Multistage Mate Choice Game with Age Preferences

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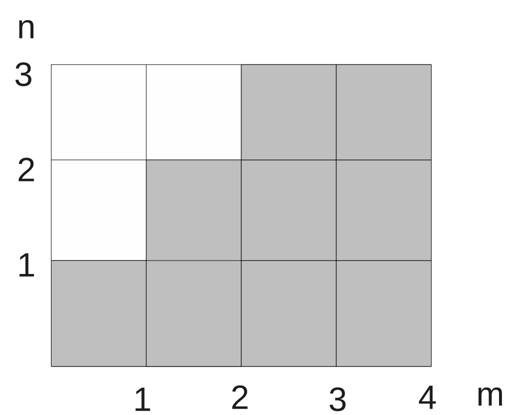
Alpern S., Katrantzi I., Ramsey D. (2010)

- Males have lifetime m, females have lifetime n, $m > n$.
- It is assumed that the total number of unmated males is greater than the total number of unmated females.
- Each group has steady state distribution for the age of individuals.
- In the game unmated individuals from different groups randomly meet each other in each period. If they accept each other, they form a couple and leave the game, otherwise they go into the next period unmated and older.
- Payoff of mated player is the number of future joint periods with selected partner: payoff of male age i and female age j is equal to min$\{m - i + 1; n - j + 1\}$
- The aim of each player is to maximize the expected payoff.

Mutual choice problem

- Alpern S., Reyniers D.J. (1999, 2005) Homotypic and common preferences
- Mazalov V., Falko A. (2008) Common preferences, arriving flow
- Alpern S., Katrantzi I., Ramsey D. (2010) Age preferences: discrete time model
- Alpern S., Katrantzi I., Ramsey D. (2013) Age preferences: continuous time model

- $a = (a_1, ..., a_m),\ b = (b_1, ..., b_n)$.
- a_i — the number of unmated males of age i relative to the number of females of age 1.
- b_j — the number of unmated females of age j relative to the number of females of age 1 ($b_1 = 1$).
- R — the ratio of the rates at which males and females enter the adult population $R = \frac{a_1}{b_1} = a_1$.
- $A = \sum_{i=1}^{m} a_i,\ B = \sum_{i=1}^{n} b_j,\ r = \frac{A}{B},\ r > 1$.
- $F = [f_1, ..., f_m],\ G = [g_1, ..., g_n]$
- $f_i = k,\ k = 1, ..., n$ — to accept a female of age $1, ..., k$
- $g_j = l,\ l = 1, ..., m$ — to accept a male of age $1, ..., l$



F=[1,2,3,3], G=[4,4,4]

- U_i, $i = 1, ..., m$ — the expected payoff of male of age i.
- V_j, $j = 1, ..., n$ — the expected payoff of female of age j.
- $\dfrac{a_i}{A}$ — the probability a female is matched with a male of age i,
- $\dfrac{B}{A}$ — the probability a male is matched.
- $\dfrac{b_j}{B}$ — the probability a male is matched with a female of age j, given that a male is mated.
- $\dfrac{b_j}{A} = \dfrac{b_j}{B} \cdot \dfrac{B}{A}$ — the probability a male is matched with a female of age j.

Case $n = 2$, $m > 2$: strategies $F = [f_1, ..., f_m]$, $G = [g_1, g_2]$

The expected payoffs of females are equal to

$$\begin{cases} V_2 = \sum_{i=1}^{m-1} \frac{a_i}{A} I\{f_i = 2\} + \frac{a_m}{A} \le 1, \\ V_1 = \sum_{i=1}^{m-1} 2\frac{a_i}{A} + \frac{a_m}{A} \max\{1, V_2\} = 2 - \frac{a_m}{A}. \end{cases}$$

$G = [m, m]$: $f_i = 1$ if $U_{i+1} > 1$, $i = 1, ..., m-2$; $f_i = 2$ if $U_{i+1} \le 1$, $i = 1, ..., m-2$

$$b = (1, 0) \; a = \left(R, R\left(1 - \frac{1}{r}\right), ..., R\left(1 - \frac{1}{r}\right)^{m-1} \right)$$

$$\begin{cases} U_m = \frac{1}{r}, \\ U_{m-i} = \frac{2}{r} + \left(1 - \frac{1}{r}\right) U_{m-i+1}, \; i = 1, ..., m-2. \end{cases}$$

Equilibrium $m = 4$	$r = \frac{A}{B}$
$([1, 1, 2, 2], [4, 4])$	$(1, 2.618)$
$([1, 2, 2, 2], [4, 4])$	$[2.618, 4.079)$
$([2, 2, 2, 2], [4, 4])$	$[4.079, +\infty)$

Case $n=3$, $m>3$: $F=[f_1,...f_{m-2},3,3]$, $G_1=[m-1,m,m]$, $G_2=[m,m,m]$

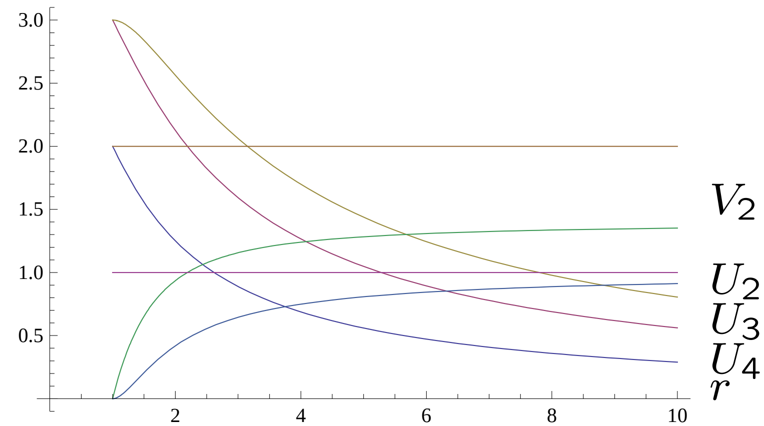
I. **Theorem.** *If players use strategy profile* (F, G_2), *where* $G_2 = [m,m,m]$, $F = [\underbrace{1,...,1}_{k}, \underbrace{2,...,2}_{l} \underbrace{3,...,3}_{m-k-l}]$, *then male's payoffs are equal to*

$$\begin{cases} U_m = 1 - z, \\ U_{m-1} = 2 - z^2 - z, \\ U_{m-i} = 3 - z^{i+1} - z^i - z^{i-1},\ i = 2,...,m-2, \end{cases}$$

for $z = 1 - 1/r$.

Equilibrium distributions are equal to

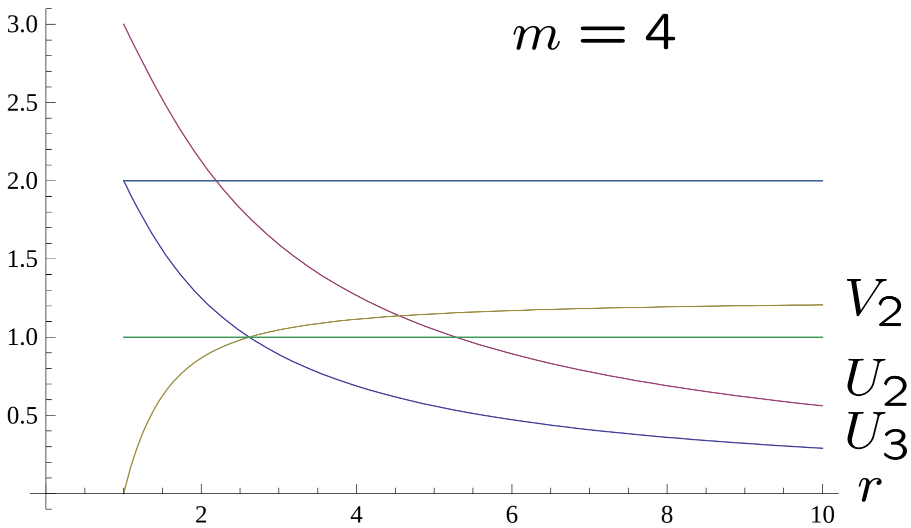
$m = 5$

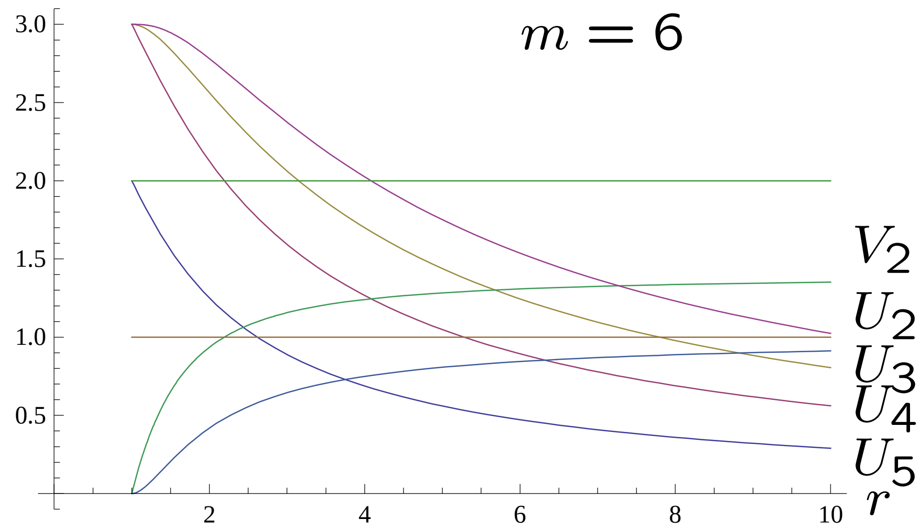


$$b = (1,0,0);\ a = (R, Rz, Rz^2, ..., Rz^{m-1}),$$

$$R = \frac{1}{(1-z)(1+z+z^2+...+z^{m-1})},$$

$$A = r = 1/(1-z).$$





$$r = 2,\ a = \left(\frac{16}{15}, \frac{8}{15}, \frac{4}{15}, \frac{2}{15}\right),\ b = (1, 0, 0)$$

$F_1 = [1, ..., 1, 2, 3, 3]$ for $r \in (1; 2.191)$ and $m \geq 4$,

$F_1 = [1, ..., 1, 2, 2, 3, 3]$ for $r \in [2.191; 2.618)$ and $m \geq 6$,

$F_1 = [1, ..., 1, 2, 3, 3, 3]$ for $r \in [2.618; 3.14)$ and $m \geq 6$,

$F_1 = [1, ..., 1, 2, 2, 3, 3, 3]$ for $r \in [3.14; 4.079)$ and $m \geq 7$.

II. Female's strategy is $G_1 = [m-1, m, m]$ $(V_2 \geq 1)$

male's strategy is $F = [\underbrace{2,...,2}_{k}, \underbrace{3,...,3}_{m-k}]$

$b = \left(1, \frac{a_m}{A}, 0\right)$; $a = \left(R, R(1-\frac{1}{r}), R(1-\frac{1}{r})^2, ..., R(1-\frac{1}{r})^{m-1}\right)$.

$$R = \frac{r(1 + (1-1/r) + (1-1/r)^2 + ... + (1-1/r)^{m-2} + 2(1-1/r)^{m-1})}{(1 + (1-1/r) + (1-1/r)^2 + ... + (1-1/r)^{m-1})^2}$$

Equilibrium for $m = 5$	$r = \frac{A}{B}$
$([2,2,3,3,3],[4,5,5])$	$[2.85, 4.517)$
$([2,3,3,3,3],[4,5,5])$	$[4.517, 6.87)$
$([3,3,3,3,3],[4,5,5])$	$[6.87, +\infty)$

III. Female's strategy is $G_1 = [m-1, m, m]$ $(V_2 \geq 1)$,

male's strategy is $F = [\underbrace{1, ..., 1}_{k}, \underbrace{2, ..., 2}_{l}, \underbrace{3, ..., 3}_{m-k-l}]$

$$V_2 = 2 - \frac{a_m}{A} - 2\sum_{i=1}^{k} \frac{a_i}{A} < 1$$

$$b = \left(1, \frac{a_m}{A}, \frac{a_m}{A}\sum_{i=1}^{k}\frac{a_i}{A}\right);\ a = (a_1, ..., a_m)$$

$a_1 = R$, $a_i = a_{i-1}(1 - 1/A)$, $i = 1, ..., k+1$,

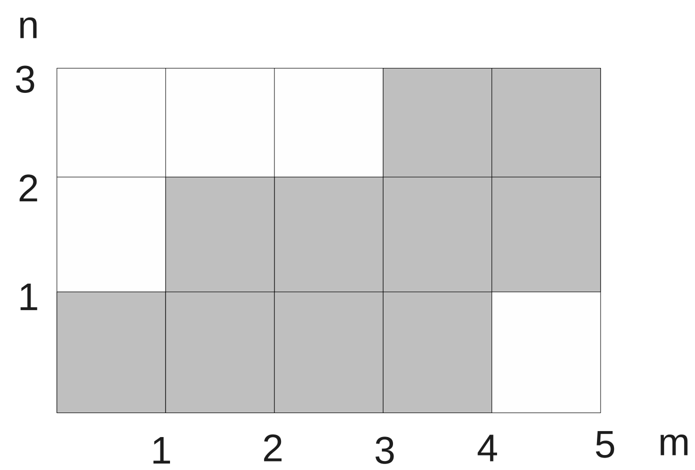
$$a_i = a_{i-1}\left(\frac{b_3}{A} + 1 - \frac{1}{r}\right),\ i = k+2, ..., k+l+2,$$

$$a_i = a_{i-1}\left(1 - \frac{1}{r}\right),\ i = k+l+3, ..., m$$

Equilibrium for $m = 5$	$r = \frac{A}{B}$
$([1,2,3,3,3],[4,5,5])$	$[2.016, 2.901)$

Table. $m = 5$

Equilibrium	$r = \frac{A}{B}$	R
$([1,1,2,3,3],[5,5,5])$	$(1, 2.191)$	$(1, 1.049)$
$([1,2,3,3,3],[4,5,5])$	$[2.016, 2.901)$	$[1.081, 1.191)$
$([2,2,3,3,3],[4,5,5])$	$[2.85, 4.517)$	$[1.209, 1.560)$
$([2,3,3,3,3],[4,5,5])$	$[4.517, 6.87)$	$[1.560, 2.097)$
$([3,3,3,3,3],[4,5,5])$	$[6.87, +\infty)$	$[2.097, +\infty)$



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THANK YOU FOR YOUR ATTENTION