Bioresource management problem with asymmetric players

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1. History of the models of "fish wars"

Levhari and Mirman (1980)

The biological growth rule is given by

$$x_{t+1} = (x_t)^{\alpha}, \;\; x_0 = x,$$

where $x_t \geq 0$ – size of the population, $0 < \alpha < 1$ – natural birth rate.

Two players exploit the fish stock and the utility functions are logarithmic. The players' net revenue over infinite time horizon:

$$\bar{J}_i = \sum_{t=0}^{\infty} \beta_i^t \ln(u_t^i),$$

where $u_t^i \geq 0$ – players' catch at time t, $0 < \beta_i < 1$ – the discount factor for player i.

Our model with many players

The dynamics of the fishery is described by the equation

$$x_{t+1} = (\varepsilon x_t - \sum_{i=1}^{n} u_{it})^{\alpha}, \;\; x_0 = x,$$

where $x_t \geq 0$ – size of population at a time t, $\varepsilon \in (0,1)$ – natural death rate, $\alpha \in (0,1)$ – natural birth rate, $u_{it} \geq 0$ – the catch of player i, $i = 1, \ldots, n$.

The players' net revenues over infinite time horizon are:

$$J_i = \sum_{t=0}^{\infty} \delta^t \ln(u_{it}),$$

where $0 < \delta < 1$ – the common discount factor.

Fisher and Mirman (1992)

The biological growth rule is given by

$$x_{t+1} = f((x_t - c_{1t}), (y_t - c_{2t}))\,,$$
$$y_{t+1} = g((x_t - c_{1t}), (y_t - c_{2t}))\,,$$

where $x_t \geq 0$ – size of the population in the first region, $y_t \geq 0$ – size of the population in the second region, $0 \leq c_{1t} \leq x_t$, $0 \leq c_{2t} \leq y_t$ – players' catch at time t.

Players wish to maximize the sum of discounted utility

$$\sum_{t=1}^{\infty} \delta_1^t \ln(c_{1t}), \quad \sum_{t=1}^{\infty} \delta_2^t \ln(c_{2t})\,,$$

where $0 < \delta_i < 1$ – the discount factors ($i = 1, 2$).

Our model of bioresource sharing problem

The center (referee) shares a reservoir between the competitors and there are migratory exchanges between the regions of the reservoir.

The dynamics is of the form

$$\begin{cases} x_{t+1} = (x_t - u_{1t})^{\alpha_1 - \beta_1 s}(y_t - u_{2t})^{\beta_1 s}, \\ y_{t+1} = (y_t - u_{2t})^{\alpha_2 - \beta_2(1-s)}(x_t - u_{1t})^{\beta_2(1-s)}, \end{cases}$$

where $x_t \geq 0$ – size of the population in the first region, $y_t \geq 0$ – size of the population in the second region, $0 < \alpha_i < 1$ – natural birth rate, $0 < \beta_i < 1$ – coefficients of migration between the regions ($i = 1, 2$), $0 \leq u_{1t} \leq x_t$, $0 \leq u_{2t} \leq y_t$ – countries' catch at time t, $0 < \delta_i < 1$ – the discount factor for country i ($i = 1, 2$).

2. Model with asymmetric players

Two players exploit the fish stock during infinite time horizon. The dynamics of the fishery is

$$x_{t+1} = (\varepsilon x_t - u_{1t} - u_{2t})^{\alpha}, \;\; x_0 = x, \tag{1}$$

where $x_t \geq 0$ – the size of population at a time t, $\varepsilon \in (0,1)$ – natural death rate, $\alpha \in (0,1)$ – natural birth rate, $u_{it} \geq 0$ – the catch of player i, $i = 1,2$.

The players' net revenues over infinite time horizon are

$$J_i = \sum_{t=0}^{\infty} \delta_i^t \ln(u_{it}), \tag{2}$$

where $0 < \delta_i < 1$ – the discount factor for country i, $i = 1,2$.

2.1. Nash equilibrium

(u_1^N, u_2^N) – Nash equilibrium if

$$J_1(u_1^N, u_2^N) \geq J_1(u_1, u_2^N)\,,\; J_2(u_1^N, u_2^N) \geq J_2(u_1^N, u_2)\,,\; \forall u_1, u_2\,.$$

The Nash equilibrium of the problem (1), (2) is

$$u_1^N = \frac{a_2(1-a_1)}{a_1 + a_2 - a_1 a_2}\varepsilon x\,,\; u_2^N = \frac{a_1(1-a_2)}{a_1 + a_2 - a_1 a_2}\varepsilon x\,,$$

where $a_i = \alpha\delta_i\,,\; i = 1, 2$. And the payoffs are

$$V_i(x, \delta_i) = A_i \ln x + B_i = \frac{1}{1-a_i}\ln x + B_i\,. \tag{3}$$

2.2. Cooperative equilibrium

The objective is to maximize the sum of the players' utilities:

$$J = \sum_{t=0}^{\infty} \delta^t \Big[\ln(u_{1t}) + \ln(u_{2t}) \Big] , \tag{4}$$

where δ is unknown common discount factor.

The cooperative equilibrium of the problem (1), (4) is

$$u_1^c = u_2^c = \frac{1 - \alpha\delta}{2} \varepsilon x \, .$$

And the joint payoff is

$$V(x, \delta) = A \ln x + B = \frac{2}{1 - \alpha\delta} \ln x + B \, . \tag{5}$$

3. The joint discount factor

First, we show that the joint discount factor for the case when cooperative payoff is distributed proportionally among the players exists.

Second, we suppose that the cooperative payoff is distributed in the portion $\gamma V(x,\delta)$ and $(1-\gamma)V(x,\delta)$ and find the conditions on δ and γ to satisfy the inequalities

$$\gamma V(x,\delta) \geq V_1(x,\delta_1)\,,\ (1-\gamma)V(x,\delta) \geq V_2(x,\delta_2)\,.$$

To construct the solution we propose to use Nash bargaining scheme, so

$$(\gamma V(x,\delta) - V_1(x,\delta_1))((1-\gamma)V(x,\delta) - V_2(x,\delta_2)) \to \max_{\delta,\gamma}\,.$$

3.1. Proportional distribution

The conditions on δ to satisfy the inequalities

$$\frac{\delta_i}{\delta_1+\delta_2}V(x,\delta) \geq V_i(x,\delta_i)\,,\ i=1,2.$$

are: if $\delta_1 V_2(x,\delta_2) - \delta_2 V_1(x,\delta_1) < 0$ then the common discount factor satisfy the inequality $\delta < usl_1$, otherwise $\delta < usl_2$, where

$$usl_i = \frac{K_i + (1+\alpha)M_i}{2\alpha M_i} +$$

$$+\frac{\sqrt{(K_i+(1+\alpha)M_i)^2 + 8a_i(1-a_i)M_i(\ln(\varepsilon) - 1 - (1-\alpha)\ln(2))}}{2\alpha M_i},$$

$M_i = (\delta_1+\delta_2)(\ln(x)+B_i(1-a_i))\,, K_i = 2\delta_i(1-a_i)(\alpha\ln(2)-\ln(x))\,.$

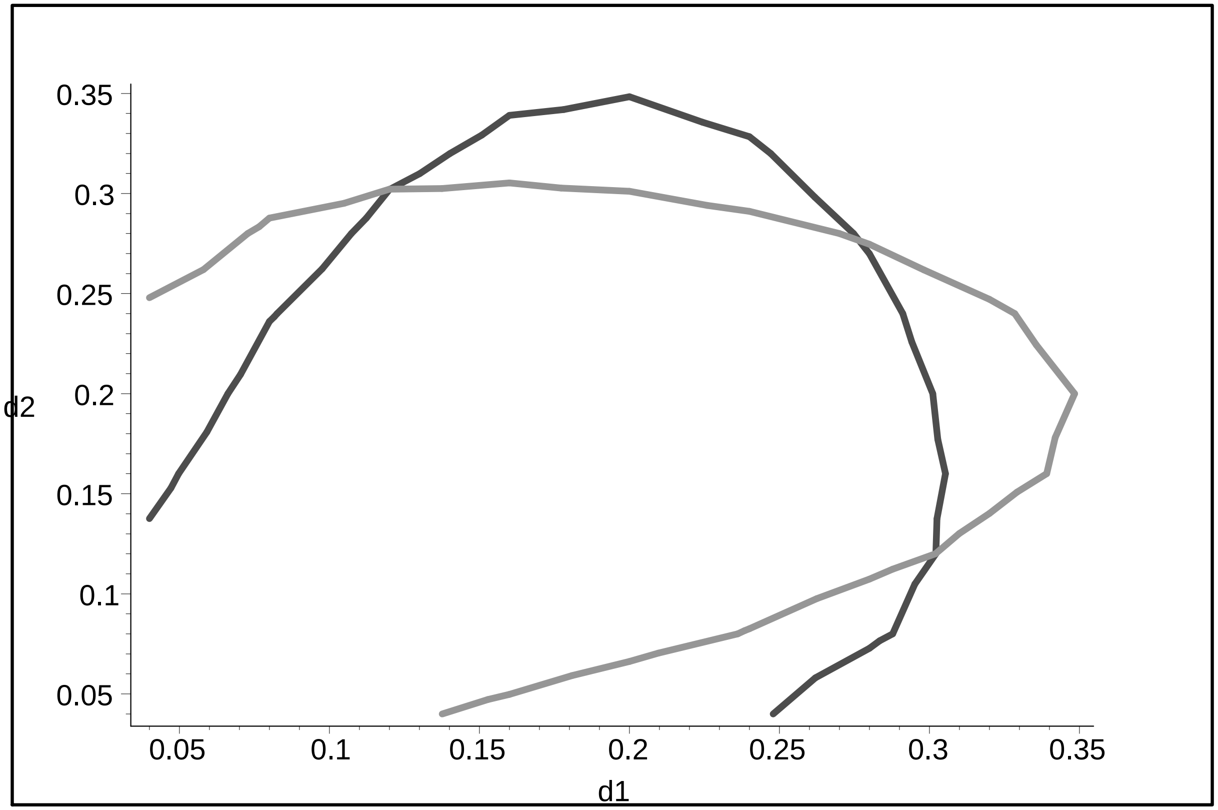

Fig. 1. Conditions on δ: dark – usl_1, light – usl_2

3.2. Proportion and bargaining solution

We suppose that the cooperative payoff is distributed in the portion $\gamma V(x,\delta)$ and $(1-\gamma)V(x,\delta)$, where γ is a parameter. We find the conditions on δ and γ to satisfy the rationality conditions

$$\gamma V(x,\delta) \geq V_1(x,\delta_1)\,,\ (1-\gamma)V(x,\delta) \geq V_2(x,\delta_2)\,. \tag{6}$$

We have the set of admissible parameters δ and γ. To construct the solution we use Nash bargaining scheme, so

$$g = (\gamma V(x,\delta) - V_1(x,\delta_1))((1-\gamma)V(x,\delta) - V_2(x,\delta_2)) \to \max_{\delta,\gamma}\,.$$

For the analytical solution $\delta \to 0\,, \; \gamma = \gamma^*$

the next conditions should be fulfilled

$$V_1(x,\delta_1) + V_2(x,\delta_2) < 2\ln(\frac{\varepsilon x}{2})\,,$$
$$2\ln(\frac{\varepsilon x}{2}) < V_1(x,\delta_1) - V_2(x,\delta_2) < 2\ln(\frac{2}{\varepsilon x})\,. \qquad (7)$$

In other cases the solution can be found numerically.

$\delta_1 = 0.1$, $\delta_2 = 0.2$ (8) takes the form $0.070 < \gamma < 0.494$. Therefore the analytical solution exists: $\delta = 0$, $\gamma = 0.183$. The players' payoffs: $V_1^c = -0.390$, $V_2^c = -1.560$.

Let $\delta_1 = 0.8$ and $\delta_2 = 0.9$. (8) is not fulfilled and we find the solution numerically. We obtain $\delta = 0.001$, $\gamma = 0.1$ and cooperative payoffs $V_1^c = -0.195$, $V_2^c = -1.755$.

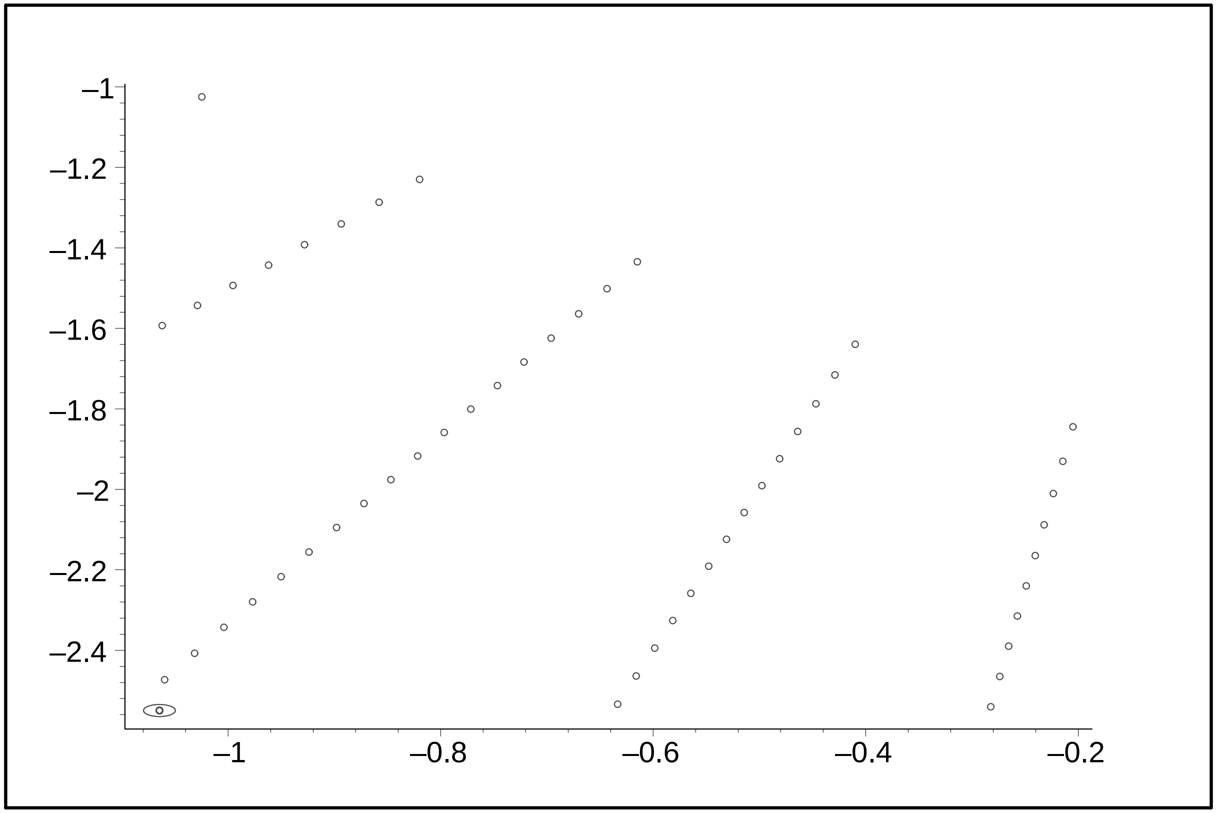

Fig. 2. Bargaining set $\delta_1 = 0.1$, $\delta_2 = 0.2$

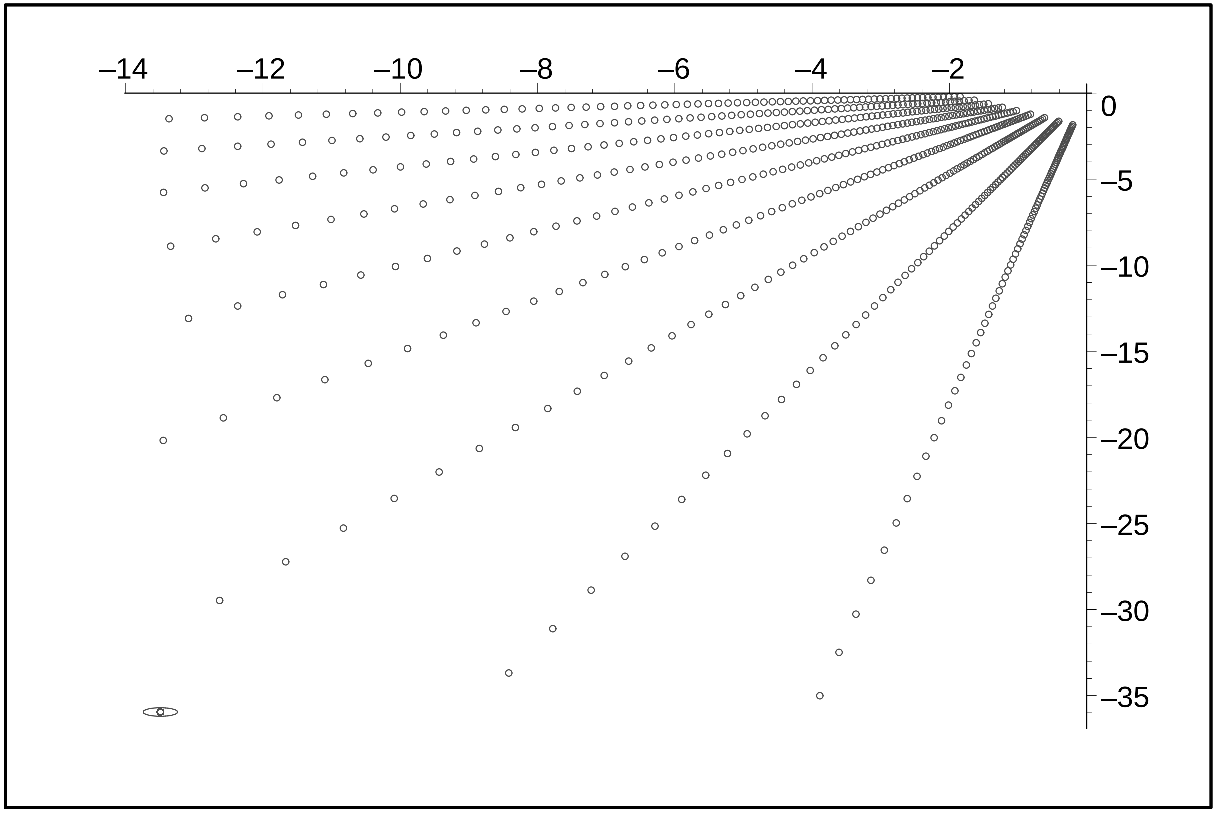

Fig. 3. Bargaining set $\delta_1 = 0.8$, $\delta_2 = 0.9$

4. Bargaining procedure

Nash equilibrium payoffs for n step game are:

$$V_i^N(x,\delta_i) = \sum_{j=0}^{n} (a_i)^j \ln(x) + \sum_{j=1}^{n} (\delta_i)^{n-j} A_i^j - (\delta_i)^n \ln(2)\,. \qquad (8)$$

Here we obtain the cooperative strategies without determining the joint discount factor using recursive Nash bargaining procedure.

We consider two different approaches of bargaining procedure:

1. The cooperative strategies are determined as the Nash bargaining solution for the whole planning horizon.

2. We use recursive Nash bargaining procedure determining the cooperative strategies on each time step.

4.1. Nash bargaining for the whole game

We construct cooperative strategies and the payoff maximizing the Nash product for the whole game, so we need to solve the next problem

$$(V_1^{nc}(x,\delta_1) - V_1^N(x,\delta_1))(V_2^{nc}(x,\delta_2) - V_2^N(x,\delta_2)) =$$

$$= (\sum_{t=0}^{n} \delta_1^t \ln(u_{1t}^c) - V_1^N(x,\delta_1))(\sum_{t=0}^{n} \delta_2^t \ln(u_{2t}^c) - V_2^N(x,\delta_2)) \to \max,$$

where $V_i^N(x,\delta_i)$ – noncooperative payoffs (8).

Cooperative payoffs for n step game take the forms:

$$H_1^n(\gamma_1^1,\dots,\gamma_1^n,\gamma_2^1,\dots,\gamma_2^n) = \frac{1-a_1^{n+1}}{1-a_1}\ln(x) +$$

$$\sum_{j=1}^{n} \delta_1^{n-j} \ln(\gamma_1^j) + \sum_{j=1}^{n} \delta_1^{n-j} \frac{a_1(1-a_1^j)}{1-a_1} \ln(\varepsilon - \gamma_1^j - \gamma_2^j) - \delta_1^n \ln(2) \quad (9)$$

and

$$H_2^n(\gamma_1^1, \ldots, \gamma_1^n, \gamma_2^1, \ldots, \gamma_2^n) = \frac{1 - a_2^{n+1}}{1 - a_2} \ln(x) +$$

$$\sum_{j=1}^{n} \delta_2^{n-j} \ln(\gamma_2^j) + \sum_{j=1}^{n} \delta_2^{n-j} \frac{a_2(1 - a_2^j)}{1 - a_2} \ln(\varepsilon - \gamma_1^j - \gamma_2^j) - \delta_2^n \ln(2). \quad (10)$$

The cooperative strategies are connected as

$$\gamma_1^n = \frac{\varepsilon \gamma_1^1 a_2^{n-1}(1 - a_2^2)(1 - a_1)}{\varepsilon a_1^{n-1}(1 - a_1)(1 - a_2^{n+1}) + \gamma_1^1 (a_2^{n-1}(1 - a_2^2) - a_1^{n-1}(1 - a_1^2) + a_1^{n-1} a_2^{n-1}(a_2^2 - a_1^2))},$$

$$\gamma_2^n = \frac{\varepsilon(1 - a_1)(1 - a_2) - \gamma_1^n (1 - a_2)(1 - a_1^{n+1})}{(1 - a_1)(1 - a_2^{n+1})}.$$

And γ_1^1 can be determined from one of the first order conditions, for example, the last one

$$a_1^{n-1}(\varepsilon - \gamma_1^1(1+a_1))(H_1^n - V_1) - a_2^{n-1}(1+a_2)\gamma_1^1(H_2^n - V_2) = 0\,.$$

Statement. *The Nash bargaining scheme for infinite time horizon gives the advantages to the player with lower discount factor.*

If $\delta_1 < \delta_2$ then as $n \to \infty$

$$\gamma_1^n \to \varepsilon(1-a_1)\,,\ \gamma_2^n \to 0\,.$$

If $\delta_2 < \delta_1$ then as $n \to \infty$

$$\gamma_1^n \to 0\,,\ \gamma_2^n \to \varepsilon(1-a_2)\,.$$

Modelling

We present the results of numerical modelling for 20-stage game with the next parameters:

$$\varepsilon = 0.6\,, \quad \alpha = 0.3\,, \quad x_0 = 0.8\,,$$
$$\delta_1 = 0.85\,, \quad \delta_2 = 0.9\,.$$

We obtain $\gamma_1^1 = 0.1778$. The cooperative and Nash gains are

$$V_1^{nc}(x,\delta_1) = -13.2103 > V_1^N(x,\delta_1) = -14.6439\,,$$

$$V_2^{nc}(x,\delta_2) = -20.5328 > V_2^N(x,\delta_2) = -23.2596\,.$$

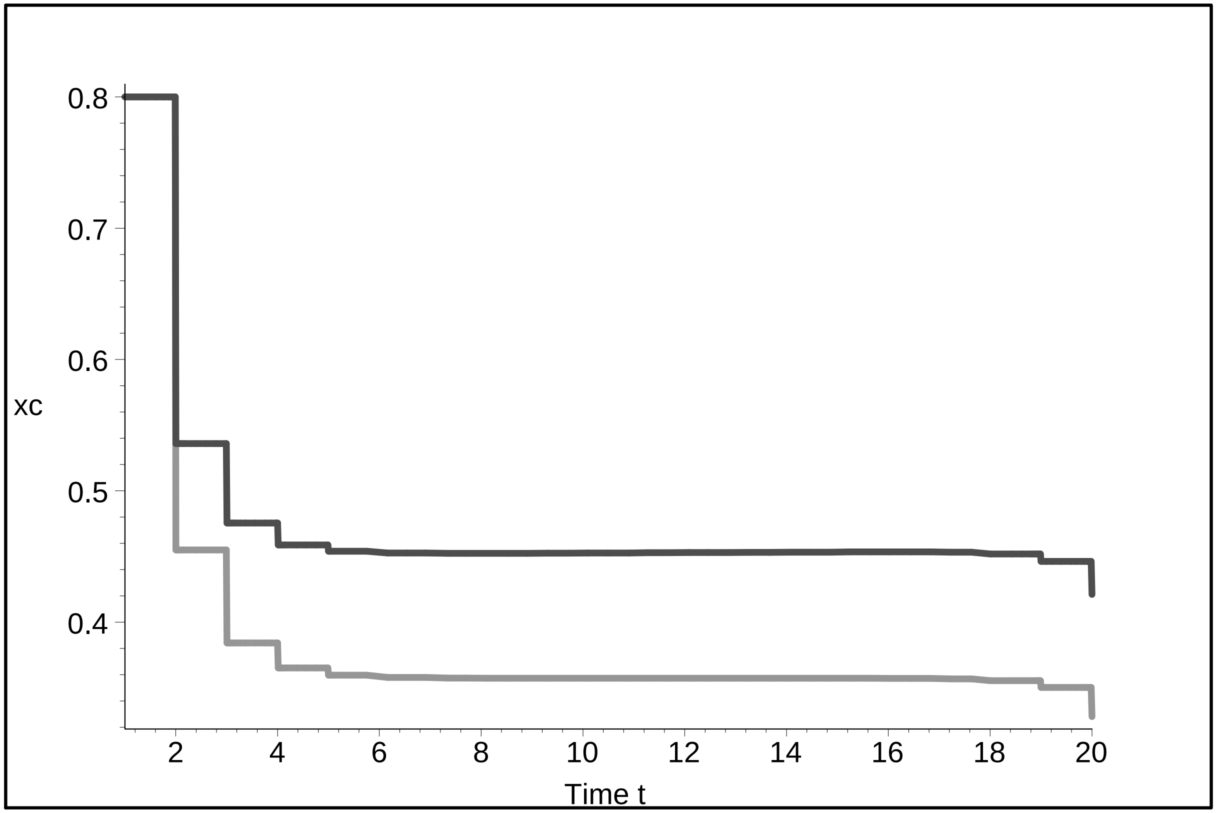

Fig. 4. The population size: dark – cooperative, light – Nash

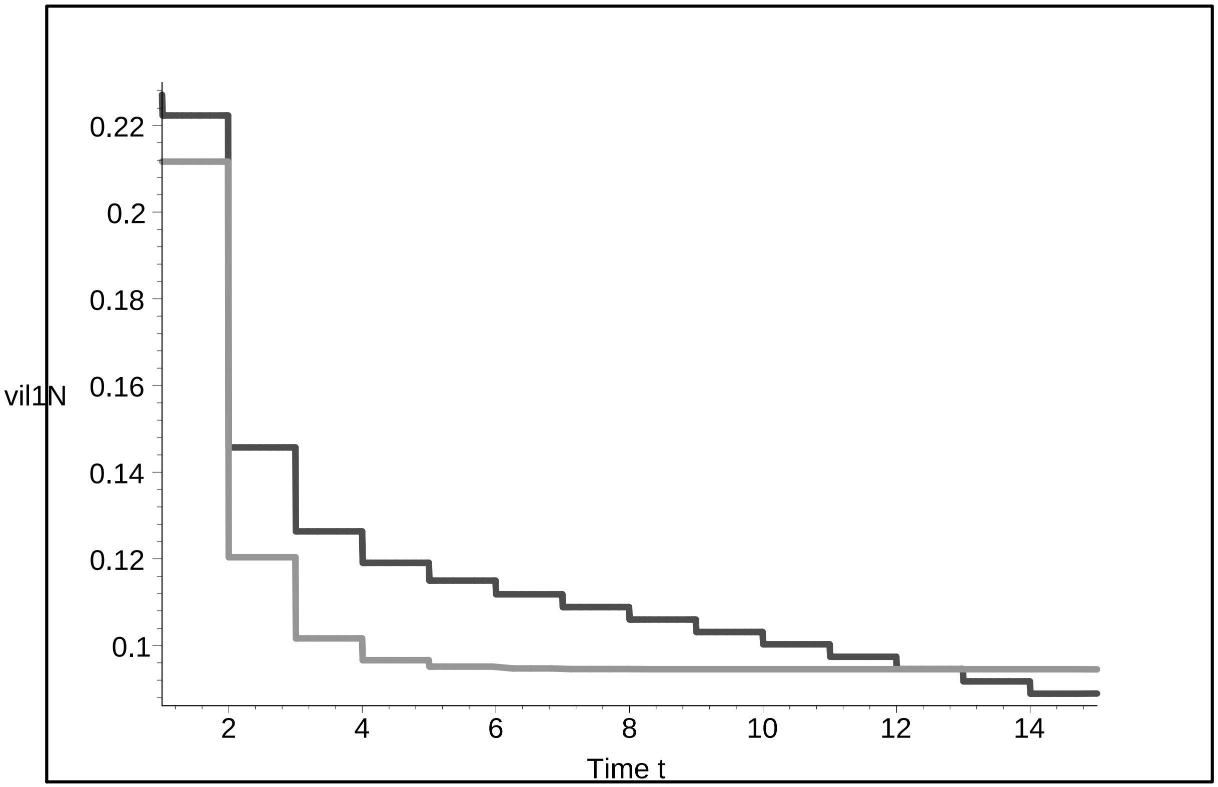

Fig. 5. The catch of player 1: dark – cooperative, light – Nash

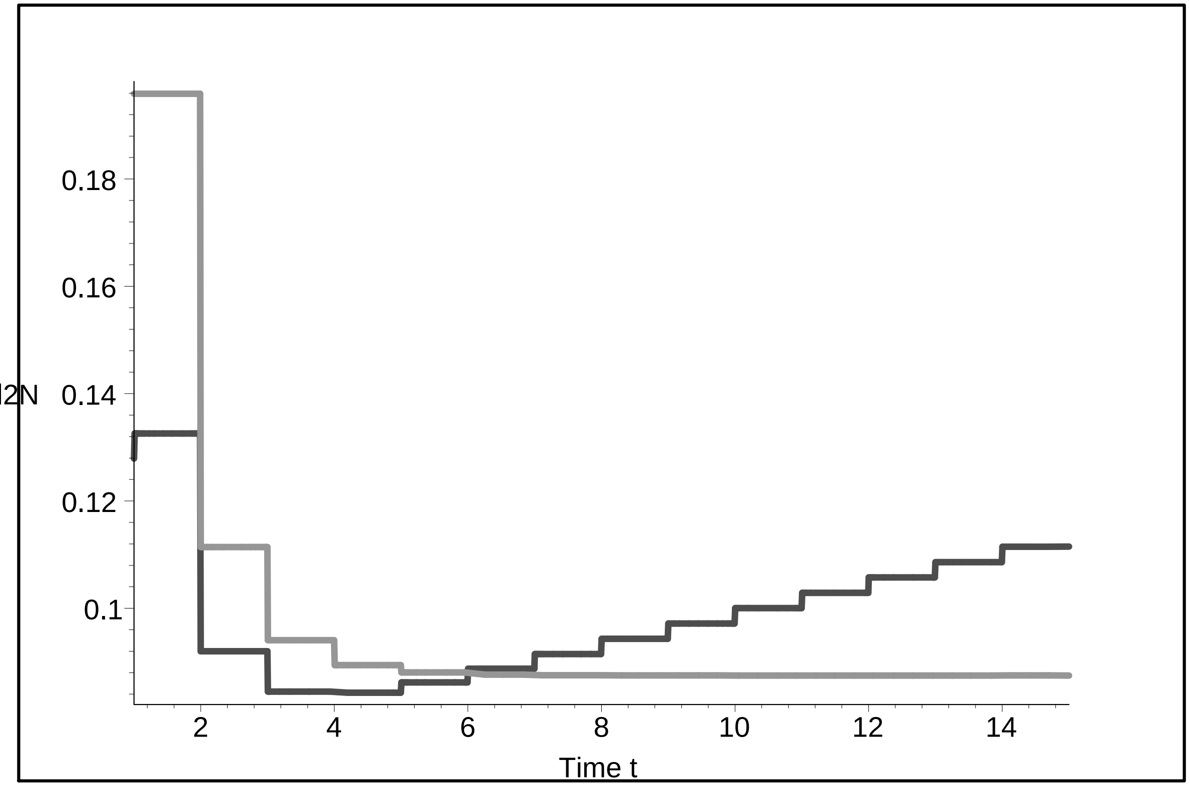

Fig. 6. The catch of player 2: dark – cooperative, light – Nash

4.2. Recursive Nash bargaining solution

On each time moment the cooperative strategies are determined as the Nash bargaining solution taking the non-cooperative profits as a status-quo point.

We start with the one-step game and assume that if there were no future period, the countries would get the remaining fish in the ratio $1:1$. Let the initial size of the population be x.

Noncooperative gains are

$$H_1^{1N} = (1 + a_1)\,\mathsf{ln}(x) + A_1^1 - \delta_1\,\mathsf{ln}(2)\,, \tag{11}$$

$$H_2^{1N} = (1 + a_2)\,\mathsf{ln}(x) + A_2^1 - \delta_2\,\mathsf{ln}(2)\,. \tag{12}$$

The cooperative strategies are determined maximizing the Nash product

$$
\begin{aligned}
H^{1c} = (\ln(u_1) + a_1 \ln(\varepsilon x - u_1 - u_2) - \delta_1 \ln(2) - H_1^{1N}) \cdot \\
\cdot(\ln(u_2) + a_2 \ln(\varepsilon x - u_1 - u_2) - \delta_2 \ln(2) - H_2^{1N}) = \\
= (H_1^{1c} - H_1^{1N})(H_2^{1c} - H_2^{1N}) \to \max ,
\end{aligned}
$$

where H_i^{1N} are given in (11)–(12).

The cooperative strategies are can be found as the solution of the next equation

$$
\gamma_2^{1c}\Big(\ln(\gamma_2^{1c})+a_2 \ln(\varepsilon-\gamma_1^{1c}-\gamma_2^{1c})-A_2^1\Big)=\gamma_1^{1c}\Big(\ln(\gamma_1^{1c})+a_1 \ln(\varepsilon-\gamma_1^{1c}-\gamma_2^{1c})-A_1^1\Big) \tag{13}
$$

with the relation

$$
\gamma_2^{1c} = \frac{\varepsilon - \gamma_1^{1c}(1 + a_1)}{1 + a_2} .
$$

The cooperative gains for one step game have the forms

$$H_1^{1c} = (1 + a_1)\ln(x) + \ln(\gamma_1^{1c}) + a_1 \ln(\varepsilon - \gamma_1^{1c} - \gamma_2^{1c}) - \delta_1 \ln(2), \tag{14}$$

$$H_2^{1c} = (1 + a_2)\ln(x) + \ln(\gamma_2^{1c}) + a_2 \ln(\varepsilon - \gamma_1^{1c} - \gamma_2^{1c}) - \delta_2 \ln(2). \tag{15}$$

We pass to two stage game. If the players act non-cooperatively till the end of the game then the gains are

$$H_1^{2N} = (1 + a_1 + a_1^2)\ln(x) + A_1^2 + \delta_1 A_1^1 - \delta_1^2 \ln(2)\,, \tag{16}$$

$$H_2^{2N} = (1 + a_2 + a_2^2)\ln(x) + A_2^2 + \delta_2 A_2^1 - \delta_2^2 \ln(2)\,. \tag{17}$$

We determine the cooperative strategies maximizing the Nash product

$$H^{2c} = (\ln(u_1) + \delta_1 H_1^{1c} - H_1^{2N})(\ln(u_2) + \delta_2 H_2^{1c} - H_2^{2N}) = \\ = (H_1^{2c} - H_1^{2N})(H_2^{2c} - H_2^{2N}\Big) \to \max\,,$$

where H_i^{1c} are the cooperative gains for one step game and are given in (14)–(15) and H_i^{2N} are determined in (16)–(17).

Analogously we get the equation for γ_1^{2c} and γ_2^{2c}.

The process can be repeated for the n-stage game and we have the next form of the cooperative profits

$$H_1^{nc}(\gamma_1^1,\ldots,\gamma_1^n,\gamma_2^1,\ldots,\gamma_2^n) = \sum_{j=0}^{n} a_1^j \ln(x) + \sum_{j=0}^{n-1} \delta_1^{n-j}\Big[\ln(\gamma_1^{(n-j)c}) + \sum_{i=1}^{n-j} a_1^i \ln(\varepsilon - \gamma_1^{(n-j)c} - \gamma_2^{(n-j)c})\Big] - \delta_1^n \ln(2) \quad (18)$$

and

$$H_2^{nc}(\gamma_1^1,\ldots,\gamma_1^n,\gamma_2^1,\ldots,\gamma_2^n) = \sum_{j=0}^{n} a_2^j \ln(x) + \sum_{j=0}^{n-1} \delta_2^{n-j}\Big[\ln(\gamma_2^{(n-j)c}) + \sum_{i=1}^{n-j} a_2^i \ln(\varepsilon - \gamma_1^{(n-j)c} - \gamma_2^{(n-j)c})\Big] - \delta_2^n \ln(2). \quad (19)$$

Modelling

The cooperative and Nash gains are

$$V_1^{nc}(x,\delta_1) = -14.1039 > V_1^N(x,\delta_1) = -14.6439\,,$$

$$V_2^{nc}(x,\delta_2) = -20.5108 > V_2^N(x,\delta_2) = -23.2596\,.$$

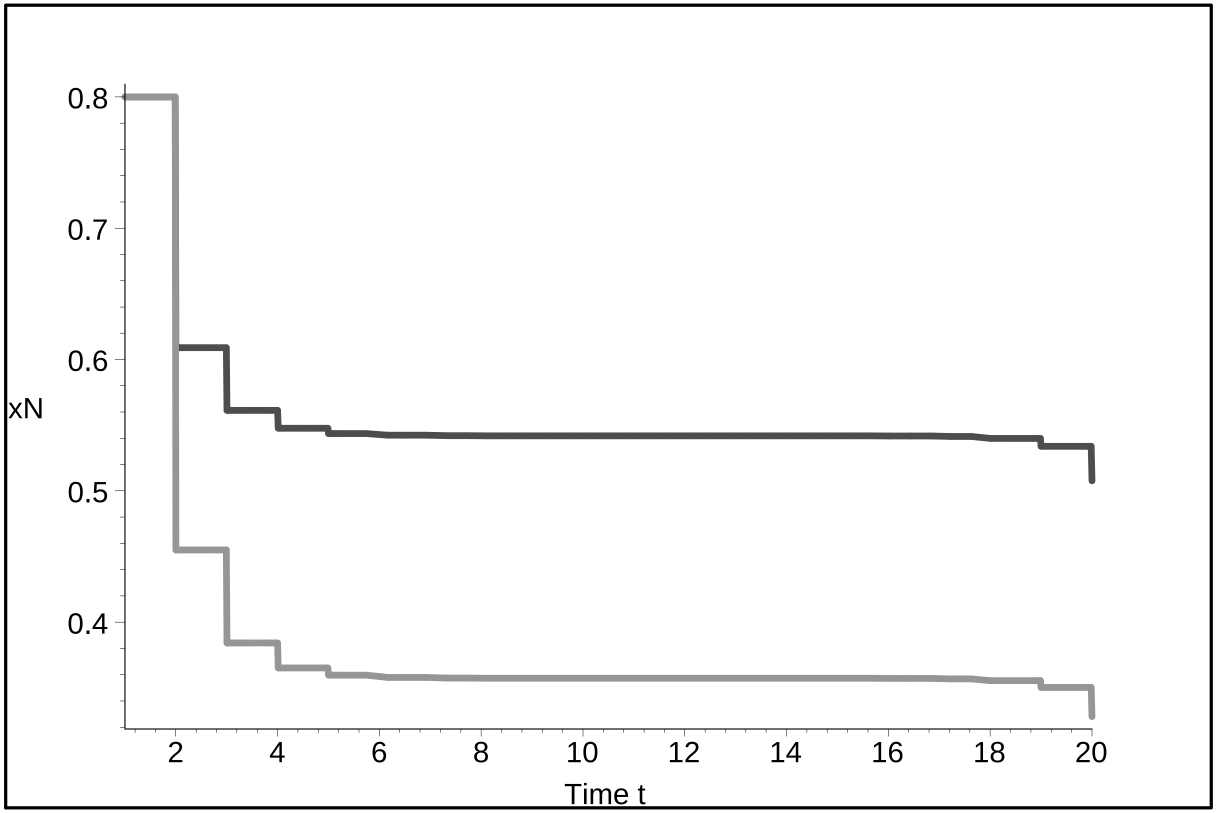

Fig. 7. The population size: dark – cooperative, light – Nash

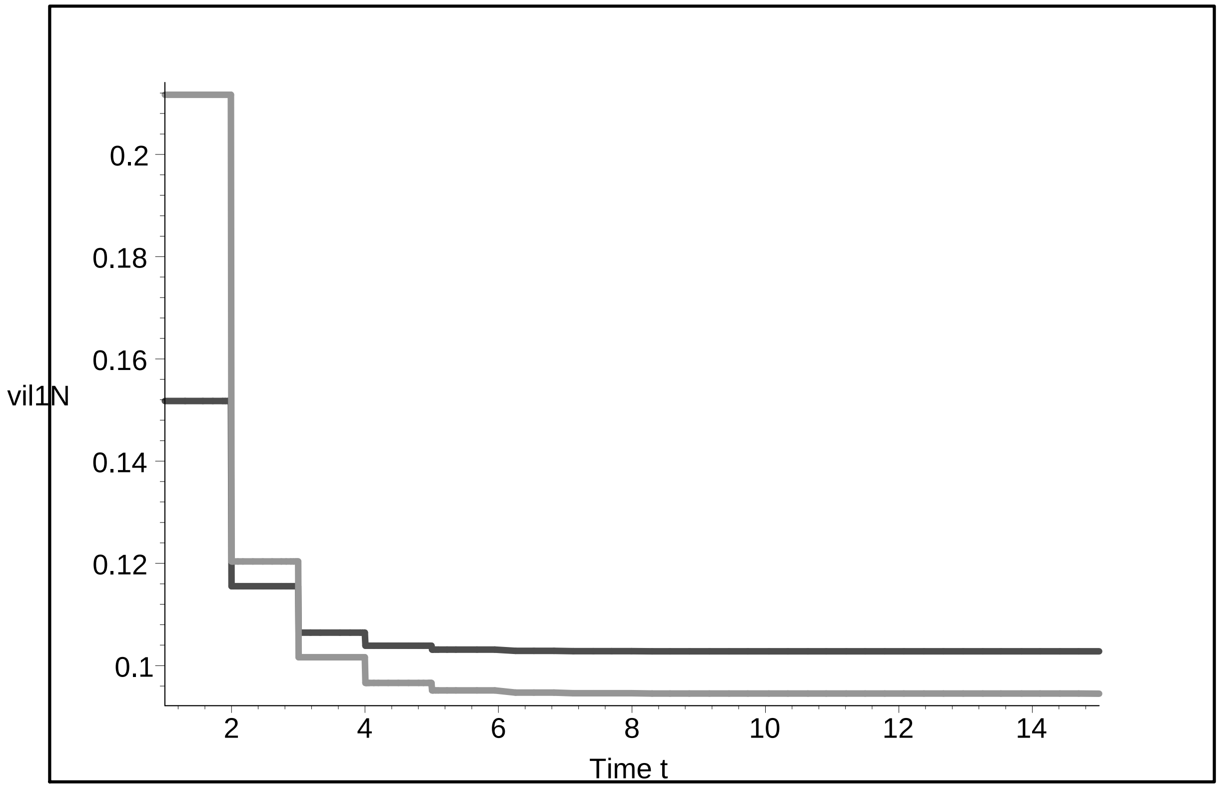

Fig. 8. The catch of player 1: dark – cooperative, light – Nash

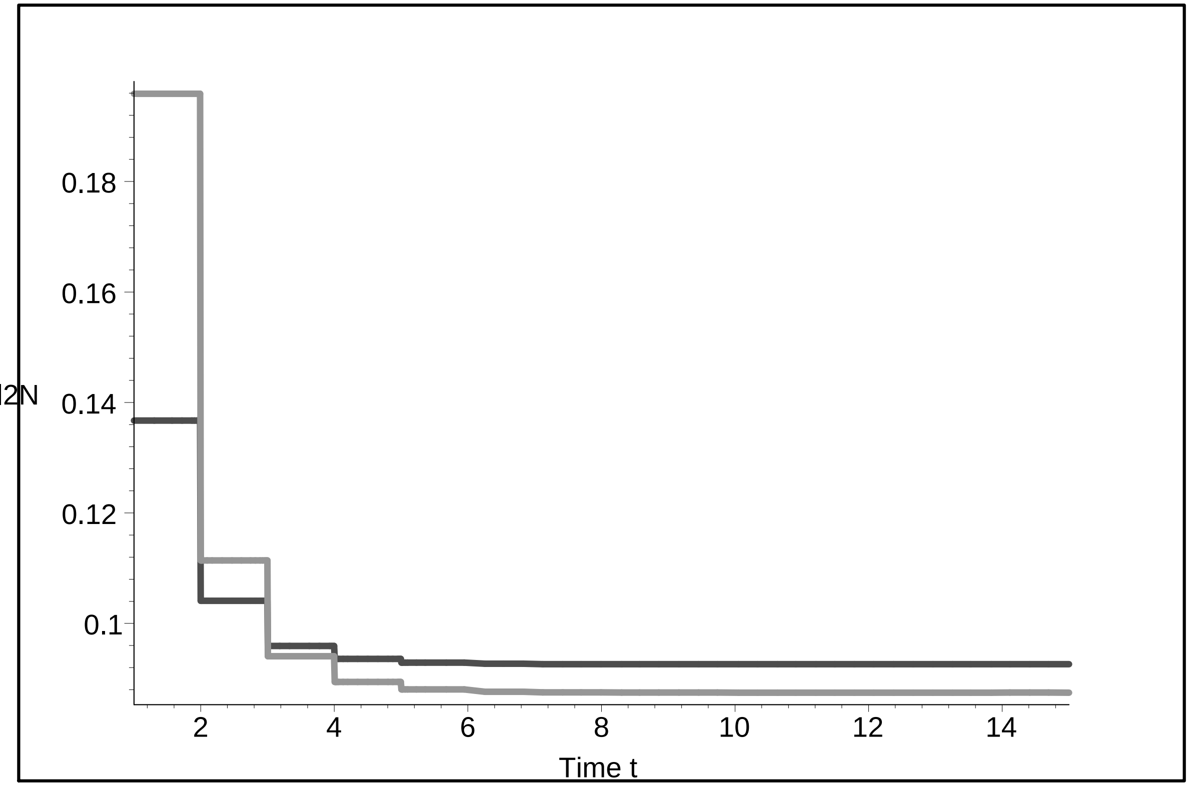

Fig. 9. The catch of player 2: dark – cooperative, light – Nash

If we compare these profits with the profits that we get in the previous scheme we can conclude that for player 1 it is smaller and for player 2 – it is almost the same. This fact shows that using the recursive Nash bargaining solution is less profitable for the player with smaller discount factor.

We considered discrete time bioresource management problem with two players which differ in their discount factors (time-preferences).

We show that the joint discount factor exists for proportional solution and the division in portion $\gamma{:}1-\gamma$. We propose to use the Nash bargaining solution to derive the joint discount factor and the portion.

Next we decline to use the joint discount factor and determine players' cooperative strategies and payoffs using Nash bargaining procedure. We present two different approaches of bargaining procedure. In the first one the cooperative strategies are determined as the Nash bargaining solution for the whole planning horizon. In the second, we use recursive Nash bargaining procedure determining the cooperative strategies on each time step.

5.1. Model with fixed times of exploitation

Let us consider the case when the first player extracts the stock n_1 time moments, and the second – n_2. Let $n_1 < n_2$. So, we have the situation when on time interval $[0, n_1]$ players cooperate and we need to determine their strategies. After n_1 till n_2 the second player acts individually.

We construct cooperative strategies and the payoff maximizing the Nash product for the whole game:

$$
\begin{aligned}
&(V_1^{nc}(x,\delta_1) - V_1^N(x,\delta_1)[0,n_1]) \cdot \\
&(V_2^{nc}(x,\delta_2) + V_2^{(n_2-n_1)} - V_2^N(x,\delta_2)[0,n_1] - V_2(x,\delta_2)[n_1,n_2]) = \\
&= (\sum_{t=0}^{n_1} \delta_1^t \ln(u_{1t}^c) - V_1^N(x,\delta_1)[0,n_1]) \cdot \\
&(\sum_{t=0}^{n_1} \delta_2^t \ln(u_{2t}^c) + \sum_{t=n_1}^{n_2} \delta_2^t \ln(u_{2t}) - V_2^N(x,\delta_2)[0,n_1] - V_2(x,\delta_2)[n_1,n_2]) \quad (20)
\end{aligned}
$$

where $V_i^N(x,\delta_i)[0,n_1]$ are the non-cooperative gains, $V_2^{(n_2-n_1)}$ – the second player's individual payoff starting from the cooperative point x, $V_2(x,\delta_2)[n_1,n_2]$ – the second player's individual payoff starting from the noncooperative point x^{Nn_1}.

We define $n = n_2 - n_1$.

To obtain the cooperative gains in the problem (20) we again start with one step game on the interval $[0, n_1]$ and so on. After n_1 the we assume that the first player gets some portion of the remaining stock $- k$ and the second H_2^{Nn} starts exploitation from the portion $(1 - k)$ of the remaining stock.

For the n_1-stage game we have the next form of the profits

$$H_1^{n_1}(\gamma_1^1, \ldots, \gamma_1^{n_1}, \gamma_2^1, \ldots, \gamma_2^{n_1}) = \frac{1 - a_1^{n_1+1}}{1 - a_1} \ln(x) +$$

$$+ \sum_{j=1}^{n_1} \delta_1^{n_1-j} \ln(\gamma_1^j) + \sum_{j=1}^{n} \delta_1^{n_1-j} \frac{a_1(1 - a_1^j)}{1 - a_1} \ln(\varepsilon - \gamma_1^j - \gamma_2^j) + \delta_1^{n_1} \ln(k) \quad (21)$$

and

$$H_2^{n_1}(\gamma_1^1, \ldots, \gamma_1^{n_1}, \gamma_2^1, \ldots, \gamma_2^{n_1}) = \frac{1 - a_2^{n_2+1}}{1 - a_2} \ln(x) +$$

$$+ \sum_{j=1}^{n_1} \delta_2^{n_1-j} \ln(\gamma_2^j) + \sum_{j=1}^{n_1} \delta_2^{n_1-j} \frac{a_2(1 - a_2^{n_2-n_1+j})}{1 - a_2} \ln(\varepsilon - \gamma_1^j - \gamma_2^j) +$$

$$+ \sum_{j=1}^{n_2-n_1} \delta_2^{n_2-j} B^j + \delta_2^{n_1} \frac{1 - a_2^{n_2-n_1+1}}{1 - a_2} \ln(1 - k). \quad (22)$$

The cooperative strategies are connected as

$$\gamma_1^{n_1} = \frac{\varepsilon\gamma_1^1 \sum\limits_{j=n_1-1}^{n+n_1} a_2^j}{\varepsilon a_1^{n_1-1} \sum\limits_{j=0}^{n+n_1} a_2^j + \gamma_1^1 (\sum\limits_{j=n_1-1}^{n+n_1} a_2^j \sum\limits_{j=0}^{n_1} a_1^j - (a_1^{n_1-1} + a_1^{n_1}) \sum\limits_{j=0}^{n+n_1} a_2^j)}, \tag{23}$$

$$\gamma_2^{n_1} = \frac{\varepsilon - \gamma_1^{n_1} \sum\limits_{j=0}^{n_1} a_1^j}{\sum\limits_{j=0}^{n+n_1} a_2^j}. \tag{24}$$

And γ_1^1 can be determined from one of the first order conditions.

Modelling

$$\varepsilon = 0.6\,, \quad \alpha_2 = 0.3\,, \quad n_2 = 20\,, \quad n_1 = 10\,,$$
$$\delta_1 = 0.85\,, \quad \delta_2 = 0.9\,, \quad x_0 = 0.8\,, \quad k = \frac{1}{3}\,.$$

We get $\gamma_1^1 = 0.272372955$. For the first player we compare the cooperative and noncooperative gains on time interval $[0, n_1]$

$$V_1^{nc}(x, \delta_1)[0, n_1] = -10.387 > V_1^N(x, \delta_1)[0, n_1] = -11.901\,,$$

For the second player we compare the cooperative gain on time interval $[0, n_1]$ plus acting individually on time interval $[n_1, n_2]$ and noncooperative gain on time interval $[0, n_1]$ plus individual gain on time interval $[n_1, n_2]$

$$V_2^{nc}(x, \delta_2)[0, n_2] = -19.637 > \tilde{V}_2^N(x, \delta_2)[0, n_2] = -23.259\,.$$

One can notice that the cooperative profits are lager that non-cooperative ones for both players.

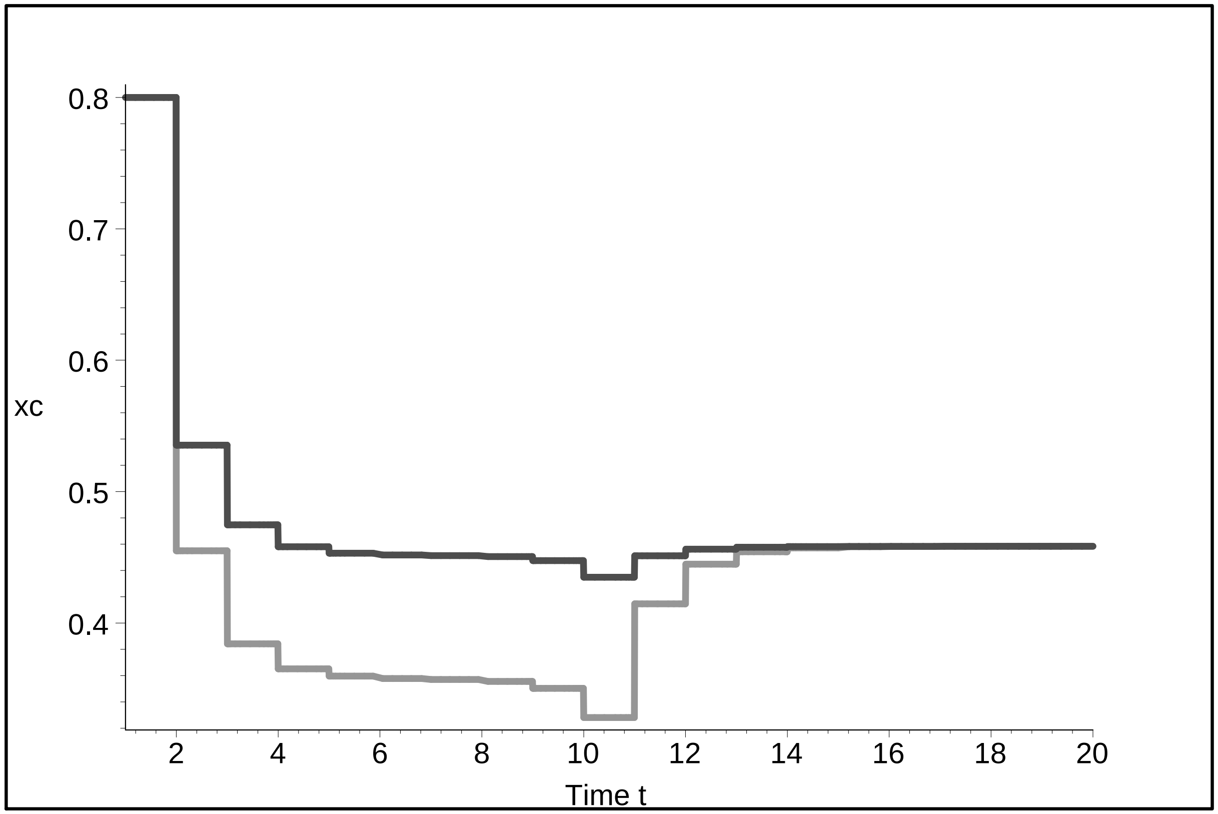

Fig. 10. The population size: dark – cooperative, light – Nash

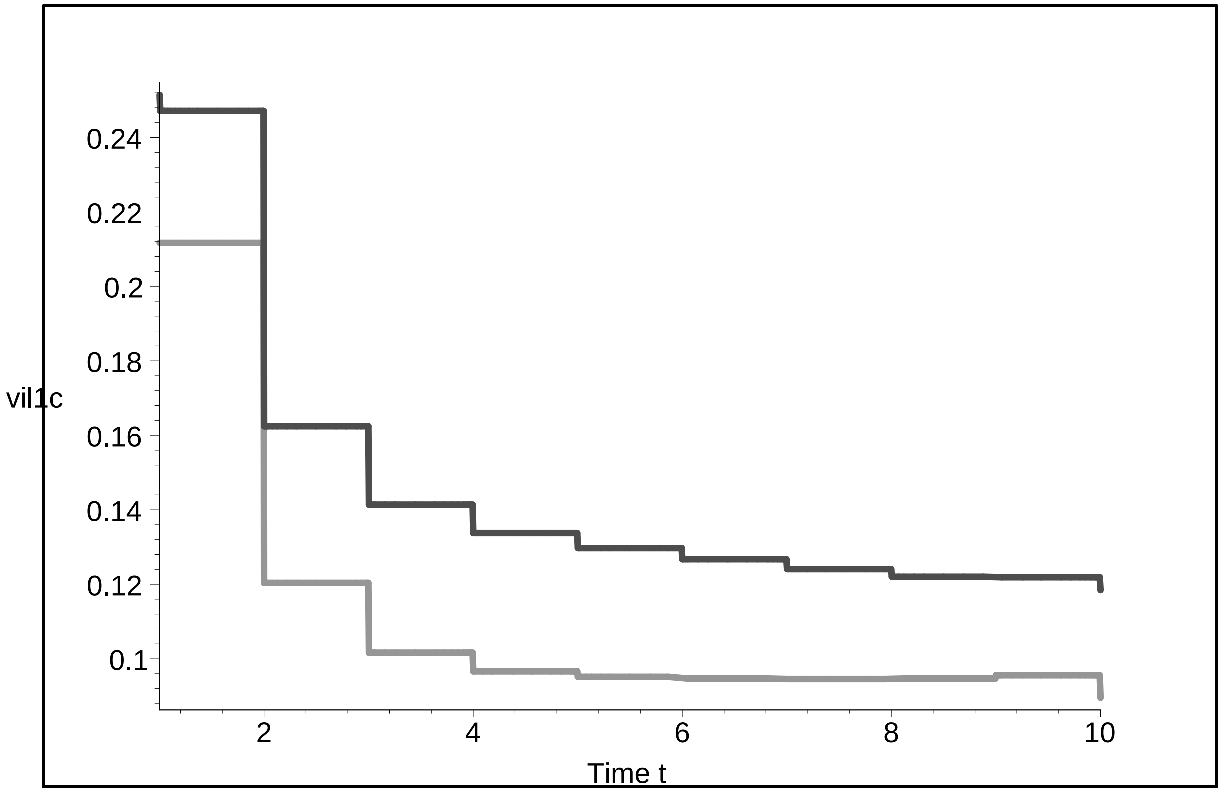

Fig. 11. The catch of player 1: dark – cooperative, light – Nash

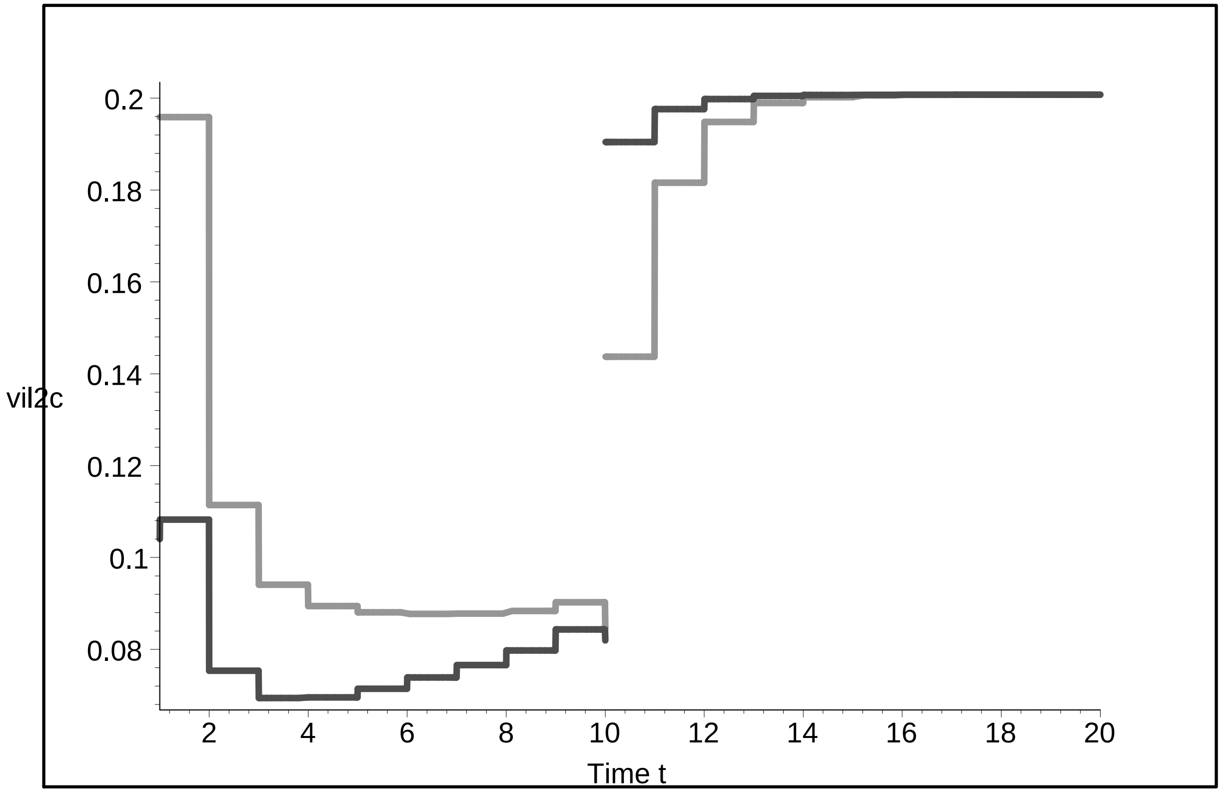

Fig. 12. The catch of player 2: dark – cooperative, light – Nash

3. The model with random times of exploitation

The first player extracts the stock n_1 time moments, and the second – n_2. n_1 is random variable with range $\{1, \ldots, N\}$ and corresponding probabilities $\{\theta_1, \ldots, \theta_N\}$. n_2 is random variable with the same range and probabilities $\{\omega_1, \ldots, \omega_N\}$.

First, we construct the players payoffs:

$$
\begin{aligned}
H_1 = E\Big\{ & \sum_{t=1}^{n_1} \delta_1^t \ln(u_{1t}) I_{\{n_1 \le n_2\}} + \\
& + \Big(\sum_{t=1}^{n_2} \delta_1^t \ln(u_{1t}) + \sum_{t=n_2}^{n_1} \delta_1^t \ln(u_{1t}^a) \Big) I_{\{n_1 > n_2\}} \Big\} = \\
& = \sum_{n_1=1}^{N} \theta_{n_1} \Big[\sum_{n_2=n_1}^{N} \omega_{n_2} \sum_{t=1}^{n_1} \delta_1^t \ln(u_{1t}) + \\
& + \sum_{n_2=1}^{n_1-1} \omega_{n_2} \Big(\sum_{t=1}^{n_2} \delta_1^t \ln(u_{1t}) + \sum_{t=n_2}^{n_1} \delta_1^t \ln(u_{1t}^a) \Big) \Big],
\end{aligned} \tag{25}
$$

$$H_2 = \sum_{n_2=1}^{N} \omega_{n_2} \Big[\sum_{n_1=n_2}^{N} \theta_{n_1} \sum_{t=1}^{n_2} \delta_2^t \ln(u_{2t}) +$$

$$+ \sum_{n_1=1}^{n_2-1} \theta_{n_1} \Big(\sum_{t=1}^{n_1} \delta_2^t \ln(u_{2t}) + \sum_{t=n_1}^{n_2} \delta_2^t \ln(u_{2t}^a) \Big) \Big], \qquad (26)$$

where u_{it}^a is a player i's strategy when his opponent quits the game, $i = 1, 2$.

Nash equilibrium

First we determine the Nash equilibrium as we use it as a status-quo point for the Nash bargaining solution.

As usually we will seek the value functions in the form $V_i^N(\tau, x) = A_i^\tau \ln x + B_i^\tau$ and the Nash strategies in the form $u_{i\tau}^N = \gamma_{i\tau}^N x$, $i = 1, 2$.

From the first order conditions we get

$$\gamma_{1\tau}^{N}=\frac{\varepsilon\delta_1^\tau A_2^\tau}{\delta_1^\tau A_2^\tau+\delta_2^\tau A_1^\tau+\alpha A_1^\tau A_2^\tau P_\tau^{\tau+1}},\ \gamma_{2\tau}^{N}=\frac{\varepsilon\delta_2^\tau A_1^\tau}{\delta_1^\tau A_2^\tau+\delta_2^\tau A_1^\tau+\alpha A_1^\tau A_2^\tau P_\tau^{\tau+1}}, \tag{27}$$

where

$$A_1^\tau=\frac{\delta_1^\tau+C_{1\tau}\sum\limits_{n_1=\tau+1}^{N}\theta_{n_1}\sum\limits_{j=0}^{n_1-\tau}a_1^j}{1-\alpha P_\tau^{\tau+1}},\ A_2^\tau=\frac{\delta_2^\tau+C_{2\tau}\sum\limits_{n_2=\tau+1}^{N}\omega_{n_2}\sum\limits_{j=0}^{n_2-\tau}a_2^j}{1-\alpha P_\tau^{\tau+1}}, \tag{28}$$

$$B_1^\tau=\frac{\delta_1^\tau\ln(\gamma_{1\tau}^N)+\alpha A_1^\tau P_\tau^{\tau+1}\ln(\varepsilon-\gamma_{1\tau}^N-\gamma_{2\tau}^N)+C_{1\tau}\sum\limits_{n_1=\tau+1}^{N}\theta_{n_1}\sum\limits_{j=1}^{n_1-\tau}\delta_1^{n_1-\tau-j}D_1^j}{1-P_\tau^{\tau+1}},$$

$$B_2^\tau = \frac{\delta_2^\tau \ln(\gamma_{2\tau}^N) + \alpha A_2^\tau P_\tau^{\tau+1} \ln(\varepsilon - \gamma_{1\tau}^N - \gamma_{2\tau}^N) + C_{2\tau} \sum\limits_{n_2=\tau+1}^{N} \omega_{n_2} \sum\limits_{j=1}^{n_2-\tau} \delta_2^{n_2-\tau-j} D_2^j}{1 - P_\tau^{\tau+1}}. \tag{29}$$

So we determined the Nash strategies and the Nash payoffs $V_i^N(\tau, x) = A_i^\tau \ln x + B_i^\tau$, $i = 1, 2$. Now we can construct the cooperative behavior.

The cooperative behavior

We construct cooperative strategies and the payoff maximizing the Nash product for the whole game, so we need to solve the next problem

$$(V_1^c(1,x) - V_1^N(1,x))(V_2^c(1,x) - V_2^N(1,x)) =$$
$$= (\sum_{n_1=1}^{N} \theta_{n_1} \Big[\sum_{n_2=n_1}^{N} \omega_{n_2} \sum_{t=1}^{n_1} \delta_1^t \ln(u_{1t}^c) +$$
$$+ \sum_{n_2=1}^{n_1-1} \omega_{n_2} (\sum_{t=1}^{n_2} \delta_1^t \ln(u_{1t}^c) + \sum_{t=n_2}^{n_1} \delta_1^t \ln(u_{1t}^a)) \Big] - V_1^N(1,x))\cdot$$

$$\cdot\Big(\sum_{n_2=1}^{N}\omega_{n_2}\Big[\sum_{n_1=n_2}^{N}\theta_{n_1}\sum_{t=1}^{n_2}\delta_2^t\ln(u_{2t}^c)+$$

$$+\sum_{n_1=1}^{n_2-1}\theta_{n_1}\Big(\sum_{t=1}^{n_1}\delta_2^t\ln(u_{2t}^c)+\sum_{t=n_1}^{n_2}\delta_2^t\ln(u_{2t}^a)\Big)\Big]-V_2^N(1,x)\Big)\to\max, \quad (30)$$

where $V_i^N(1,x)=A_i^1\ln x+B_i^1$, $i=1,2$ are the non-cooperative gains determined in (27)-(29).

The process can be repeated till the case when step 1 has arrived and we have the next form of the profits:

$$V_i^c(N-k,x) =$$
$$= \delta_i^{N-k}\ln(u_{iN-k}^c) + \alpha P_{N-k}^{N-k+1} G_{N-k+1}^i \ln(\varepsilon x - u_{1N-k}^c - u_{2N-k}^c) +$$
$$+ \sum_{l=2}^{k-1} P_{N-k}^{N-l}[\delta_i^{N-l}\ln(\gamma_{iN-l}^c) + \alpha P_{N-l}^{N-l+1}\ln(\varepsilon - \gamma_{1N-l}^c - \gamma_{2N-l}^c)] +$$
$$+ P_{N-k}^{N-1}[\delta_i^{N-1}\ln(\gamma_{iN-1}^c) + P_{N-1}^{N}\alpha A_i \ln(\varepsilon - \gamma_{1N-1}^c - \gamma_{2N-1}^c) + P_{N-1}^{N} B_i] +$$
$$+ \sum_{l=1}^{k} P_{N-k}^{N-l} C_{iN-l} H_i^l(n_i). \quad (31)$$

where

$$H_1^l(n_1) = \sum_{n_1=N-l+1}^{N} \theta_{n_1} \sum_{t=N-l}^{n_1} \delta_1^t \ln(u_{1t}^a)\,,$$

$$H_2^l(n_2) = \sum_{n_2=N-l+1}^{N} \omega_{n_2} \sum_{t=N-l}^{n_2} \delta_2^t \ln(u_{2t}^a)\,,$$

$$G_k^1 = \sum_{l=1}^{k} \delta_1^{N-l} \alpha^{k-l} P_{N-k}^{N-l} + \alpha^k A_1 P_{N-k}^N\,,$$

$$G_k^2 = \sum_{l=1}^{k} \delta_2^{N-l} \alpha^{k-l} P_{N-k}^{N-l} + \alpha^k A_2 P_{N-k}^N\,.$$

The cooperative strategies are connected as

$$\gamma_{2N-k}^c = \frac{\delta_1^{N-k}\delta_2^{N-k}\varepsilon - \delta_2^{N-k}\gamma_{1N-k}^c G_k^1}{\delta_1^{N-k} G_k^2}\,, \tag{32}$$

$$\gamma^c_{1N-k} = \frac{\delta_1^{N-k}\varepsilon\gamma^c_{1N-1}G_1^2}{\delta_1^{N-1}\varepsilon G_k^2 + \gamma^c_{1N-1}(G_k^1G_1^2 - G_1^1G_k^2)}\,. \tag{33}$$

And γ^c_{1N-1} can be determined from one of the first order conditions, for example, the last one

$$-\frac{\alpha A_1 P^N_{N-1}}{\varepsilon - \gamma^c_{1N-1} - \gamma^c_{2N-1}}(V_2^c(1,x) - V_2^N(1,x)) +$$
$$+\Big(\frac{\delta_2^{N-1}}{\gamma^c_{2N-1}} - \frac{\alpha A_2 P^N_{N-1}}{\varepsilon - \gamma^c_{1N-1} - \gamma^c_{2N-1}}\Big)(V_1^c(1,x) - V_1^N(1,x)) = 0\,. \tag{34}$$

Modelling

We use Monte-Carlo scheme for the simulation. $N = 10$.

For the same parameters and the next probabilities

$$\theta_i = 0.1\,,\ \omega_i = 0.005i + 0.0725$$

we get the expected cooperative and Nash payoffs

$$V_1^c(1,x) = -6.2151 > V_1^N(1,x) = -10.1958\,,$$

$$V_2^c(1,x) = -7.3256 > V_2^N(1,x) = -12.8829\,.$$

The fig. presents the results of modelling with 50 simulations.

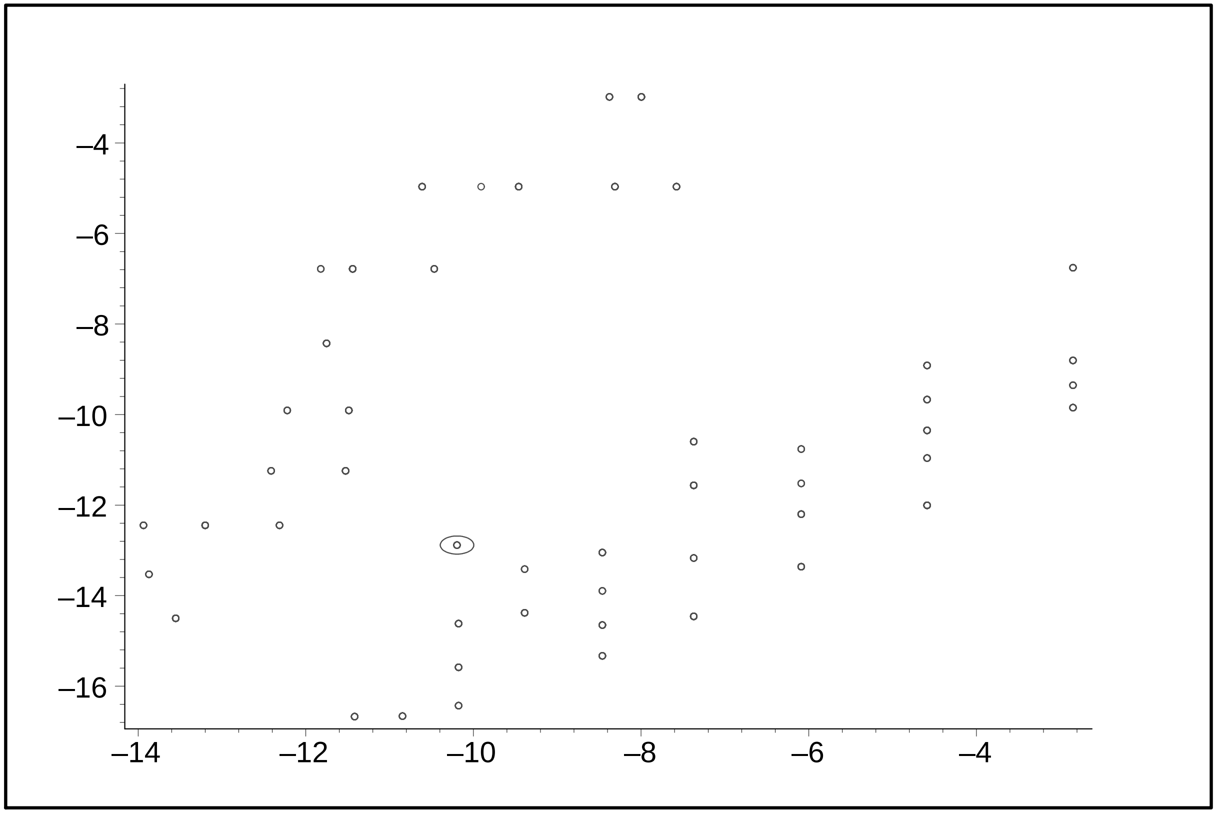

Fig. 13. Nash equilibrium

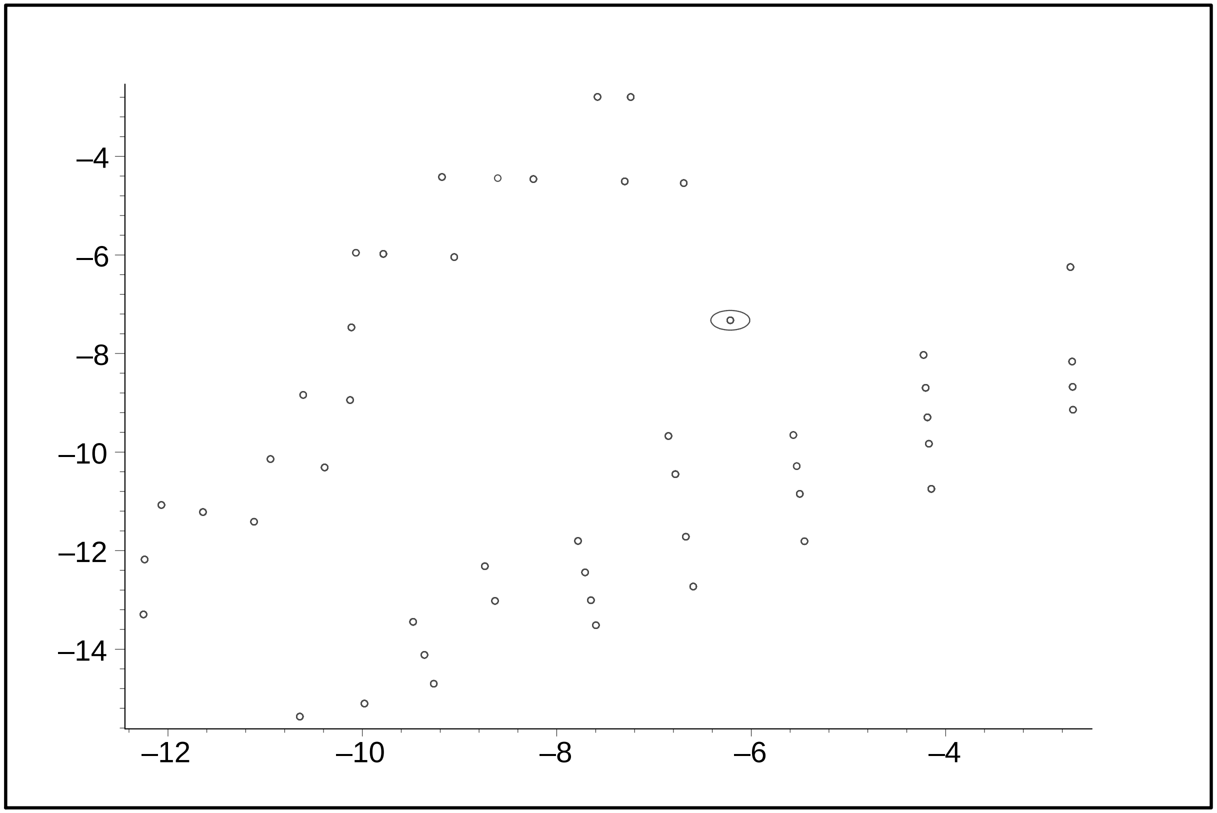

Fig. 14. Cooperative equilibrium

We considered discrete time bioresource management problem with two players which differ not only in discount factors, but in times of exploitation.

In the first model, participations' planning horizons are known. Here one player leaves the game at the fixed time moment and receives some portion of the remaining stock as compensation. The second player continues exploitation till the end of the game individually. To construct the cooperative strategies we use Nash bargaining scheme for the whole planning horizon.

In the second model, the times of exploitation are random variables with known discrete distribution. First, we construct Nash equilibrium and take it as a status-quo point. Second, we determine the cooperative strategies using recursive Nash bargaining procedure.

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