S_S

MATHEMATICAL MODELING OF NITRIFICATION AND ORGANIC MATTER OXIDATION PROCESSES IN THE BIOLOGICAL TREATMENT SYSTEM

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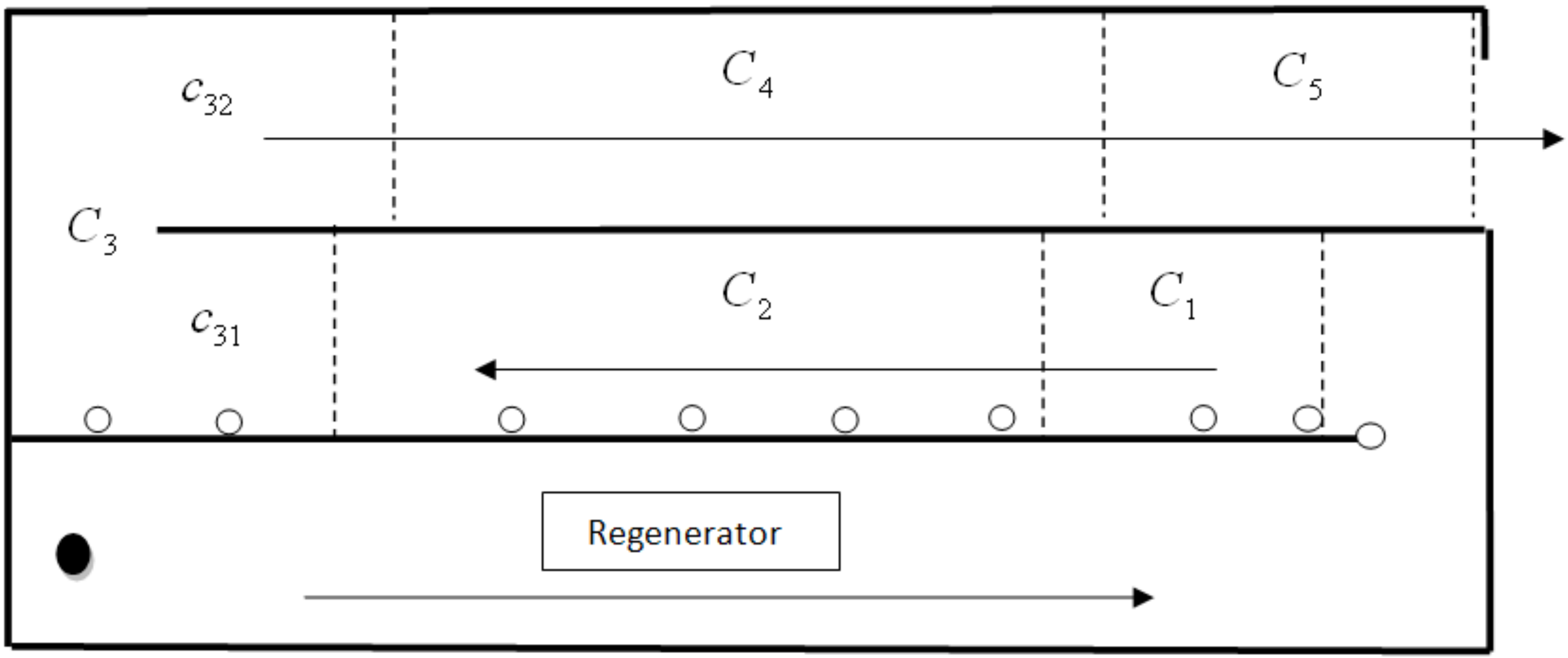
Scheme of the aeration tank



○ – wastewater entrance to the aeration tank

● – activated sludge entrance to the aeration tank

→ – direction of activated sludge flow

---- – border of compartments

Concentrations of the model:

S_S - readily biodegradable organic matter

S_{NH} - ammonia nitrogen

X_H - active heterotrophic biomass

X_A - active autotrophic biomass

Data: S_S , S_{NH} , $X = X_A + X_H$.

Mathematical model

$$f(x,p) = \frac{x}{x+p}$$ - Mono's trophic function

$$f_1(S_S) = \frac{1}{1+e^{10^6\left(\frac{c+\delta}{S_S}-1\right)}},$$

$$f_2(S_S, S_{NH}, K_\alpha) = \frac{S_{NH}/(S_S-c)}{K_\alpha + S_{NH}/(S_S-c)}$$

- Switching functions.

$$\dot{S}_S = Q(S_S^{in} - S_S) - \frac{\mu_H}{Y_H} f(S_S, K_S) f(S_O, K_{O,H}) \frac{1}{1+e^{10^6(\frac{c+\delta}{S_S}-1)}} X_H, \quad (1)$$

$$\dot{S}_{NH} = Q(S_{NH}^{in} - S_{NH}) - \frac{\mu_A}{Y_A} f(S_{NH}, K_{NH}) f(S_O, K_{O,A}) \frac{S_{NH}/(S_S-c)}{K_\alpha + S_{NH}/(S_S-c)} X_A, \quad (2)$$

$$\dot{X}_H = Q(X_H^{in} - X_H) + \left(\mu_H f(S_S, K_S) f(S_O, K_{O,H}) \frac{1}{1+e^{10^6(\frac{c+\delta}{S_S}-1)}} - b_H \right) X_H, \quad (3)$$

$$\dot{X}_A = Q(X_A^{in} - X_A) + \left(\mu_A f(S_{NH}, K_{NH}) f(S_O, K_{O,A}) \frac{S_{NH}/(S_S-c)}{K_\alpha + S_{NH}/(S_S-c)} - b_A \right) X_A, \quad (4)$$

$$\dot{S}_S = \frac{1}{1 + e^{10^6\left(\frac{c+\delta}{S_S} - 1\right)}} X_H, \qquad (5)$$

$$\dot{X}_H = \beta \frac{V_{cv}}{V_{il}} X_H. \qquad (6)$$

$\delta = 10^{-3}$

c – concentration of S_S in the end of C_5

V_{cv} – consumption of wastewater

V_{il} – consumption of activated sludge

β – specific growth rate for heterotrophic biomass

Parameter estimation

$P^0 = (p_1^0, \ldots, p_{n_p}^0)$, n_p - number of parameters.

Least squares criterion:

$$J = \sum_{i=1}^{n_y} sc_i^2 (y_i - y_i^m)^T (y_i - y_i^m)$$, sc_i - scale factor,

y_i - output observed from the real plant,

y_i^m - model output,

n_y - number of measured concentrations.

Sensitivity of the model

$$s_{ij} = sc_i \frac{y_i^m(P^0 + \delta P_j) - y_i^m(P^0)}{\delta p_j}$$

- sensitivity function

$$\delta_j^{msqr} = \sqrt{\frac{1}{n_y} \sum_{i=1}^{n} s_{ij}^2}$$

- sensitivity measure[1]

Table of values of sensitivity measure

Parameter	C_1	C_2	c_{32}	C_4	C_5
Y_H	0.49	0.25	-	-	-
Y_A	3.06	3.72	0.97	2.03	0.01
μ_H	52.53	38.67	-	-	-
μ_A	936.02	1888.1	782.85	20452.00	2341.40
b_H	9.59	23.39	-	-	-
b_A	4.80	21.90	2.78	2.93	0.80
K_S	0.00	0.00	-	-	-
K_{NH}	0.11	0.25	0.06	0.05	0.00
$K_{O,H}$	0.06	0.05	-	-	-
$K_{O,A}$	0.08	0.26	0.06	0.03	0.00
K_α	0.01	0.01	0.00	0.00	0.00
β	-	-	15.67	26.98	10.781

[1]. Brun R. et al. Practical identifiability of ASM2d parameters—systematic selection and tuning of parameter subsets. 2002.

Sensitivity of the model

$S = \{s_{ij}\}$ $\begin{matrix} i = 1, \ldots, n_y \\ j = 1, \ldots, n_p \end{matrix}$ - sensitivity matrix

$$\gamma_k = \frac{1}{min_{\|\beta\|=1} \|\tilde{S}_k \beta\|} = \frac{1}{\sqrt{\tilde{\lambda}_k}}$$ - collinearity index

$\tilde{S} = \{\tilde{s}_{ij}\}$ - normalized matrix, $\tilde{s}_{ij} = \dfrac{s_{ij}}{\|s_{ij}\|}$.

$\tilde{S}_k^T$ - submatrix $\tilde{S}$ containing those columns that correspond to the parameter subset

$\tilde{\lambda}_k$ - the smallest eigenvalue of $\tilde{S}_k^T \tilde{S}_k$

$$\rho_k = det(S_k^T S_k)^{1/2k} = \left(\prod_{j=1}^{k} \lambda_j \right)^{1/2k}$$ - determinant measure

Compartment	Parameters	γ_k	ρ_k
C_1	μ_A, b_H	1.11	132.74
C_2	μ_A, b_H	1.08	2295.59
c_{32}	μ_A, β	1.00	156.63
C_4	μ_A, β	1.00	1050.50
C_5	μ_A, β	1.00	224.69

Parameter estimation

$$(S^T S)\big|_{P^0} \Delta P = S^T e_i\big|_{P^0}$$ [1], $\Delta P = P - P^0$, $e_i = y_i - y_i^m$.

Parameter	Units	C_1	C_2	c_{32}	C_4	C_5
Y_H	(g cell X_H)·(g BOD)$^{-1}$	0.75	0.75	-	-	-
Y_A	(g cell X_A) · (g NH)$^{-1}$	0.17	0.28	0.28	0.07	0.28
μ_H	day^{-1}	10.00	7.00	-	-	-
μ_A	day^{-1}	0.80	0.80	0.66	0.01	0.001
b_H	day^{-1}	0.05	0.05	-	-	-
b_A	day^{-1}	0.05	0.05	0.05	0.09	0.06
K_S	(g BOD) · m^{-3}	200.00	40.00	-	-	-
K_{NH}	(g NH) · m^{-3}	1.00	0.70	1.00	0.30	1.00
$K_{O,H}$	(g O_2) · m^{-3}	0.20	0.01	-	-	-
$K_{O,A}$	(g O_2) · m^{-3}	0.40	0.40	1.50	0.40	1.50
K_α	(g NH) · (g BOD)$^{-1}$	5.00	5.00	5.00	5.00	5.00
β	day^{-1}	–	–	20, 306	4, 562	−24, 520

[1]. Chai Q. Modeling, Estimation, and Control of Biological Wastewater Treatment Plants. Doctoral Theses at NTNU 2008:108 at HiT, Porsgrunn. Telemark University College, 2008.

	S_{NH} ((g NH) $\cdot$ m^{-3})		
C_i	y	$y^m(P^0)$	$y^m(P)$
C_1	2.750	2.748	2.749
C_2	3.50	3.501	3.500
C_3	2.750	3.192	2.722
C_4	2.250	2.444	2.249
C_5	2.250	2.249	2.249

	X ((g X) $\cdot$ m^{-3})		
C_i	y	$y^m(P^0)$	$y^m(P)$
C_1	3300.000	3273.000	3273.000
C_2	2635.000	2631.500	2631.500
C_3	3430.000	3072.4	3081.100
C_4	3720.000	3843.600	3839.000
C_5	2760.000	2763.500	2744.800

CONCLUSION

•Mathematical model of nitrification and oxidation of readily biodegradable organic matter by active sludge is proposed.

•Subsets of identifiable parameters are determined. Parametric identifiability characteristics of the model and values of parameters are found out by means of sensitivity theory.

Thanks for your attention!