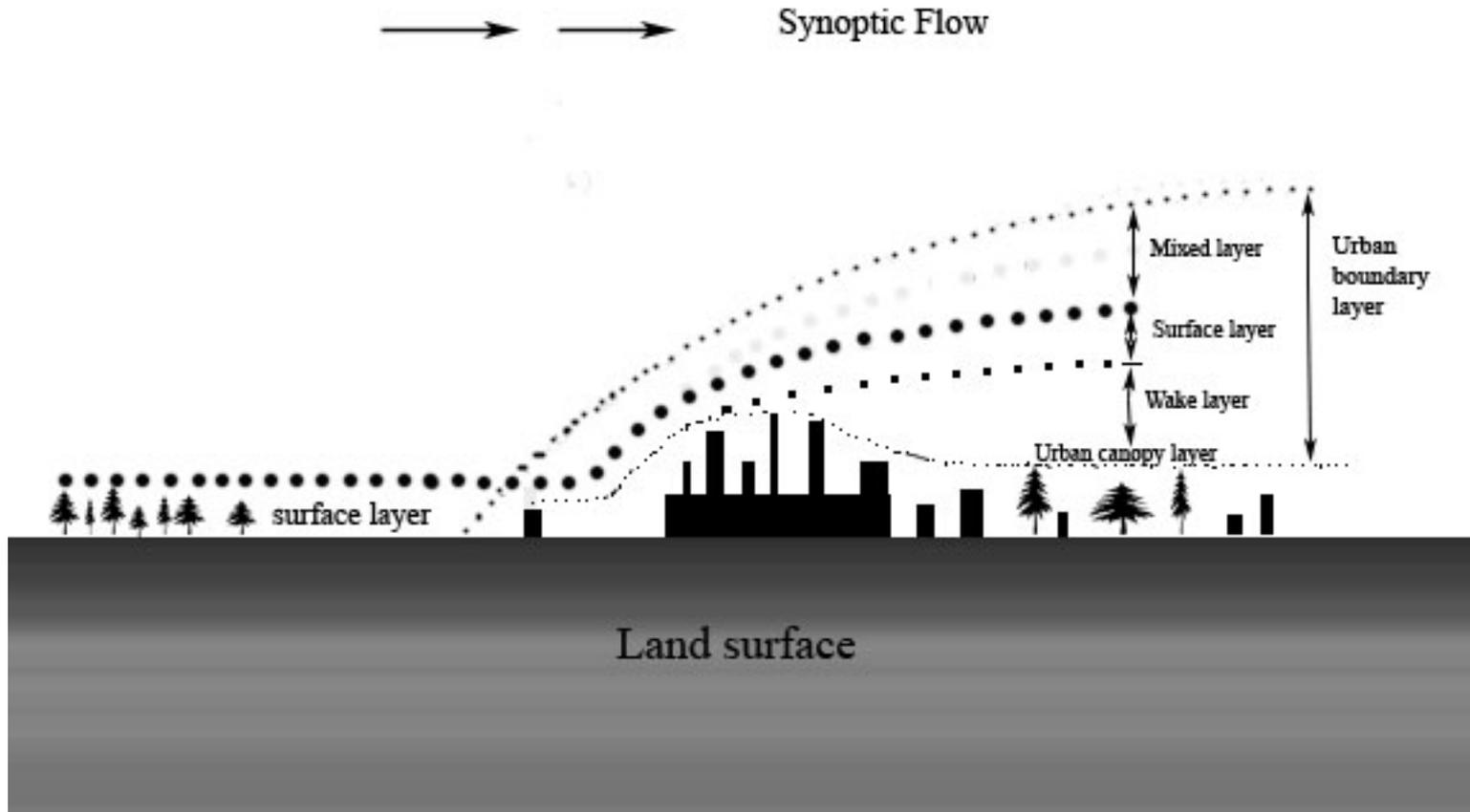
$$\frac{D\varepsilon}{Dt} + D_\varepsilon = c_{\varepsilon 1} \frac{\varepsilon}{E} \left( -\langle u_i u_k \rangle \frac{\partial U_i}{\partial x_k} + \beta g \delta_{i3} \langle u_i \theta \rangle \right) - c_{\varepsilon 2} \frac{\varepsilon^2}{E},$$

**Temperature variance,  $\overline{\theta'^2}$**

$$\frac{D\langle \theta^2 \rangle}{Dt} + D_{\theta^2} = -2h_i \frac{\partial \Theta}{\partial x_i} - 2\varepsilon_\theta,$$

# 1. Urban Boundary Layer (UBL)

A typical urban domain over flat terrain



# Scales & layers of the urban-PBL (UBL):

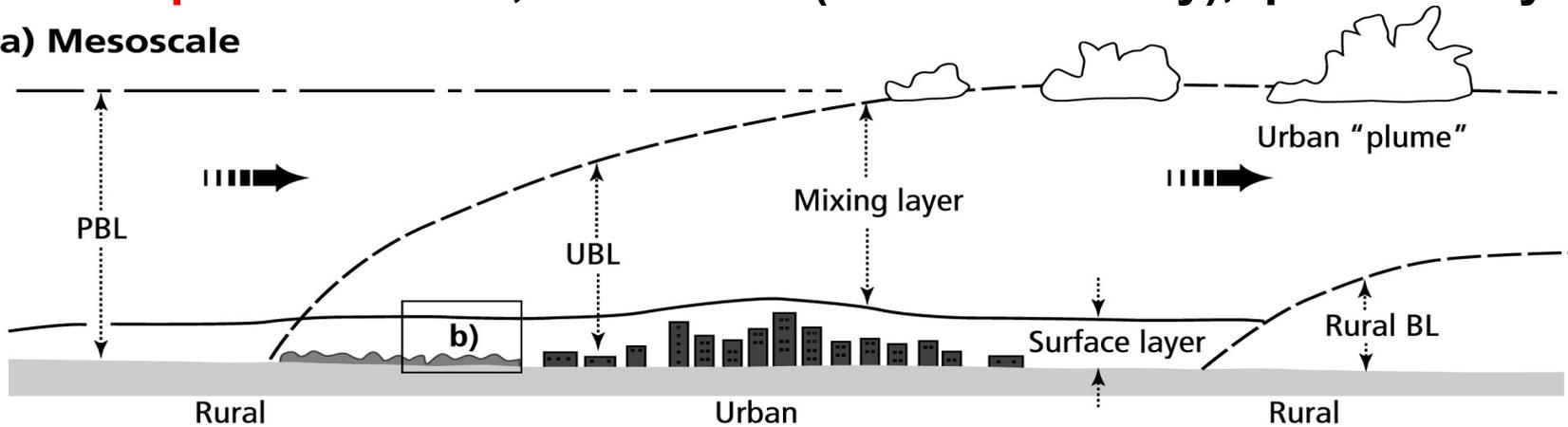
**UBL**=numerical; **UCL**=Urban-canyon layer (lower part of **RSL**)

**RSL** = non-equilibrium, non-analytical

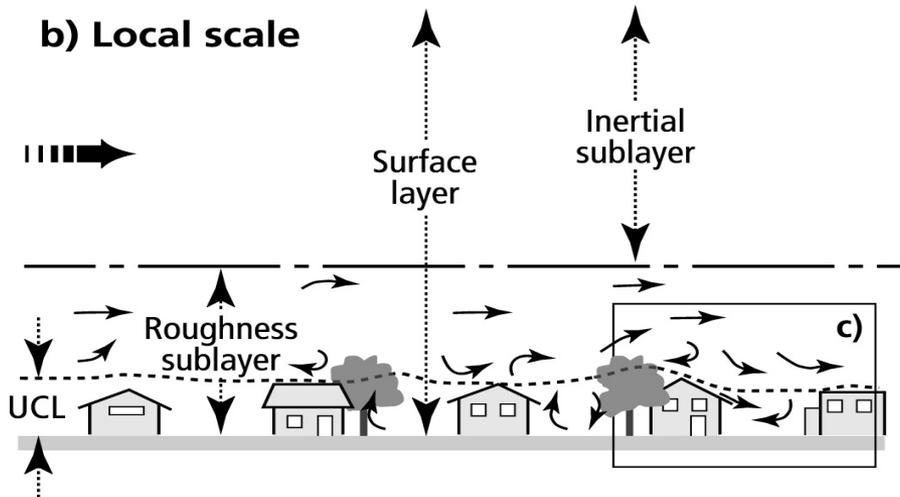
**Sfc layer** = (analytical) Inertial sub-layer + **RSL**

**urban plume** = warm, well mixed (neutral stability), polluted lay

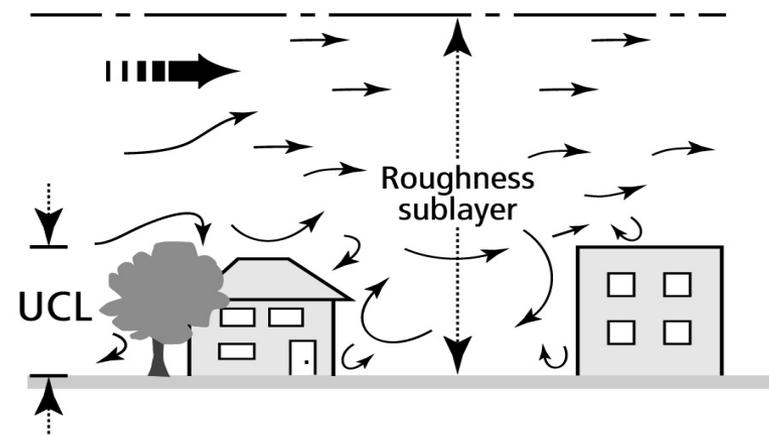
a) Mesoscale



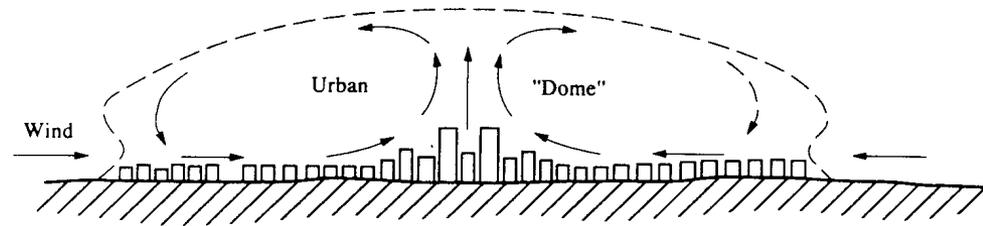
b) Local scale



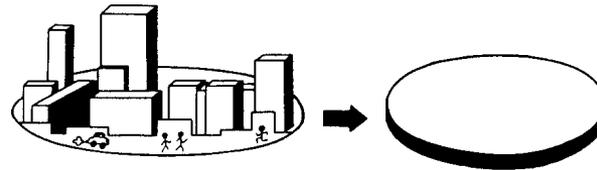
c) Microscale



# UHI: Laboratory Experiment



Oke's model of a heat island in calm condition

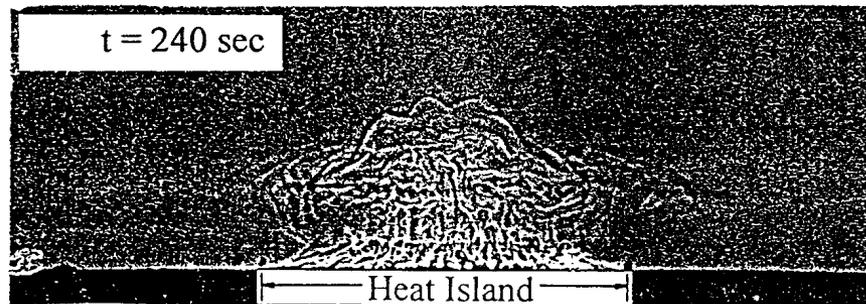


(a)

(b)

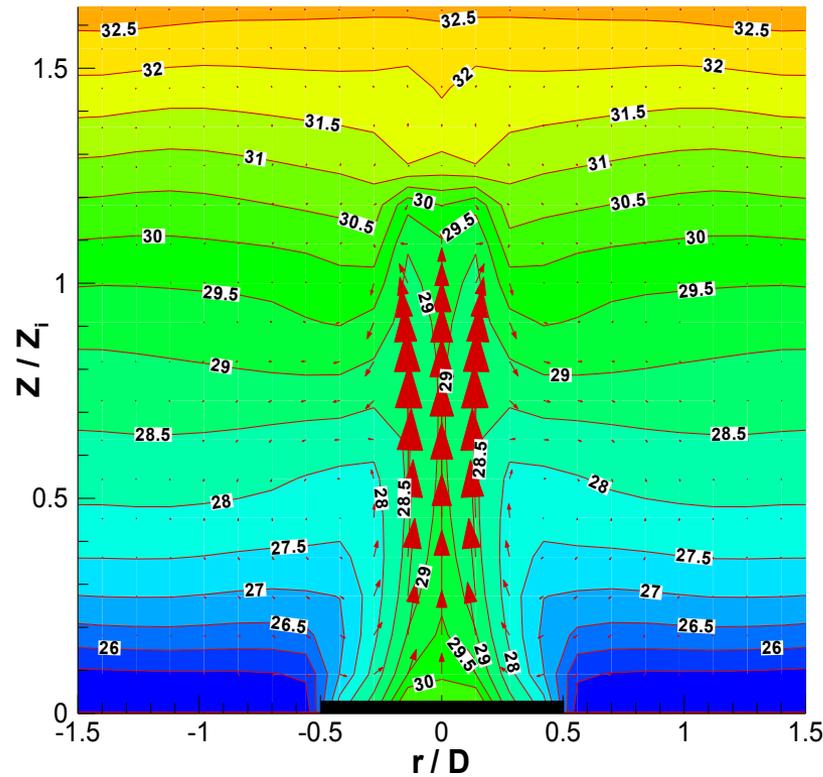
(a) real urban area

(b) heated circular disc

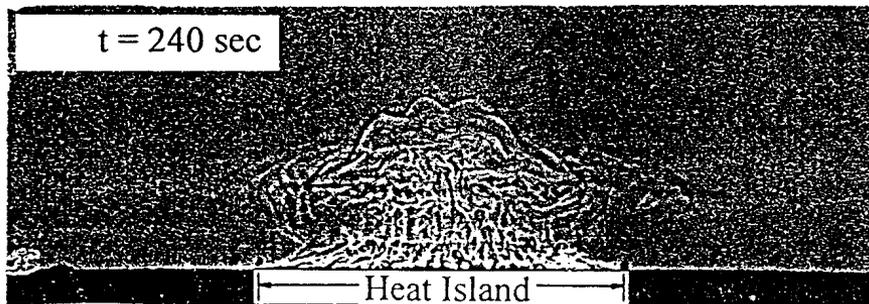


Shadowgraph picture heat island above a heated circular disc





## Simulation with three parameters RANS turbulence scheme

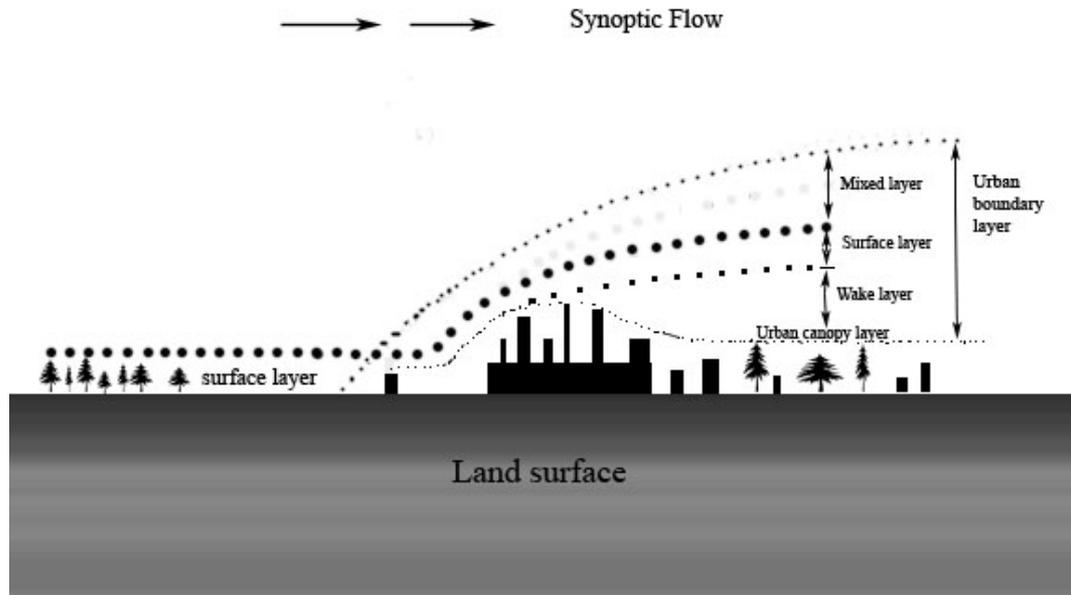


← Experiment: Lu et al.  
(J. Appl. Meteor. 1997. V. 36)

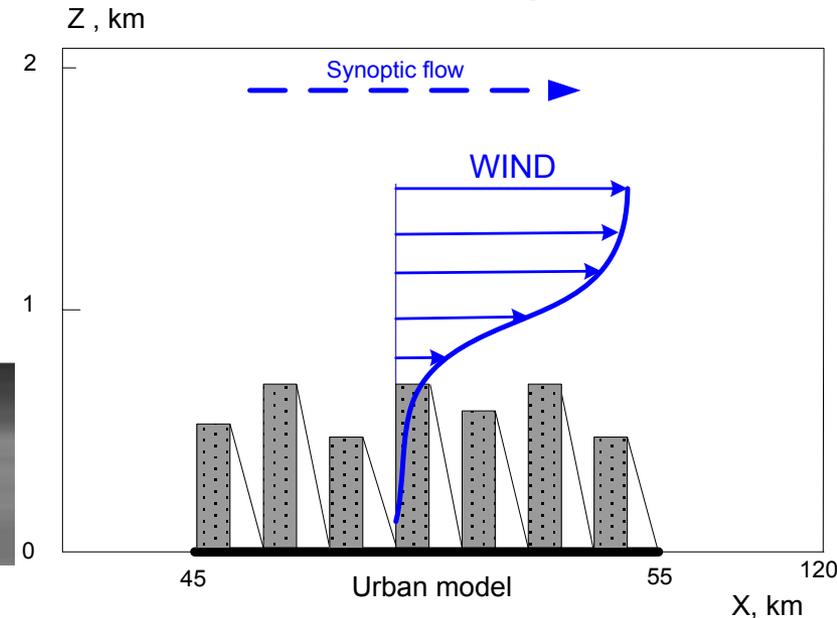
Shadowgraph picture heat island above a heated circular disc

# UBL: RANS mesoscale modelling

A typical urban domain over flat terrain



## Parameterization of Urban Roughness

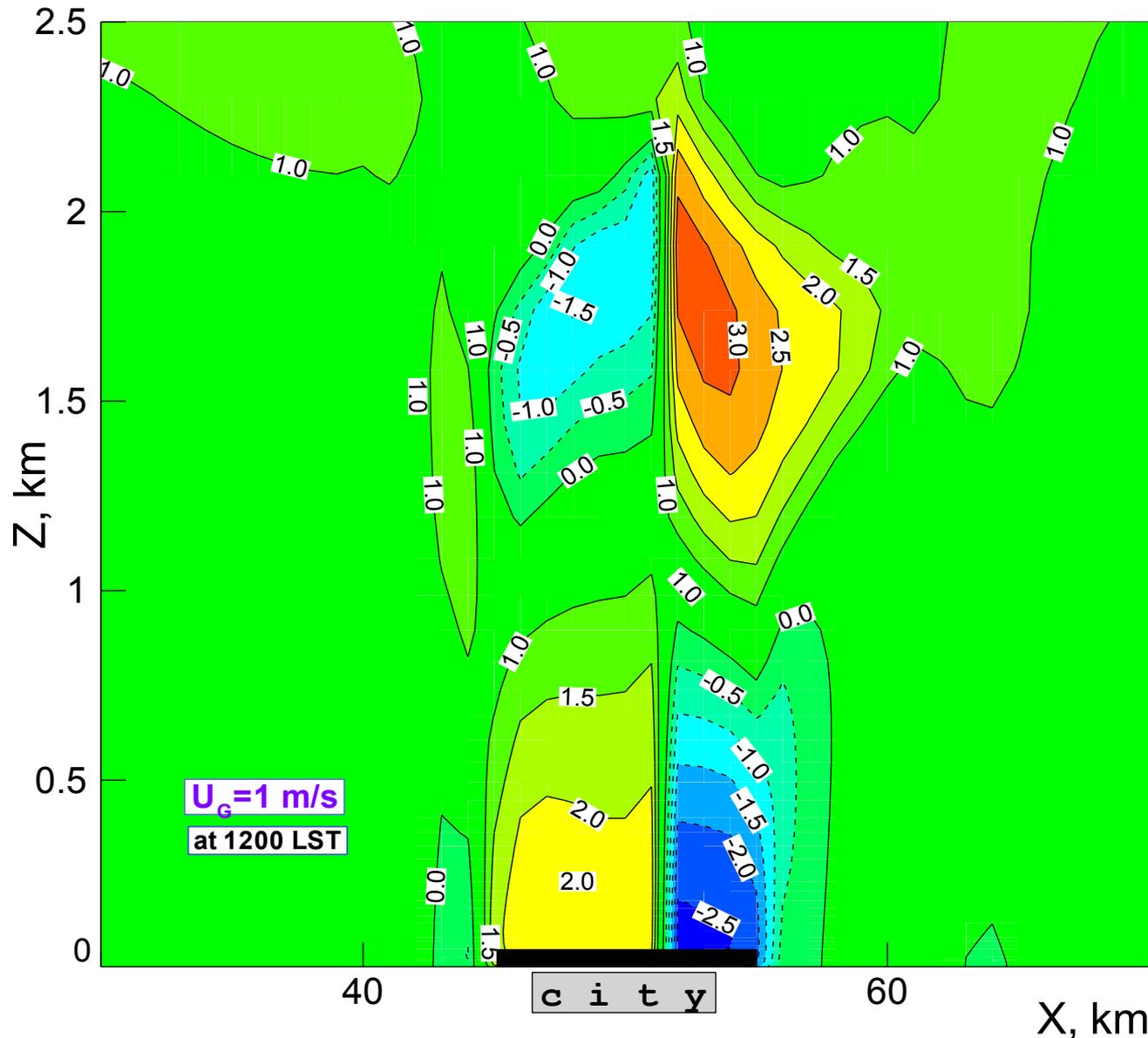


## The concept of incorporation of urban canopy model:

- city is represented as array of buildings with different heights
- impact of urban surfaces on airflow (drag induced by buildings, enhancement of the transformation of MKE into TKE and et al.) takes into account in governing equations (momentum, energy, TKE, spectral consumption of TKE, temperature variance) as the extra terms (“source”/ ”sink”).

# Thermal Circulation above UBL.

## RANS simulation

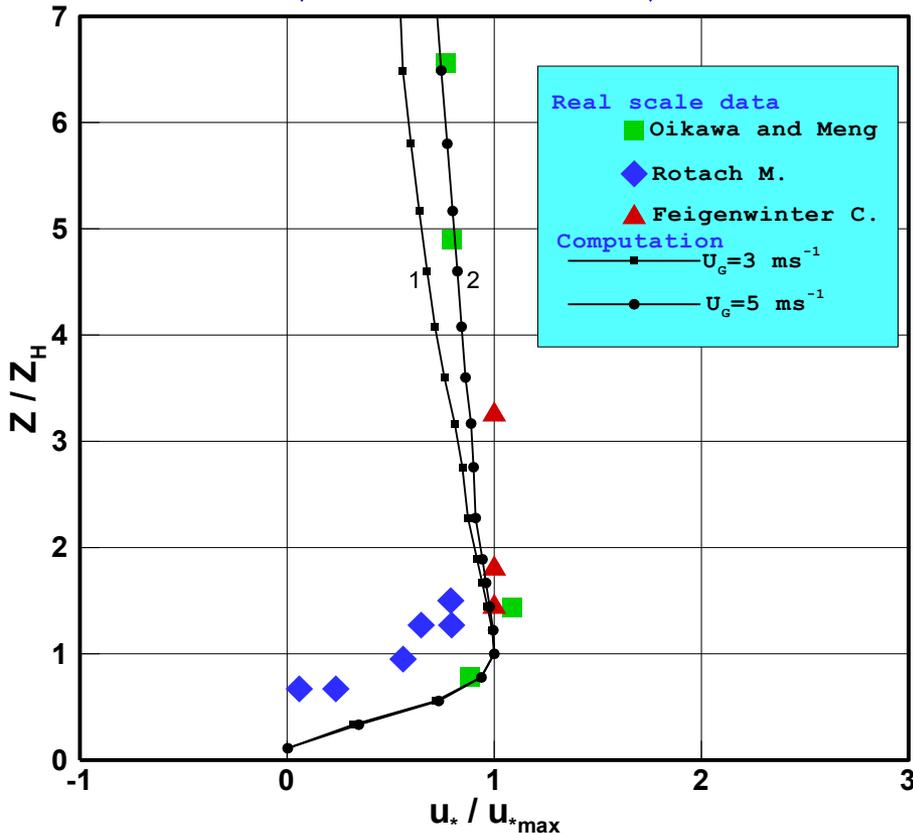


← For low synoptic speed of 1 m/sec, the horizontal gradient of temperature between air above city and air above the rural area generates a thermal circulation.

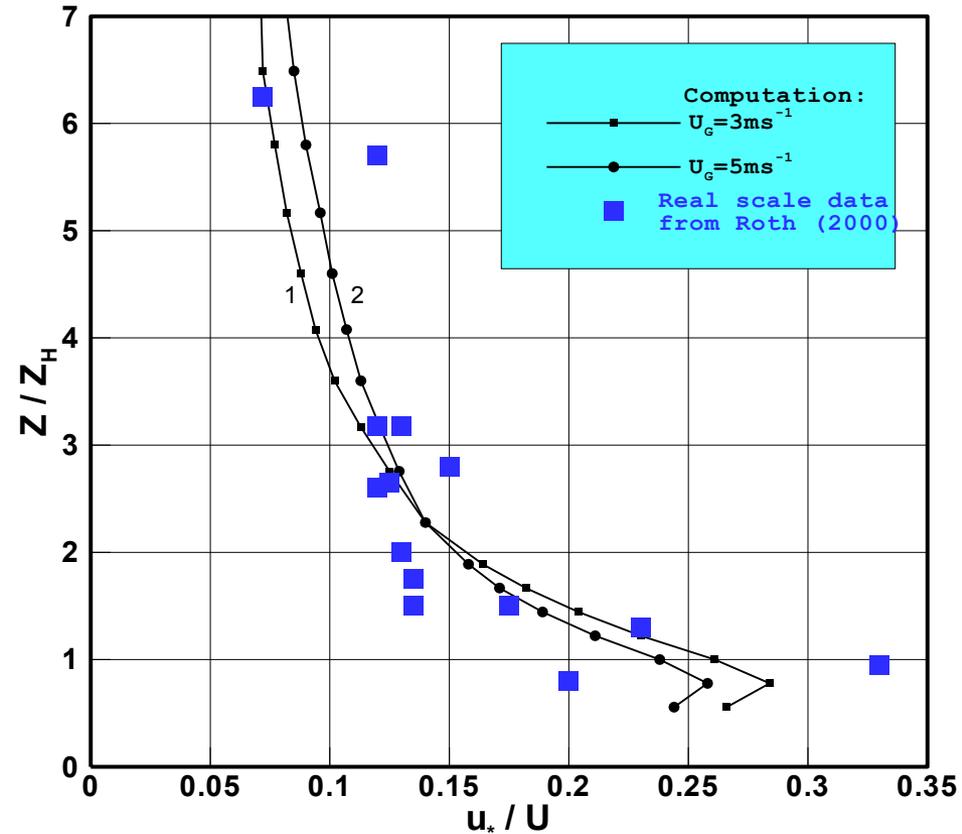
When the wind velocity increases (for example, 3 m/sec and more) the column of hot, unstable air above the city is still present, but it is advected downwind and thermal circulation it is displaced on the leeside of city.

# Vertical profiles of 'local' friction velocity

$$u_* = \left( \overline{uw}^2 + \overline{vw}^2 \right)^{1/4}$$

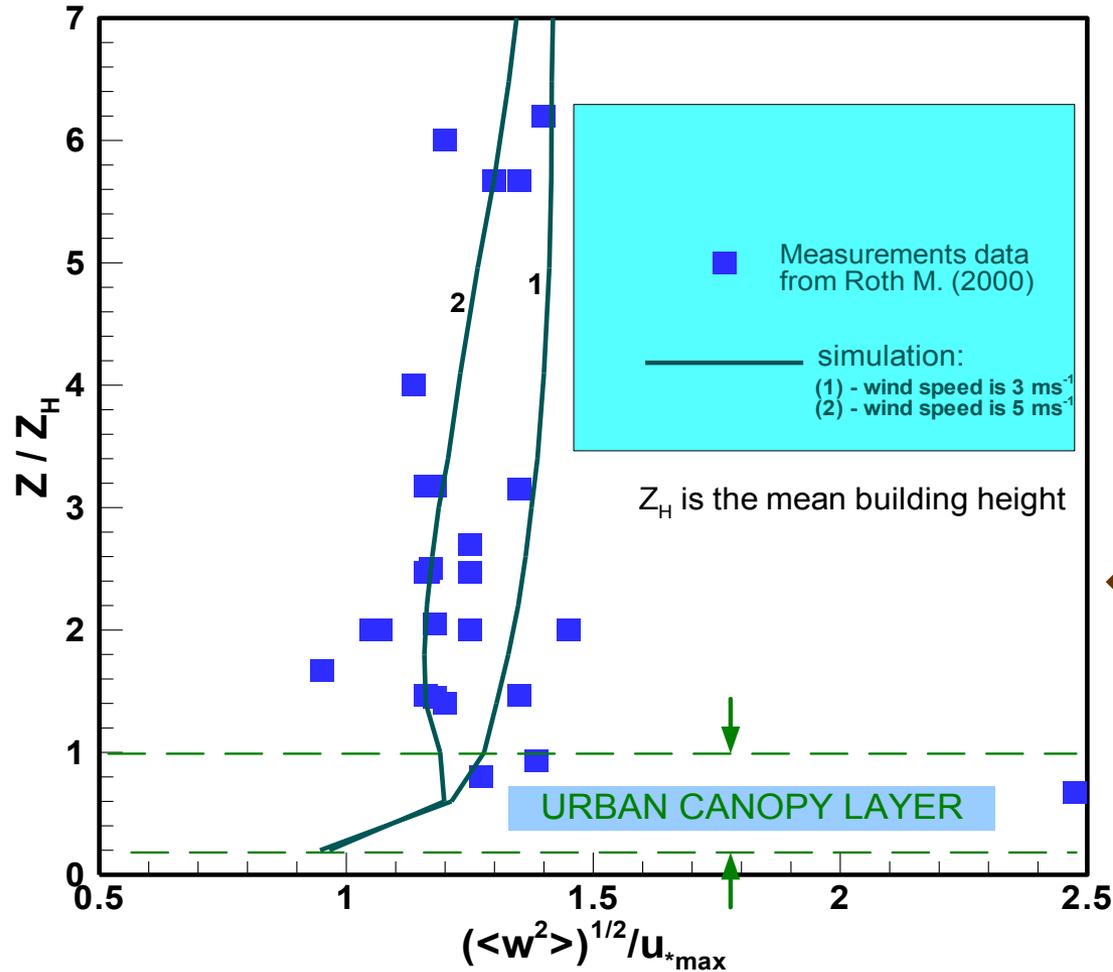


# Vertical profiles of ratio $u_*/U$



Vertical profiles of the 'local' friction velocity, normalized by its maximum value and of the ratio between the local friction velocity and the mean wind speed, for the two simulations, (1)- the case with geostrophic wind at 3 m /s, and (2 )- 5 m/s . For comparison, the fitting of different real scale data are also plotted. Since it is not certain about the recorded time and meteorological conditions of the data of different authors, computed values of frictional velocity are averaged from all computations for a 24-hr period.

# Vertical component of turbulent kinetic energy



← The symbols are data of **Roth M.**  
**Review of atmospheric  
turbulence over cities. Q. J. R.  
Meteor. Soc. 2000. Vol.126,  
941-990.**

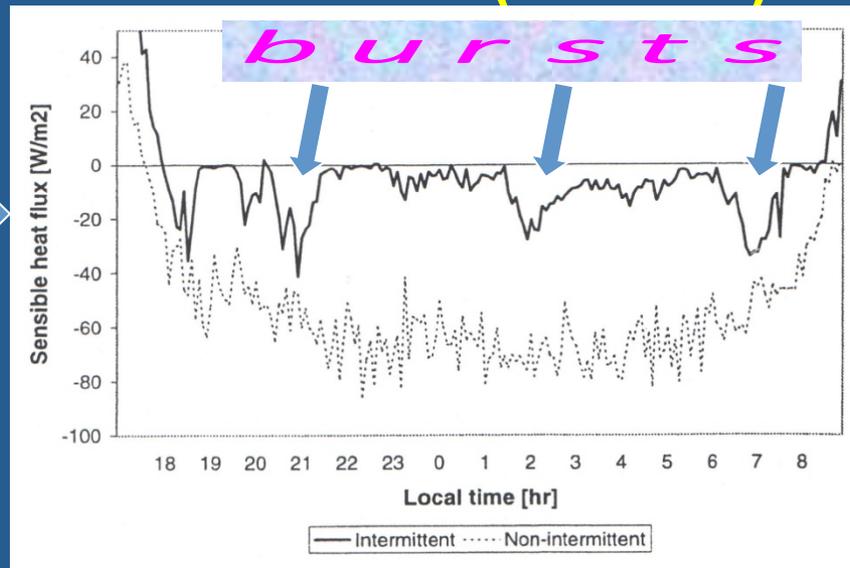
## 2. SBL. Turbulence intermittency near to surface

The SBL is formed at surface cooling.

This figure shows the development of turbulent heat flux near the surface during a clear night with relatively weak winds.

Intermittent turbulence is characterized by brief episodes of turbulence with intervening periods of relatively weak or unmeasurable small fluctuations (solid line).

Experiment :  
CASES – 99



The dashed line represents a case with continuous turbulence (strong surface winds).

# SBL. Modeling of intermittent turbulence near to surface

TKE transport equation ( $E = 1/2 \cdot \overline{u_i u_i}$ )

$$\frac{DE}{Dt} + \text{Diff}(E) = \text{generation}(shear, buoyancy) - \text{dissipation}$$

$$\text{Diff}(E) = \frac{\partial}{\partial z} \left[ w \left( \frac{u^2 + v^2 + w^2}{2} \right) + \frac{pw}{\rho_0} \right] = A + B$$

**Parameterization of third moments:**  $A = -\frac{\partial}{\partial z} \frac{K_m}{\sigma_E} \frac{\partial E}{\partial z}$

‘Prandtl number’  $\sigma_E$ :

$\sigma_E < 1$ , if both terms A and B have the same sign,  
and larger than unity otherwise.



**For SBL in near surface it follows from LES (Kosovic and Curry, 2000)**

**For CBL: Deardorff and Willis (1985), Moeng et al. (2004)**

# SBL: modeling of intermittent turbulence near to surface

TKE transport equation ( $E = 1/2 \cdot \overline{u_i u_i}$ )

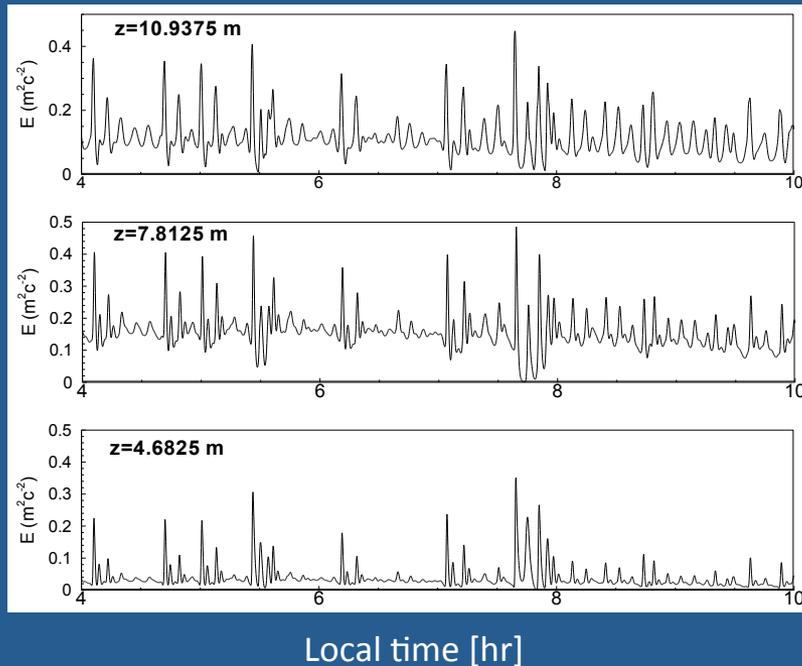
$$\frac{DE}{Dt} = \left[ \frac{\partial}{\partial z} \frac{K_m}{\sigma_E} \frac{\partial E}{\partial z} \right] + (P + G) - \varepsilon$$

- ‘Standard’ (e. g. Duynkerke, 1988),  $\sigma_E = 1$  accelerates the turbulent diffusion, smoothing out the intermittent bursts.
- $\sigma_E = 2.5$  is necessary for simulating intermittent turbulence.

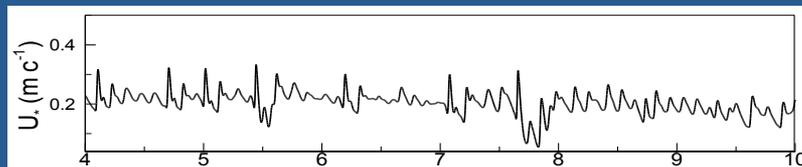
# SBL. Simulation of turbulent energy intermittency near to surface: sensitivity test of RANS-scheme

In the present study has been tested capability of RANS-scheme in reproduction of the intermittent turbulence.

Time series of TKE for a quasi-steady state of SBL



The vertical intermittent events under external conditions of low-wind and clear skies are generated at surface, propagating upwards through the turbulence transfer term in the TKE equation.



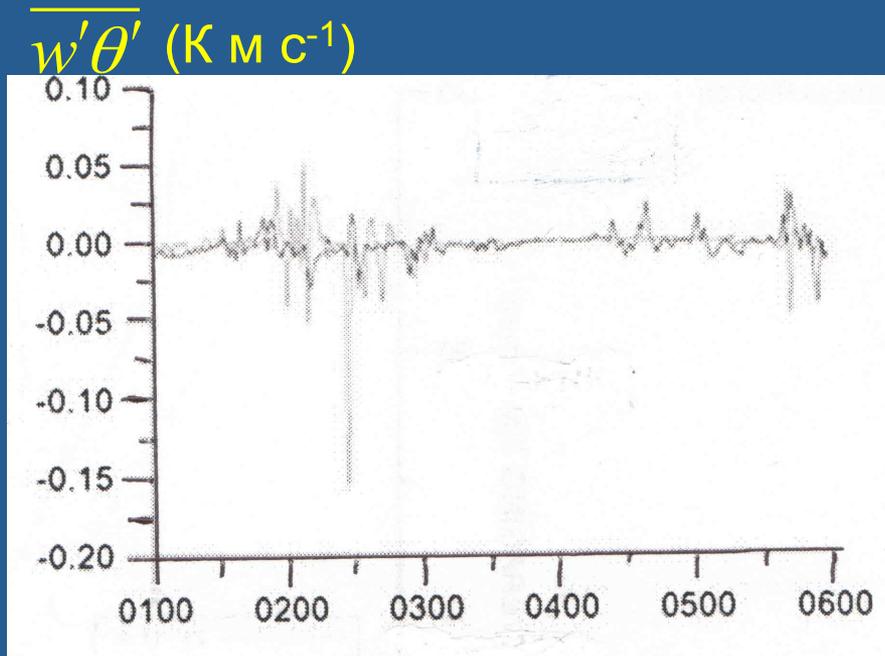
Time series of friction velocity nearest to surface

# SBL. Intermittent turbulent heat flux in near surface

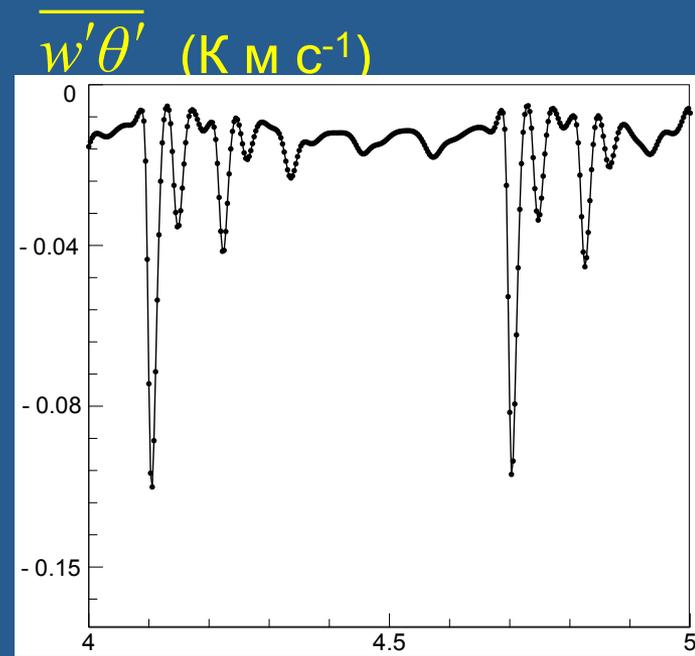
Measurements:

BLM.2010.136. 165-174

RANS simulation

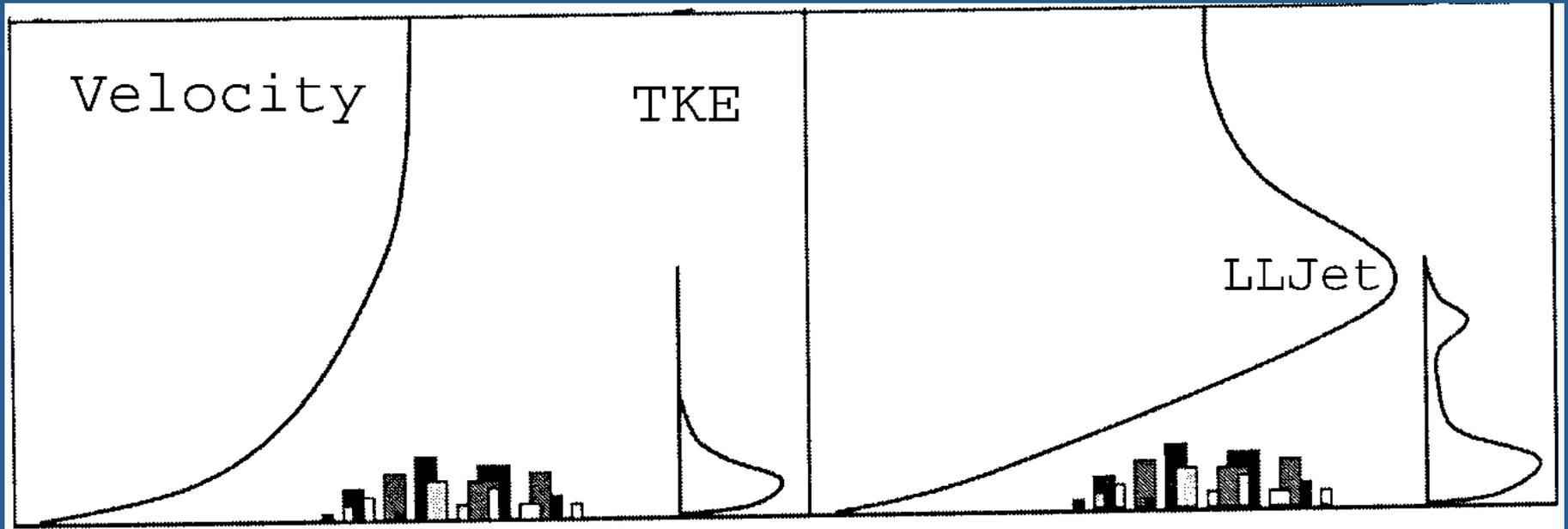


Local time

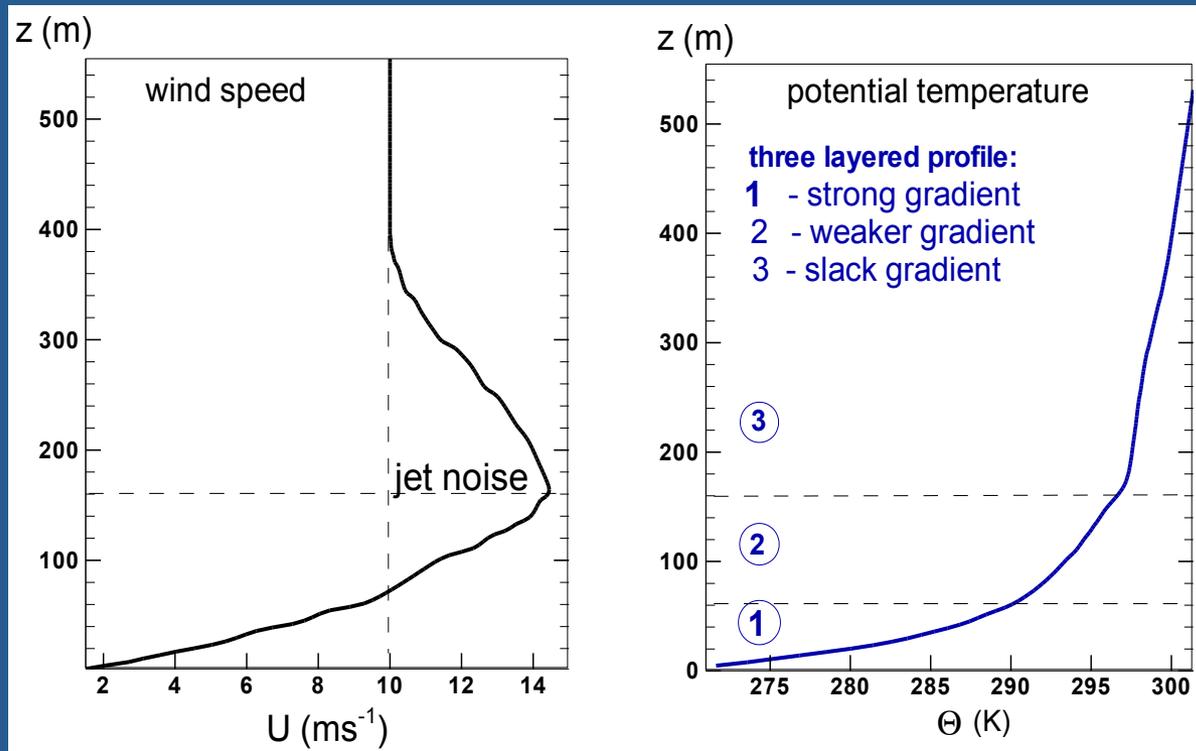


Local time (hr)

### 3. SBL. Intermittency of elevated turbulence generated by Low-level-Jet (LLJ)



# SBL. Intermittency of elevated turbulence generated by Low-level-Jet (LLJ): simulation with RANS-scheme

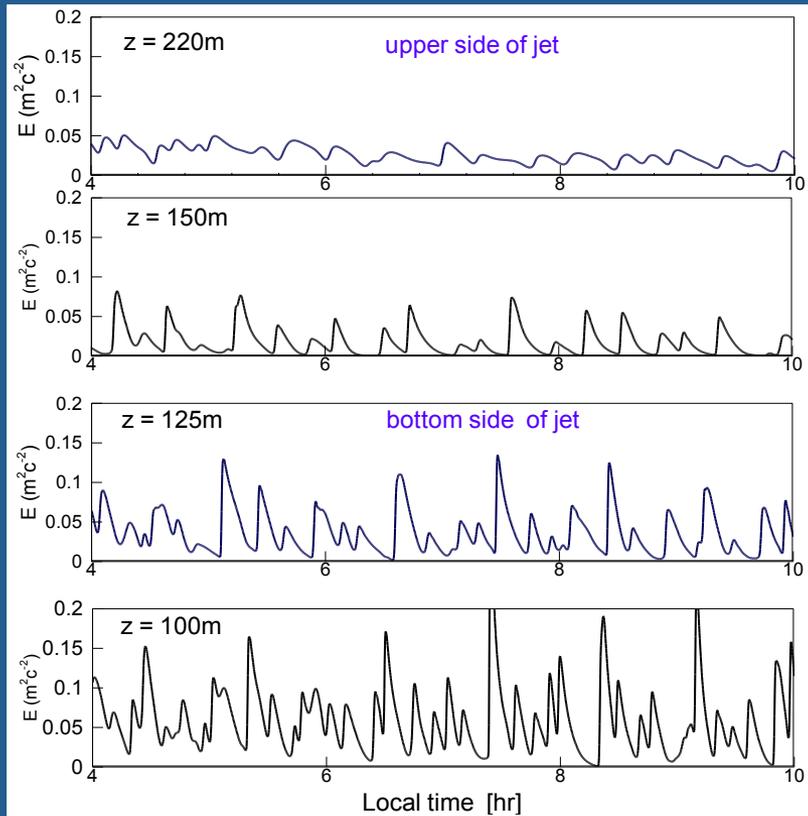


Vertical profiles (a) wind speed and (b) potential temperature for strong SBL with the LLJ.

# SBL. Intermittency of elevated turbulence generated by Low-level-Jet (LLJ): simulation with RANS-scheme

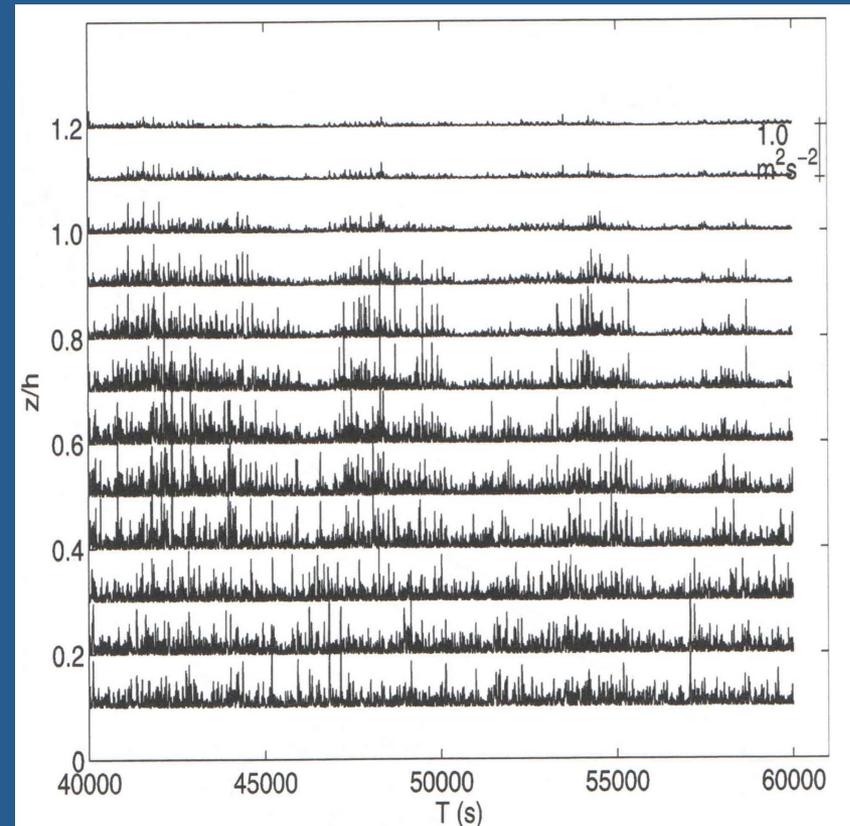
## RANS simulation

### Time series of TKE around LLJ



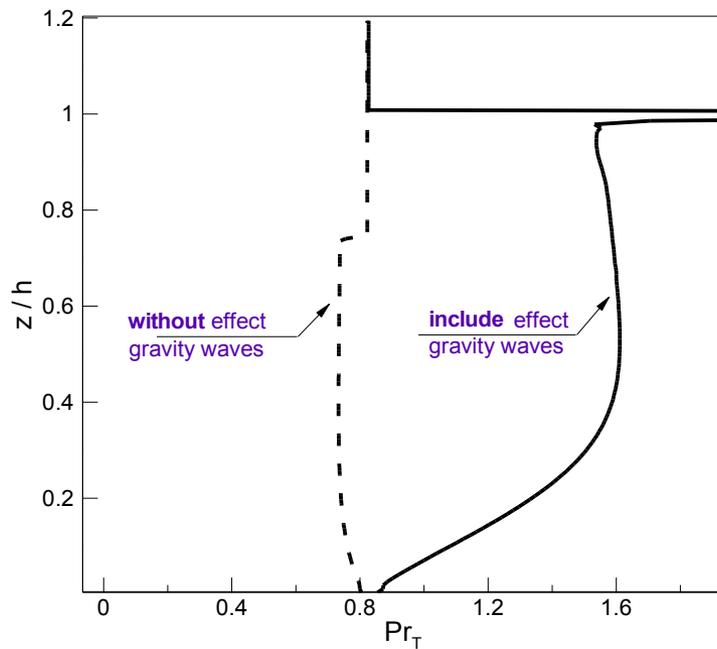
## LES : JAS 2011. V. 68. 2142-2155.

### Time series of TKE around LLJ

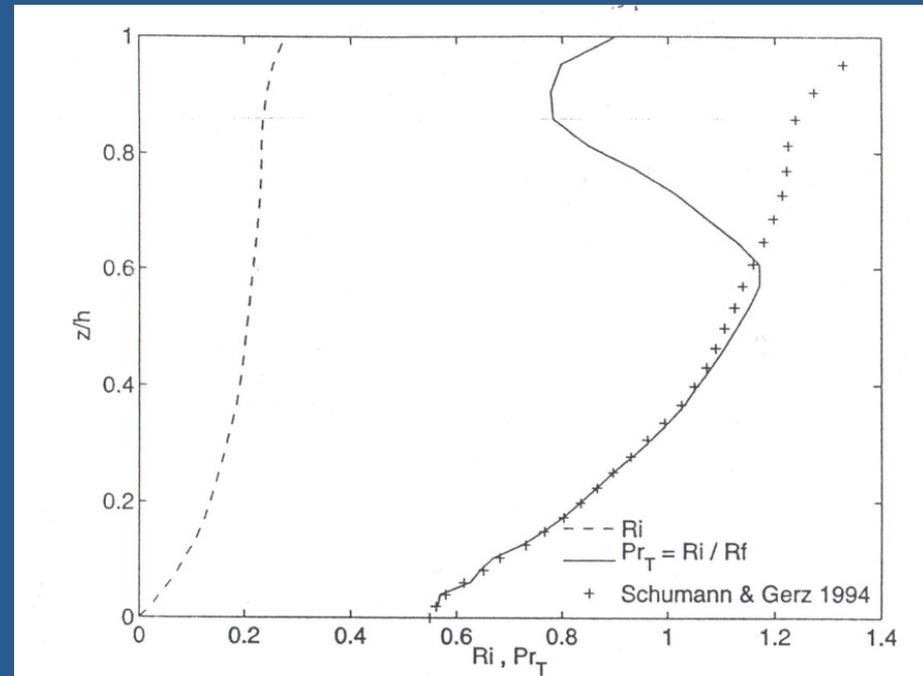


# 4. SBL: Features of Eddy Mixing. Turbulent Prandtl number $Pr_T = K_m / K_h$

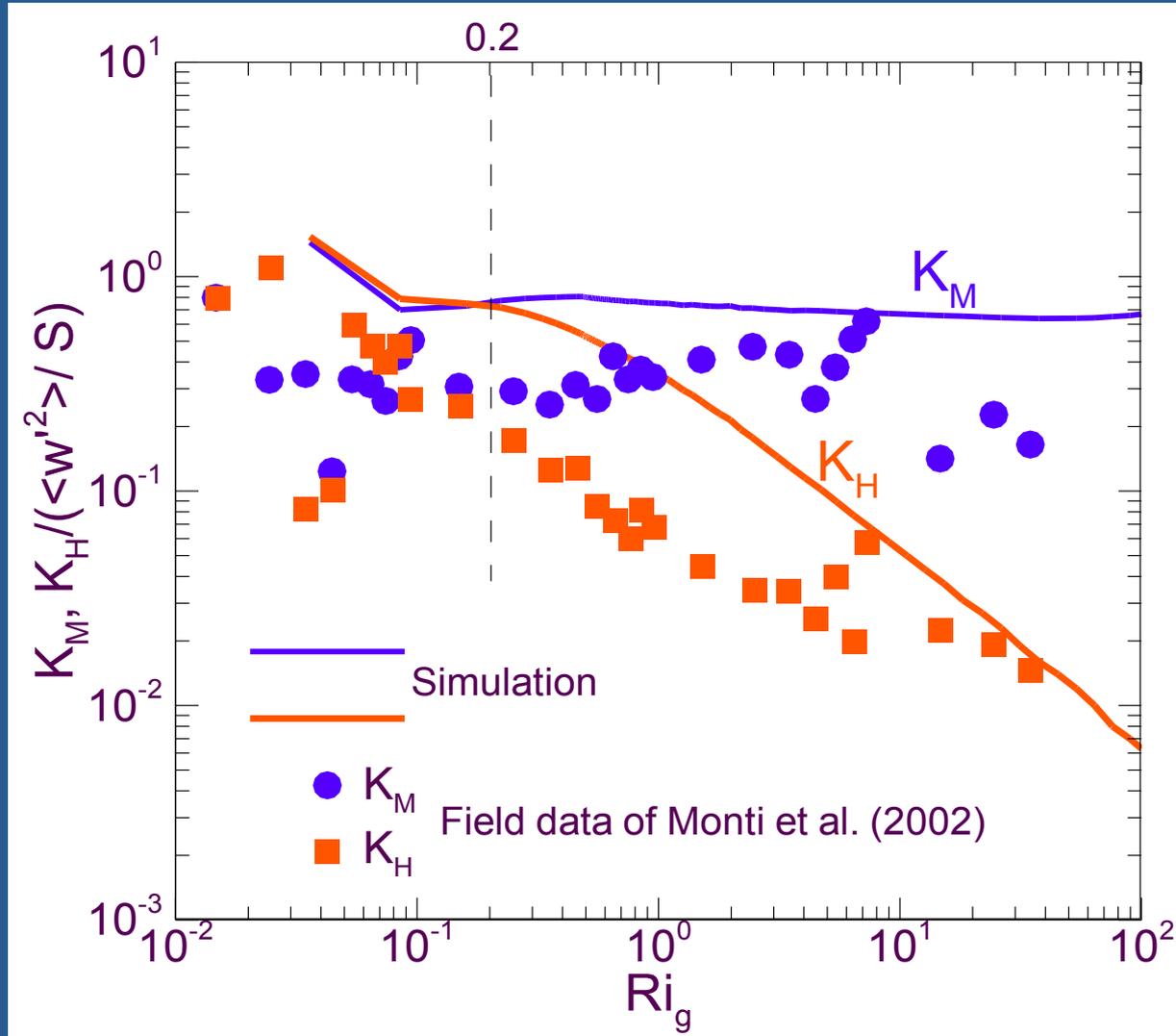
Simulation with the three-parameter RANS-scheme



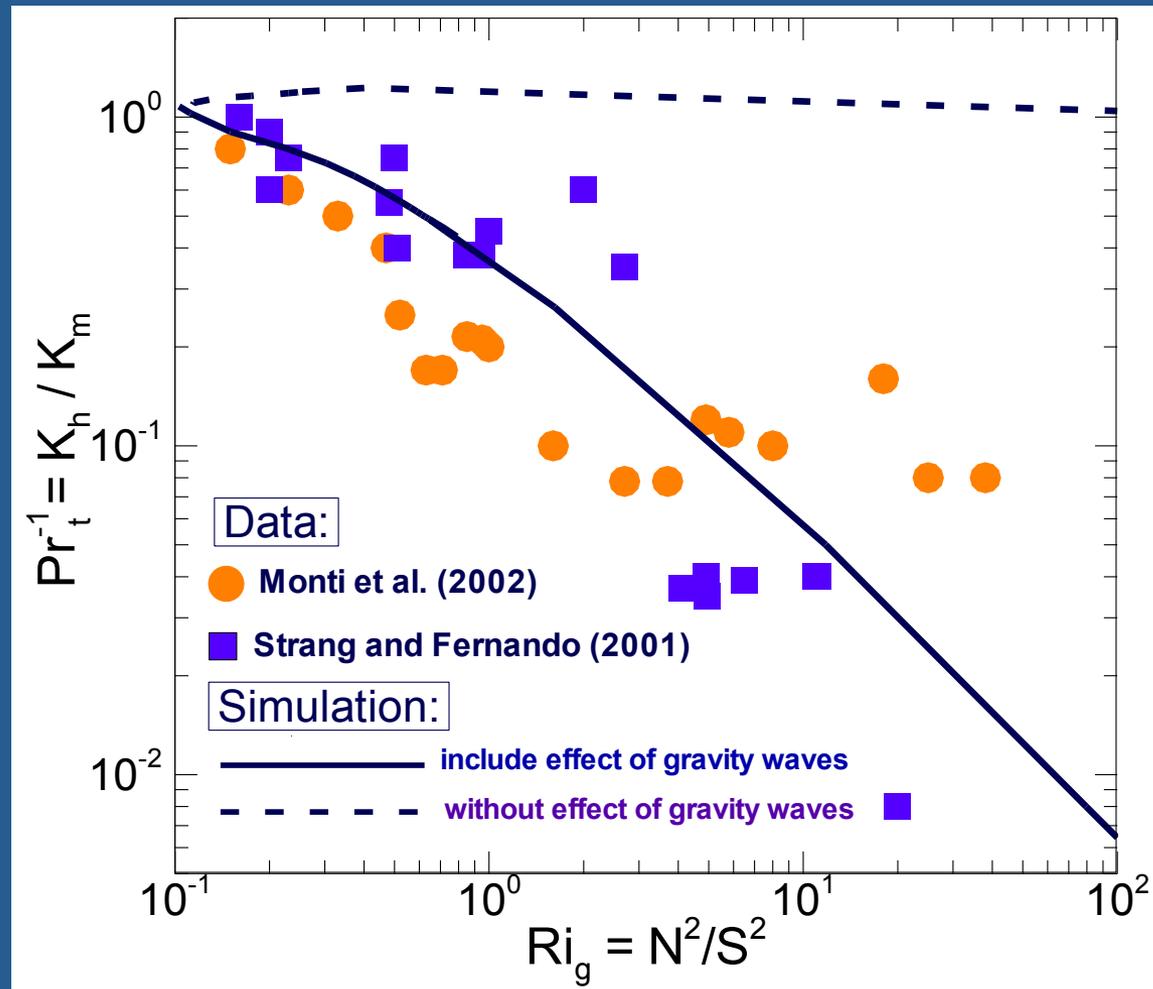
LES. Solid line: JAS, 2011, vol. 68, 2142-2155.



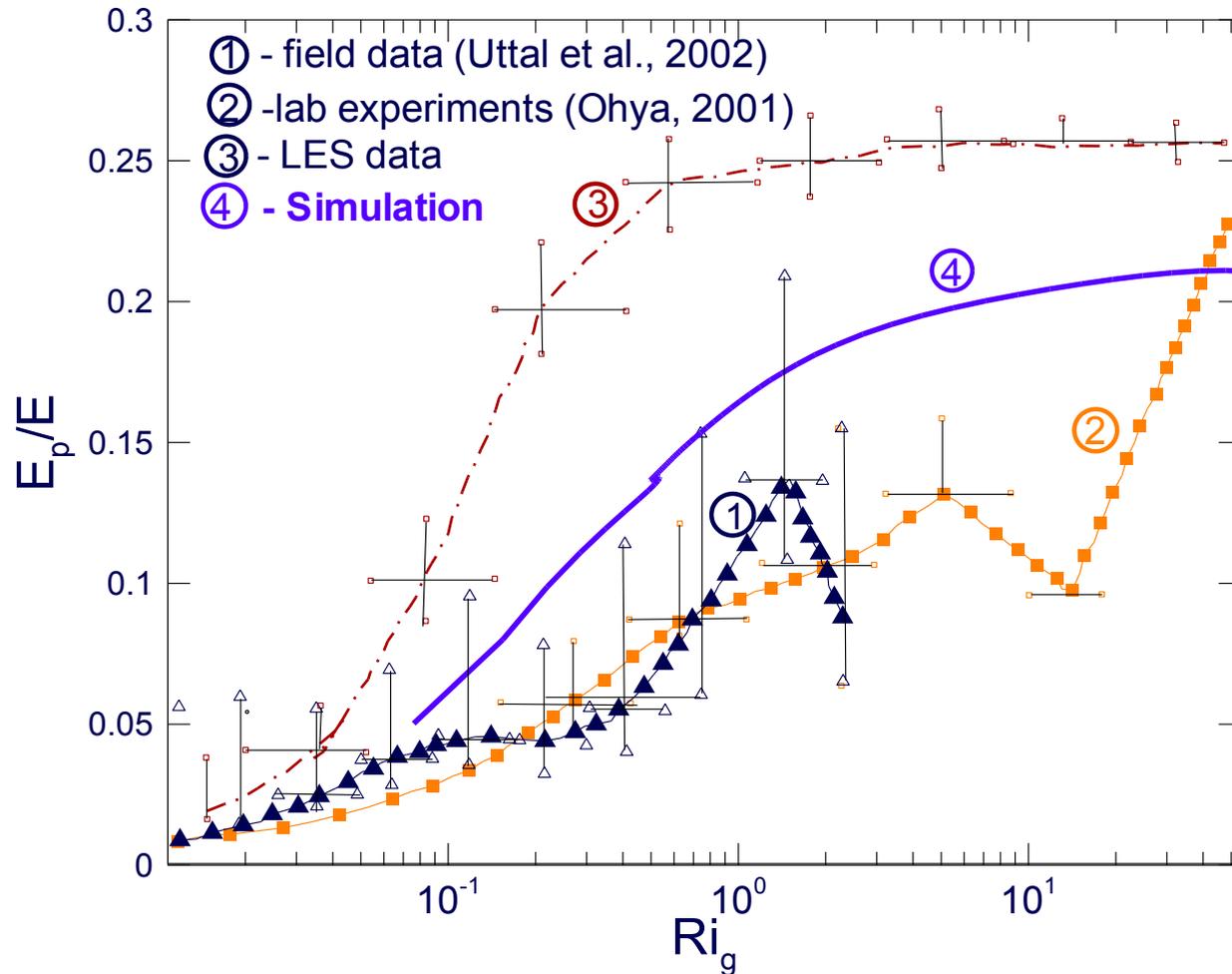
# Eddy diffusivities of momentum and heat in the SBL



# Features of eddy mixing in the SBL: behavior of inverse $Pr_T^{-1} = K_h / K_m$



# 5. Turbulent potential energy in SBL

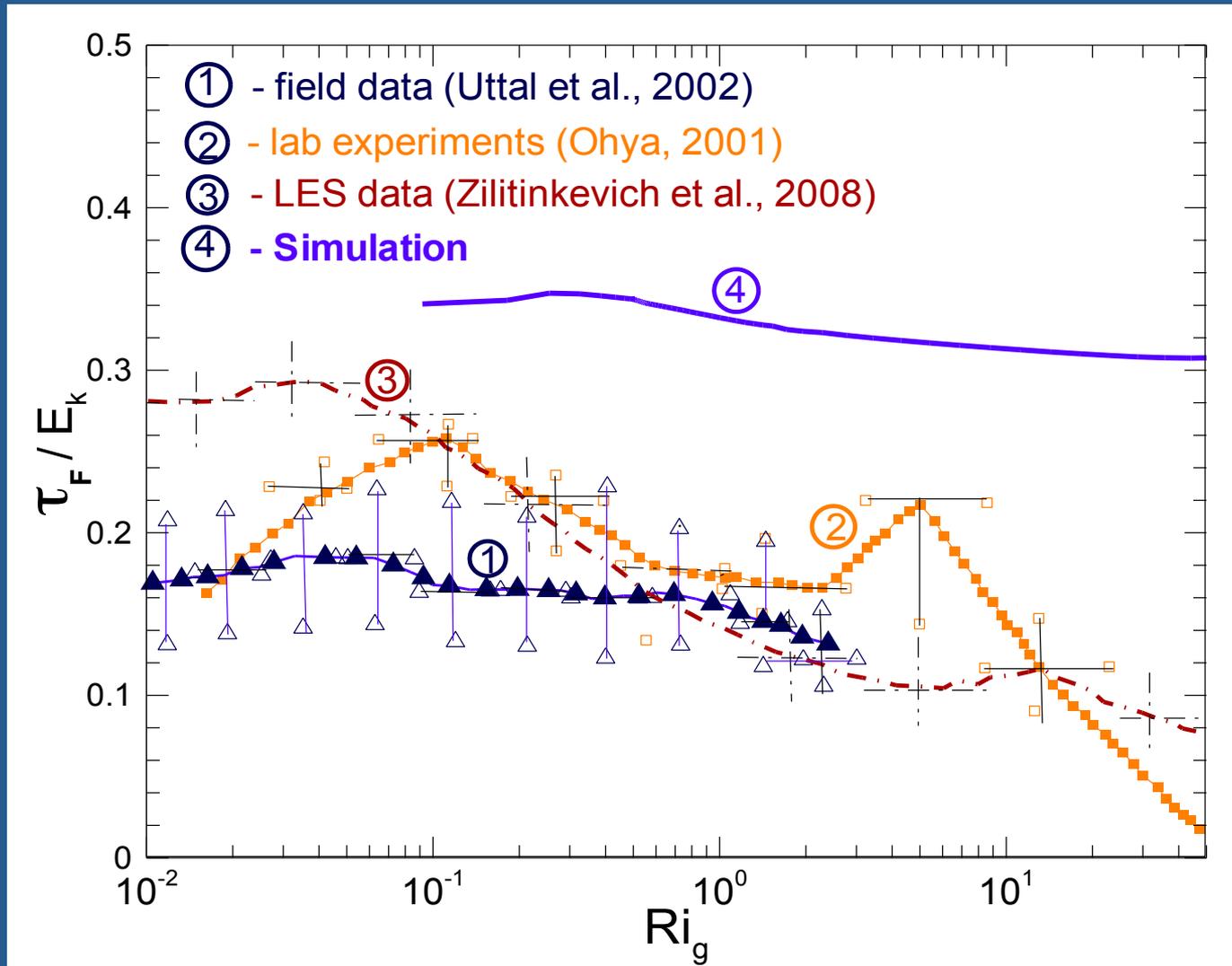


$$E_p = \frac{1}{2} \left( \frac{\beta g}{N} \right)^2 \overline{\theta'^2}$$

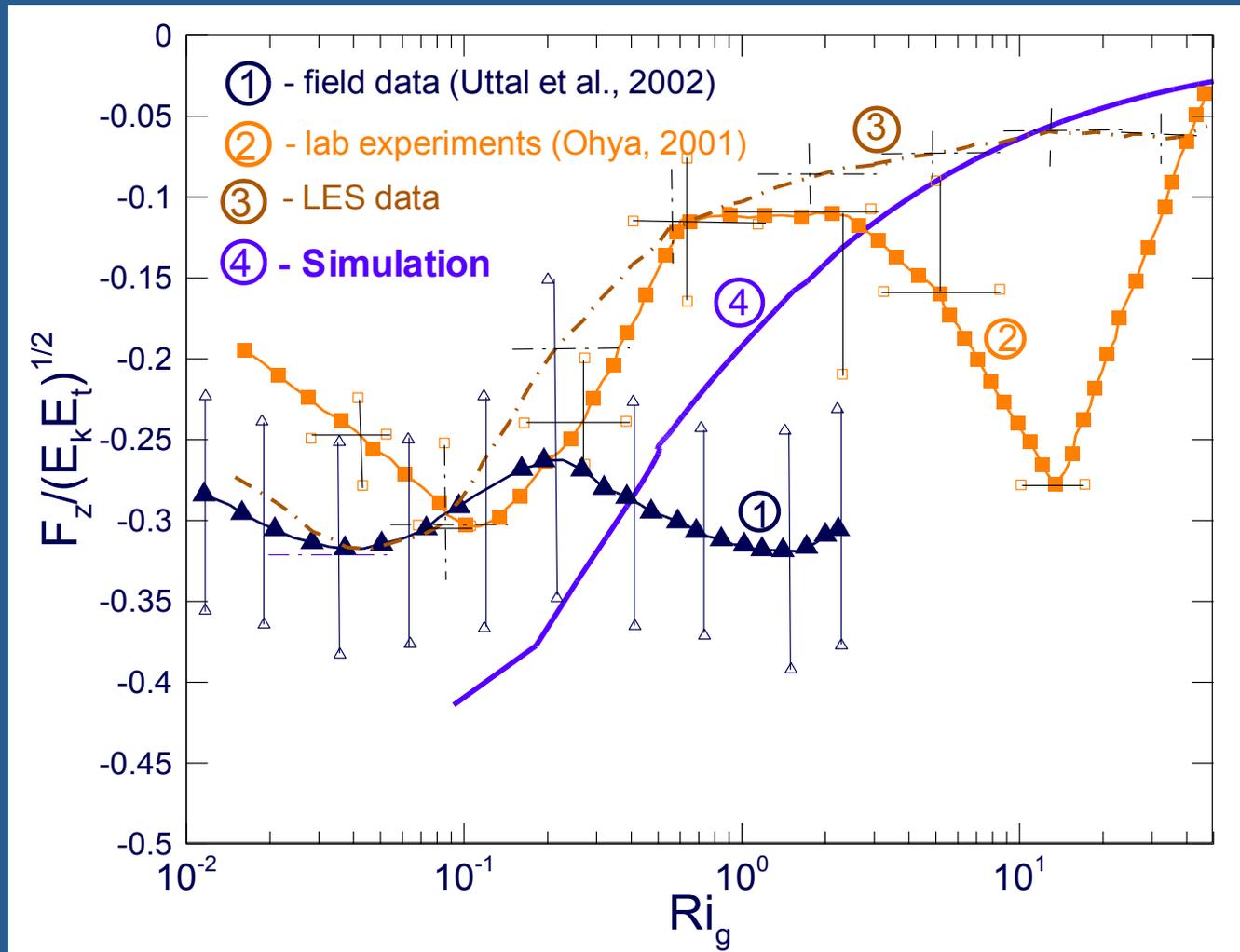
$$E_k = \frac{1}{2} \overline{u_i'^2}$$

$$E = E_k + E_p$$

# Vertical turbulent momentum flux in SBL



# Vertical turbulent heat flux in SBL



## 6. Eddy Mixing in the Free Atmosphere

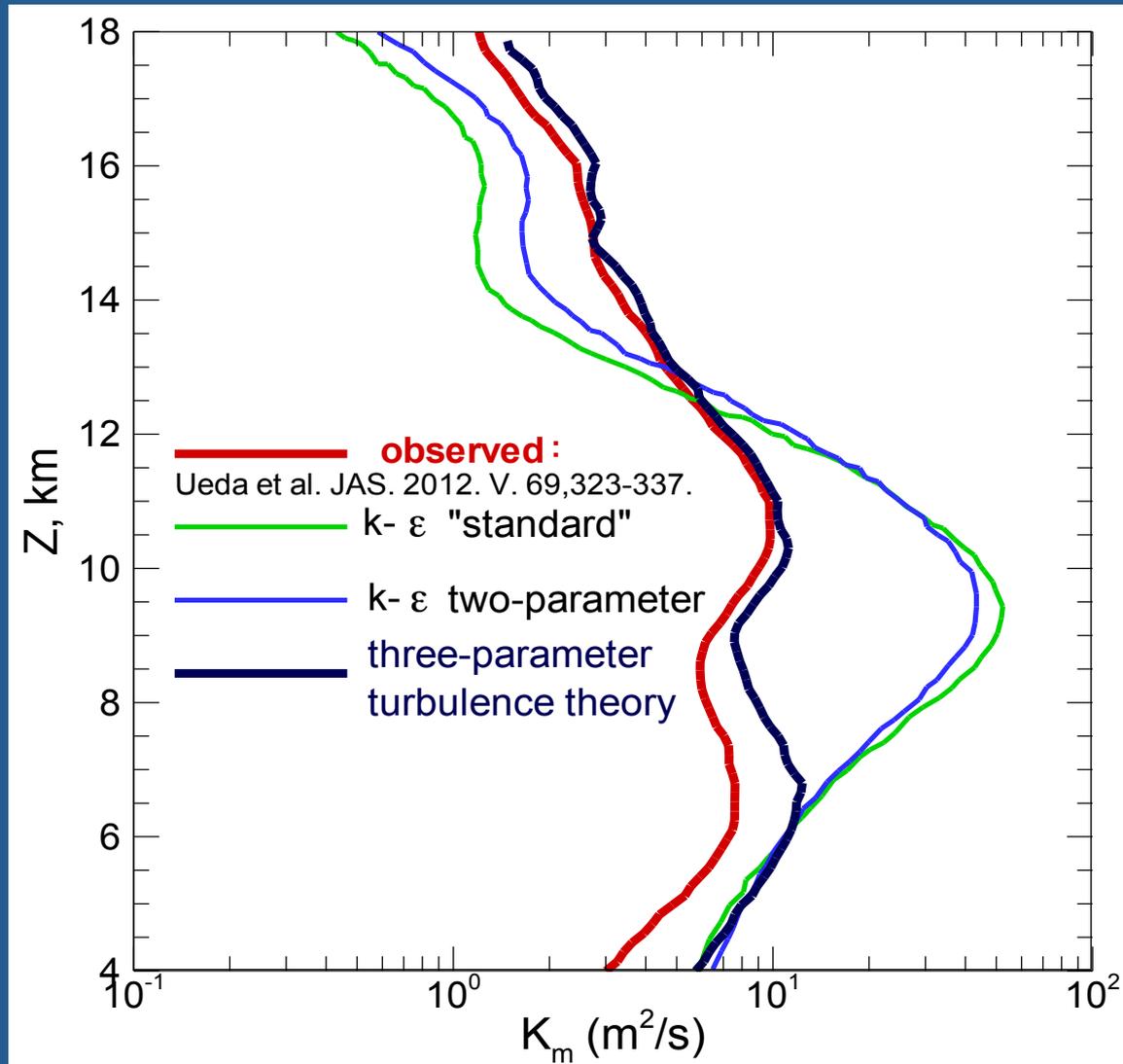
- Diffusion processes in the free atmosphere play an important role in the transport of momentum, heat, and mass on global and regional scales, although the eddy diffusivities there is much smaller than in the atmospheric boundary layer.

- In particular, diffusion processes of minor components in the upper troposphere and lower stratosphere are essential to global warming, stratospheric ozone depletion and transboundary air pollution problems because they govern the exchange of mass between the troposphere and stratosphere.

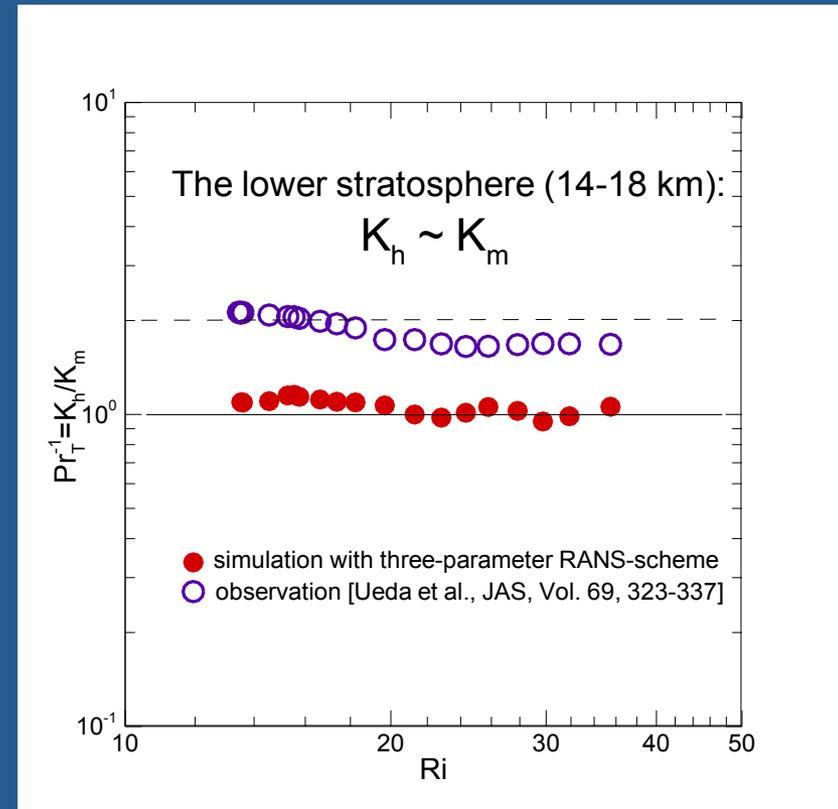
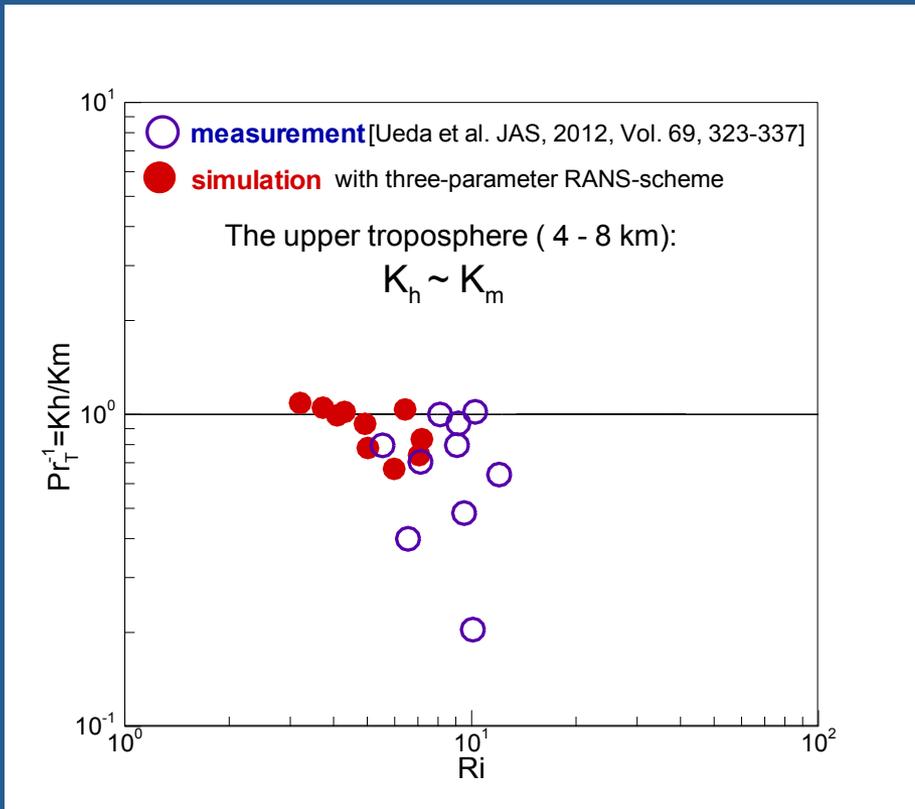
# 6. Features of Eddy Mixing in the Free Atmosphere

- In the upper troposphere and lower stratosphere, air is usually stably stratified, and internal gravity waves induced by boundary layer flow and geography are predominant.
- The turbulence eddies in these layers are generated intermittently and sporadically when gravity waves breaking and shear instability occur.
- These turbulence eddies transport heat and mass, and then they are partly destructed by buoyancy and viscous forces. Thus, turbulent motions and diffusion processes in these layers are complicated and not yet well understood!

# 4. Eddy diffusivity for momentum measured directly in the free atmosphere: comparison with turbulence models



# $Pr_T^{-1}$ in upper troposphere and lower stratosphere



# Surprisingly! $K_h \sim K_m$ : Explanation

In upper troposphere and lower stratosphere the turbulent eddies are produced by shear instability and wave breaking.

By these motions, momentum and heat are considered to be transferred simultaneously in the same manner.

After these events, turbulent eddies are quickly destroyed by buoyancy and do not contribute significantly to vertical diffusion of heat and momentum.

Therefore, it is reasonable to assume that in upper troposphere and lower stratosphere  $K_h \sim K_m$ .

**THANK YOU FOR  
ATTENTION**