Simulation of mesoscale stratified flows over steep obstacles of various shapes

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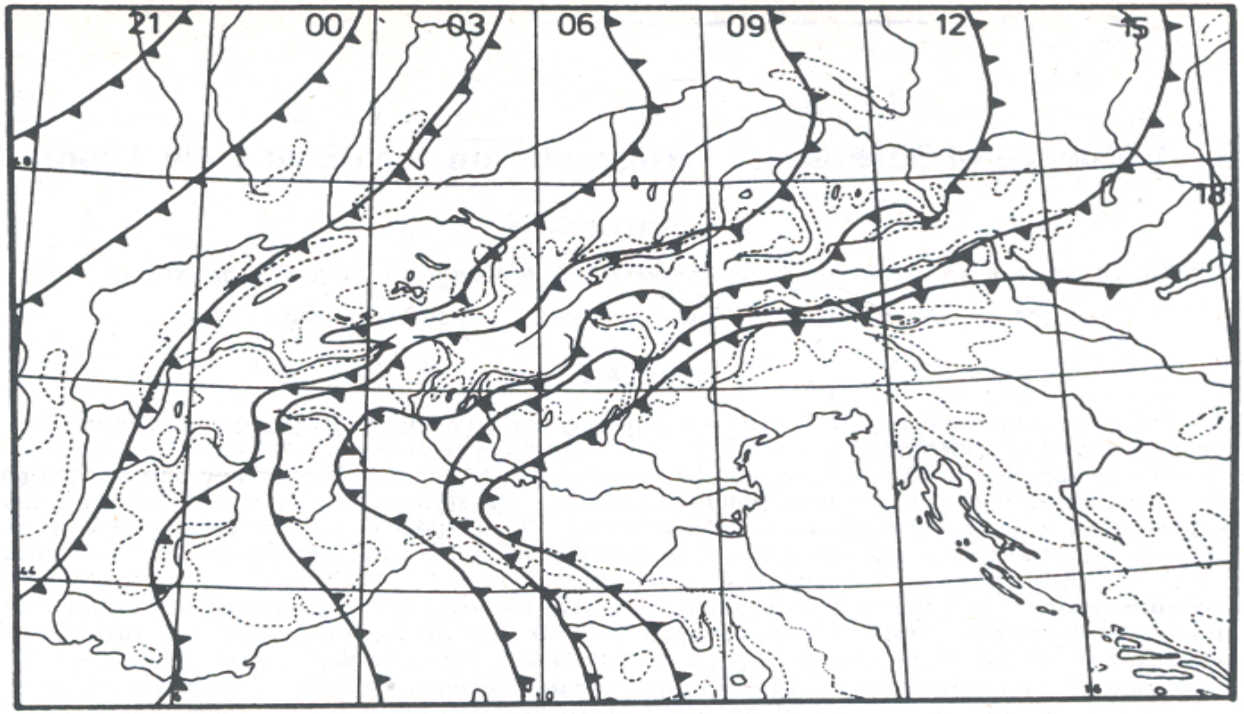


COLD FRONT PROPAGATION
(Schumann,1987)

$$\frac{dU}{dt}+\frac{\partial P}{\partial x}=f_1(V-V_g)-f_2W+R_u,$$

$$\frac{dV}{dt}+\frac{\partial P}{\partial y}=-f_1(U-U_g)+R_\upsilon,$$

$$\frac{dW}{dt}+\frac{\partial P}{\partial z}+\frac{gP}{C_s^2}=f_2U+g\,\frac{G^{1/2}\,\overline{\rho}\theta'}{\theta}+R_\varpi$$

$$\frac{d\theta}{dt}=R_\theta,$$

$$\frac{ds}{dt}=R_s,$$

$$\frac{1}{C_s^2}\frac{\partial P}{\partial t}+\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=\frac{\partial}{\partial t}\left(\frac{\overline{\rho}\theta'}{\theta}\right)$$

$$U=\overline{\rho}u, V=\overline{\rho}\upsilon, P=\overline{\rho}p',\ W=\overline{\rho}\omega$$

$$\frac{}{dt} + \frac{}{\partial x} + \frac{(\phantom{G^{13}P})}{\partial \eta} = f_1(V - V_g) - f_2 W + R_u,$$

$$\frac{dV}{dt} + \frac{\partial P}{\partial y} + \frac{\partial (G^{23} P)}{\partial \eta} = -f_1(U - U_g) + R_v,$$

$$\frac{dW}{dt} + \frac{1}{G^{1/2}}\frac{\partial P}{\partial \eta} + \frac{gP}{C_s^2} = f_2 U + g\frac{G^{1/2}\bar{\rho}\theta'}{\bar{\theta}} + R_w,$$

$$\frac{d\theta}{dt} = R_\theta,$$

$$\frac{ds}{dt} = R_s,$$

$$\frac{1}{C_s^2}\frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial}{\partial \eta}\left(G^{13}U + G^{23}V + \frac{1}{G^{1/2}}W\right) = \frac{\partial}{\partial t}\left(\frac{G^{1/2}\bar{\rho}\theta'}{\bar{\theta}}\right).$$

$\bar{\rho}G^{1/2}u$, $V = \bar{\rho}G^{1/2}v$, $W = \bar{\rho}G^{1/2}w$, $P = G^{1/2}p'$, where p', θ' are deviations from the state pressure $\bar{p}$ and potential temperature $\bar{\theta}$, s is specific humidity, C_s is the sound speed, u_g, v_g are the components of geostrophic wind representing the synoptic part of ressure, η is a terrain-following coordinate transformation:

$$\eta = \frac{H(z - z_s)}{(H - z_s)},$$

the surface height, H is the height of the top of the model domain. Here $H = \text{const}$,

$$G^{1/2} = 1 - \frac{z_s}{H}, \quad G^{13} = \frac{1}{G^{1/2}}\left(\frac{\eta}{H} - 1\right)\frac{\partial z_s}{\partial x}, \quad G^{23} = \frac{1}{G^{1/2}}\left(\frac{\eta}{H} - 1\right)\frac{\partial z_s}{\partial y}.$$

$$\frac{\partial U}{\partial t}+\frac{\partial P}{\partial x}=-\bar{U}\frac{\partial U}{\partial x}-\bar{V}\frac{\partial U}{\partial y}-\Delta G\frac{\partial P}{\partial \eta},$$
$$\frac{\partial V}{\partial t}+\frac{\partial P}{\partial y}=-\bar{U}\frac{\partial V}{\partial x}-\bar{V}\frac{\partial V}{\partial y}-\Delta G\frac{\partial P}{\partial \eta},$$
$$\frac{\partial W}{\partial t}+\frac{\partial P}{\partial \eta}=N\theta''-\Delta H\frac{\partial P}{\partial \eta}-\bar{U}\frac{\partial W}{\partial x}-\bar{V}\frac{\partial W}{\partial y},$$
$$\frac{\partial \theta''}{\partial t}=NW-\bar{U}\frac{\partial \theta''}{\partial x}-\bar{V}\frac{\partial \theta''}{\partial y},$$
$$\frac{1}{C_s^2}\frac{\partial P}{\partial t}+\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial \eta}=-\Delta H\frac{\partial W}{\partial \eta}-\Delta G\frac{\partial U}{\partial \eta}-\Delta G\frac{\partial V}{\partial \eta}.$$

Here $\Delta G \sim G^{13} \sim G^{23}$, a measure of mountain steepness; $\Delta H \sim \left(\frac{1}{G^{1/2}}-1\right)$, a measure of mountain height; $N^2=\frac{g}{\theta}\frac{\partial \theta}{\partial z}$, the squared Brunt-Vaisala frequency; $\bar{U}$ and $\bar{V}$ are constant basic state wind velocity components; and $\theta''=\frac{\rho'}{N}\frac{g\bar{\rho}}{\bar{\theta}}$.

ere $S^n = n(P^n, U^n, V^n, W^n, \theta''^{\,n})'$, and the matrices A, C, and B are as follows:

$$A = \begin{vmatrix} 1/C_2^s & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}, \quad C = \begin{vmatrix} 0 & ikx^* & iky^* & ikz^* & 0 \\ ikx^* & 0 & 0 & 0 & 0 \\ iky^* & 0 & 0 & 0 & 0 \\ ikz^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix},$$

$$B = 2\delta t \begin{vmatrix} 0 & -\Delta G ikz^{**} & -\Delta G ikz^{**} & -\Delta H ikz^* & 0 \\ -\Delta G ikz^{**} & 0 & 0 & 0 & 0 \\ -\Delta G ikz^{**} & 0 & 0 & 0 & 0 \\ -\Delta H ikz^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

ere, in addition, $ky^* = \dfrac{2\sin(ky\frac{\Delta y}{2})}{\Delta y}$.

In the three-dimensional case, it is not an easy task to obtain an analytical solution. tead of calculating the characteristic equation, the eigenvalue problem for the amplification trix is solved by using a procedure for matrices in Hessenberg form described by Wilkinson l Reinsch [7] (see also [8]). The input parameters are used as in [6]: $(\Delta x, \Delta y, \Delta\eta, C_s) =$ 00 m, 1200 m, 200 m, 340 m/s). At $\Delta t = 12$ s, we have found instability for any ΔG. ducing Δt to 2 s, the calculations have shown that, similar to the two-dimensional case sidered in [6], the necessary stability limitation on ΔG is as follows:

$$0 \le \Delta G \le \gamma < 1,$$

$$\alpha = \sum_{j=1}^{n} \phi_j(\mathrm{x})\alpha_j(t)$$

$$\mathrm{M}\dot{U} + \mathrm{N}(u)U = -\mathrm{C}\,P + \mathrm{F}U + \mathrm{K}U + f,$$

$$\mathrm{C}^T U = g,$$

$$\mathrm{M}_s\,\dot{\theta} + \mathrm{N}_s(u)\theta = \mathrm{K}\,s\theta + f_s$$

SPACE DISCRETIZATION

[

$$(\mathrm{M} - \frac{\Delta t}{2}\widetilde{\mathrm{K}}_n)U_{n+1} = (\mathrm{M} + \frac{\Delta t}{2}\widetilde{\mathrm{K}}_n)U_n + \Delta t[-\mathrm{C}\mathrm{P}_n + \mathrm{F}U_n + f_n - \mathrm{N}(u_n)U_n],$$

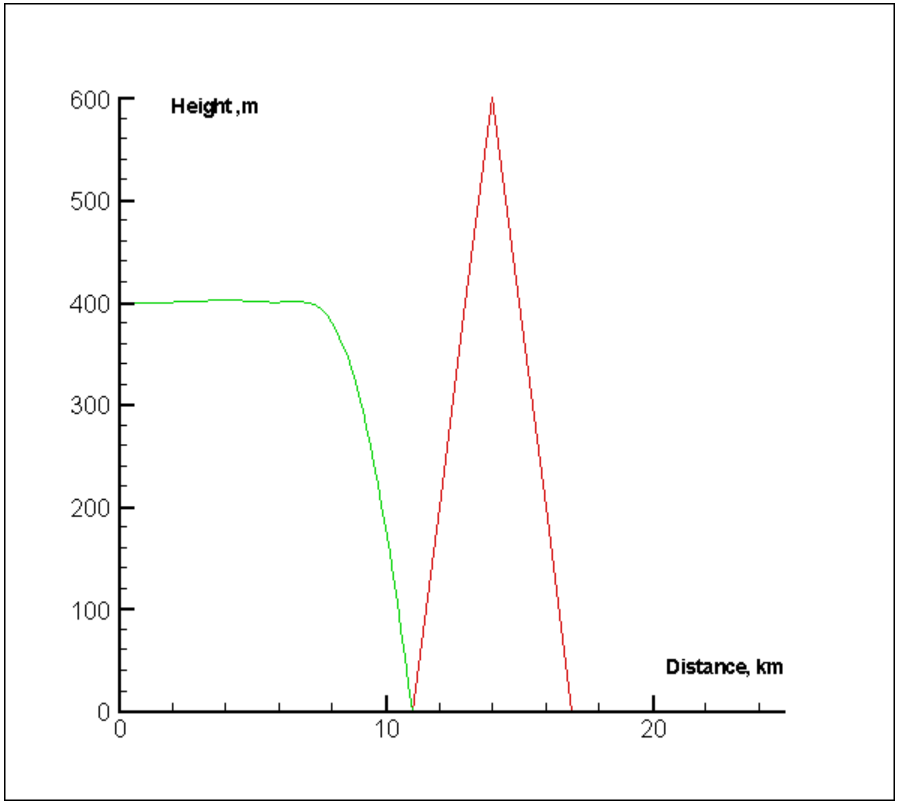
$$[\mathrm{M}_s - \frac{\Delta t}{2}(\widetilde{\mathrm{K}}_s)_n]\theta_{n+1} = [\mathrm{M}_s + \frac{\Delta t}{2}(\widetilde{\mathrm{K}}_s)_n - \Delta t\,\mathrm{N}_s\;\theta_n + \Delta t(f_s)_n$$

$$(\mathrm{C}^T\,\mathrm{M}^{-1}\mathrm{C})\,\mathrm{P}_n = \mathrm{C}^T\,\mathrm{M}^{-1}[f_n + \widetilde{\mathrm{K}}_n U_n + \mathrm{F}U_n - \mathrm{N}(u_n)U_n] + \mathrm{C}^T U_n - g_{n+1})/\Delta t$$

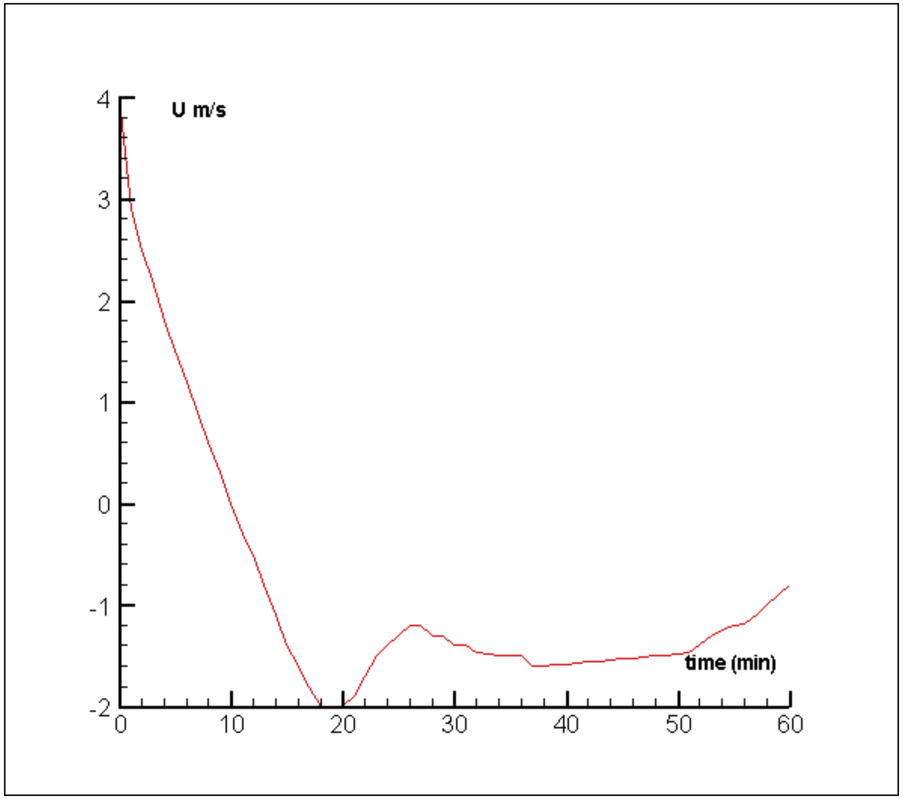
$$\widetilde{\mathrm{K}}_n = \mathrm{K}_n + \frac{\Delta t}{2}\mathrm{uu}$$

$$\mathrm{C}^T U_{n+1} = g_{n+1},$$

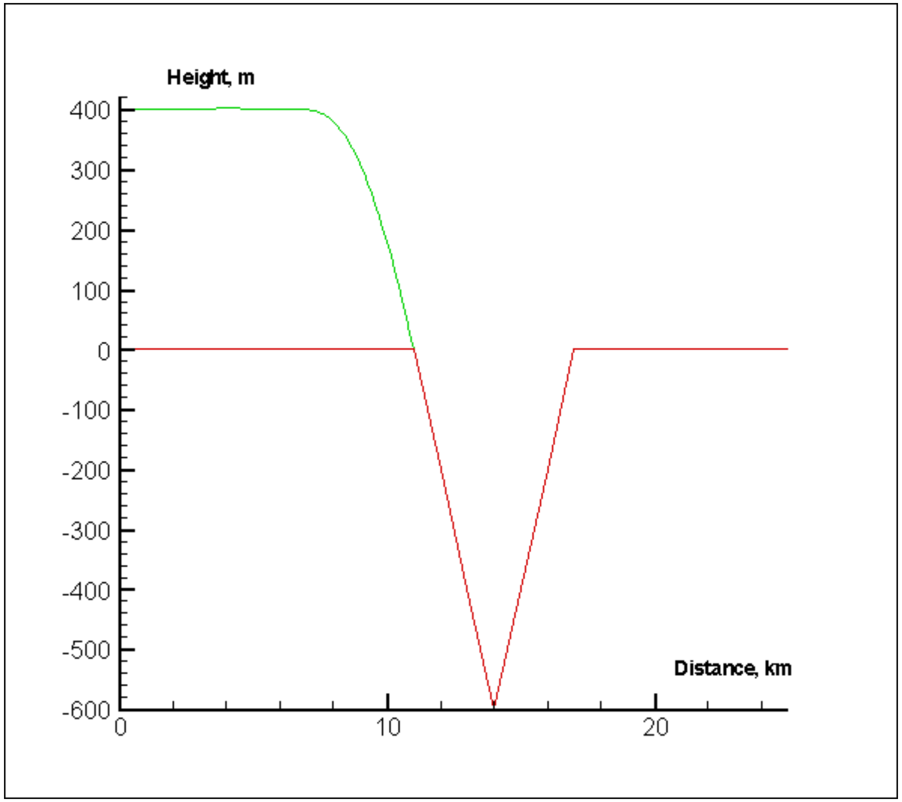
TIME DISCRETIZATION



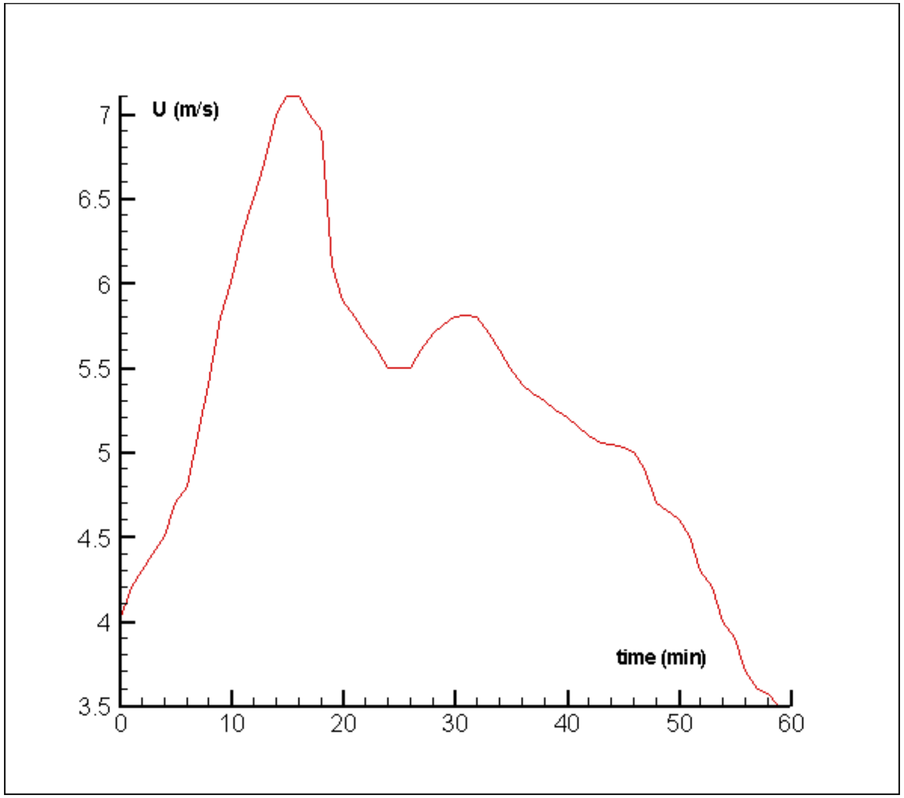
Location of the front as it meets with the obstacle : hill
Neutral stratification



Normal to the front velocity component: hill
Neutral stratification.



Location of the front as it meets with the obstacle: valley
Neutral stratification

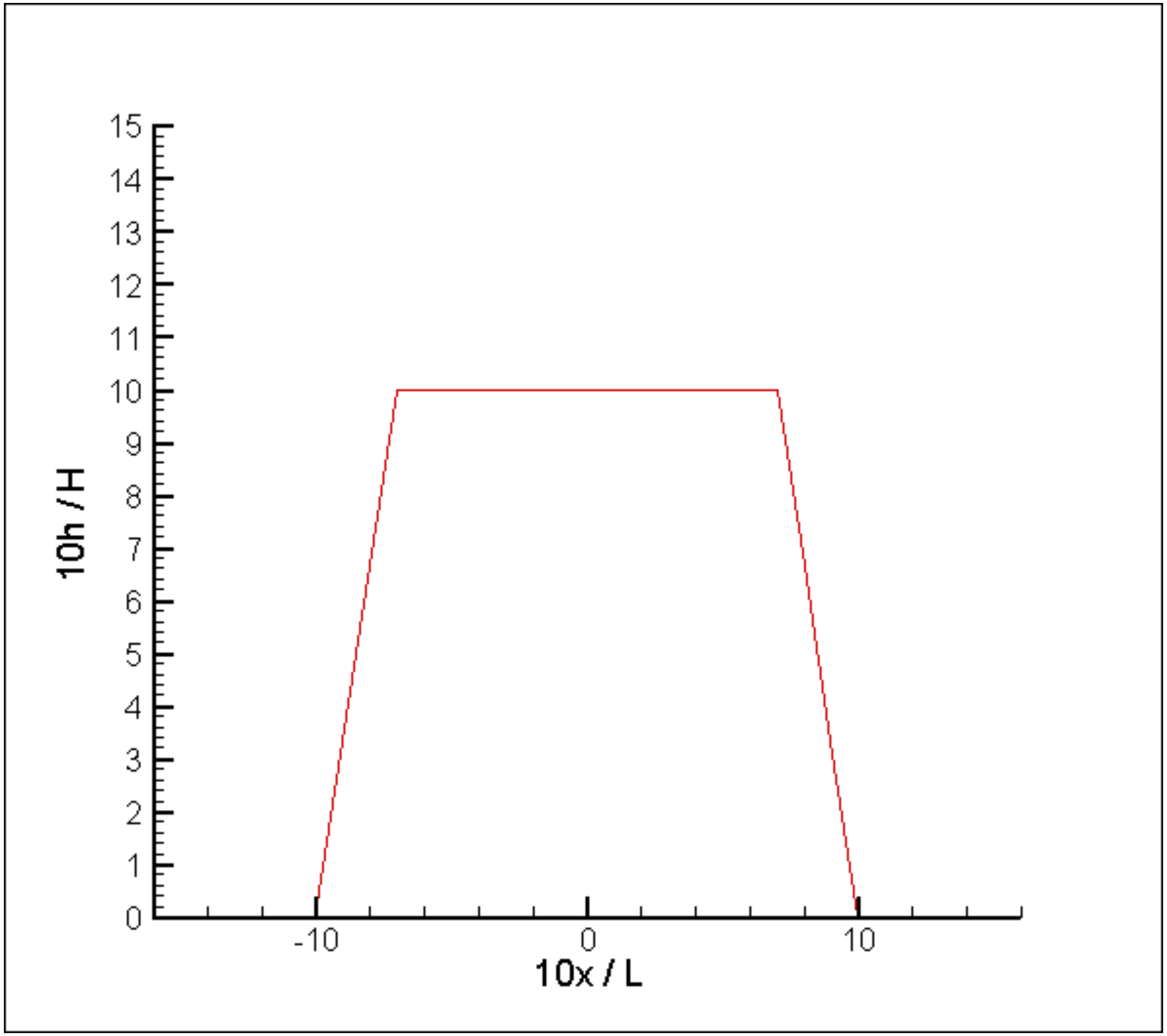


Normal to the front velocity component: valley

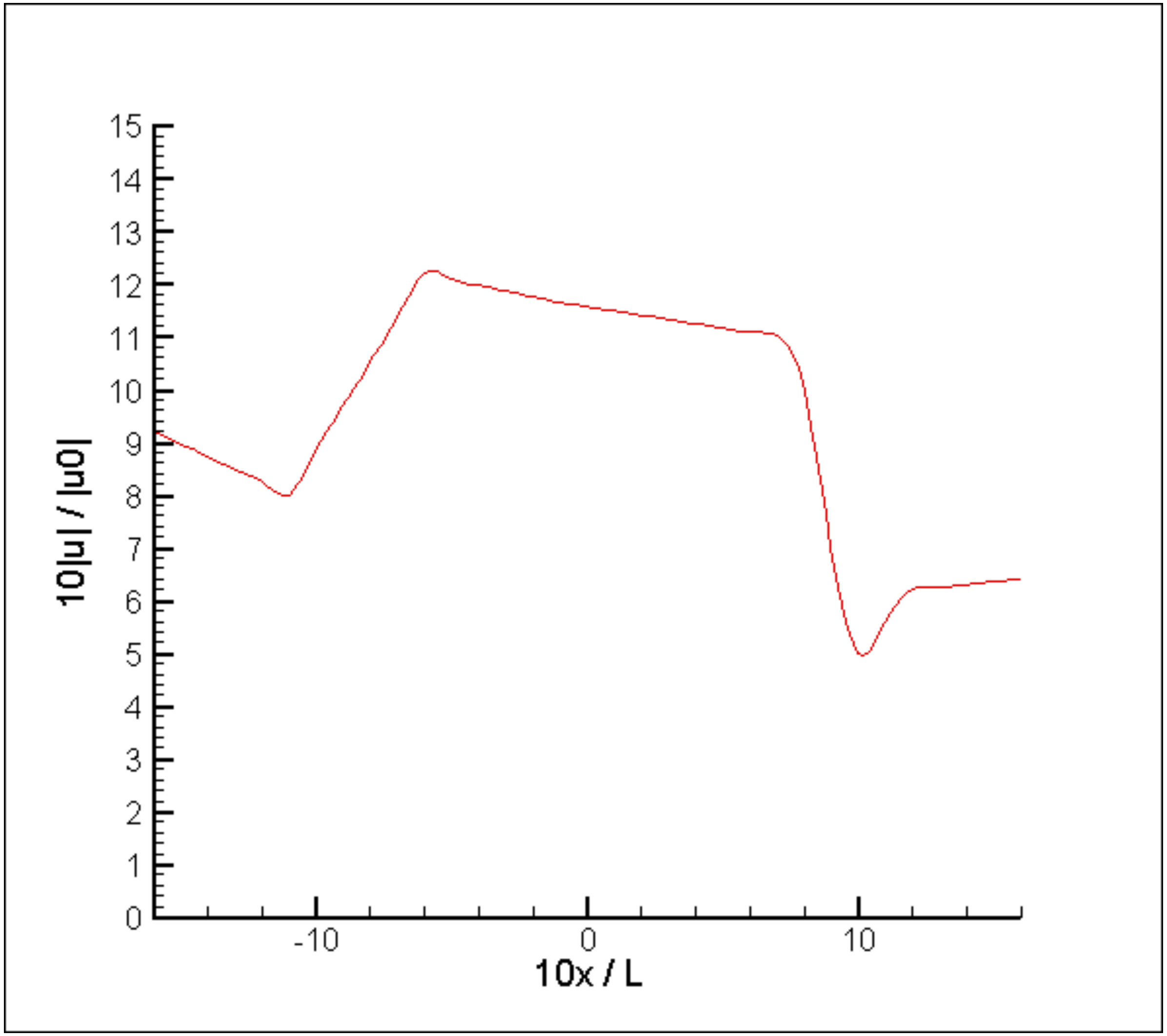
Neutral stratification.

Cold front propagation over orographic obstacles of various shapes and stratifications

OBSTACLE HEIGHT (m)	INITIAL FRONT HEIGHT (м)	STRATIFICATION (K / 100m)	WINDWARD SPEED (m /sec)	LEEWARD SPEED (m /sec)
0	400	0.0	4.5	4.5
0	400	0.35	5.1	5.1
600	400	0.0	4.4	3.7
600	400	0.35	4.9	2.7
600	100	0.35	3.0	0.0
600	700	0.35	7.5	4.5
- 600	400	0.0	4.5	3.9



Trapezoidal obstacle: topography
Neutral stratification



Trapezoidal obstacle: wind speed
Neutral stratification

CONCLUSIONS

The effects of steep orography on a gravity flow and the changes in the wind and front surface structures have been analyzed in the present study with a two-dimensional nonhydrostatic finite-element model.

In the cold front propagation simulations, important physical phenomena, e.g., the formation of an upwind-propagating hydraulic jump and near surface blocking, have been well reproduced by the model. The current is retarded on the windward side of the obstacle with a much greater reduction in the leeward speed. It has been also found that the retardation is very sensitive to the initial front height and stratification. This is in agreement with major findings obtained by a nonhydrostatic finite-difference model [1].

These preliminary results show that the finite-element model can be used for the simulation of atmospheric front propagation over steep orographic obstacles.

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