Semi-implicit semi-Lagrangian shallow-water model at the staggered reduced lat-lon grid

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Outline

- Motivation
- Model description
- Numerical experiments

Global semi-Lagrangian atmosphere model SL-AV

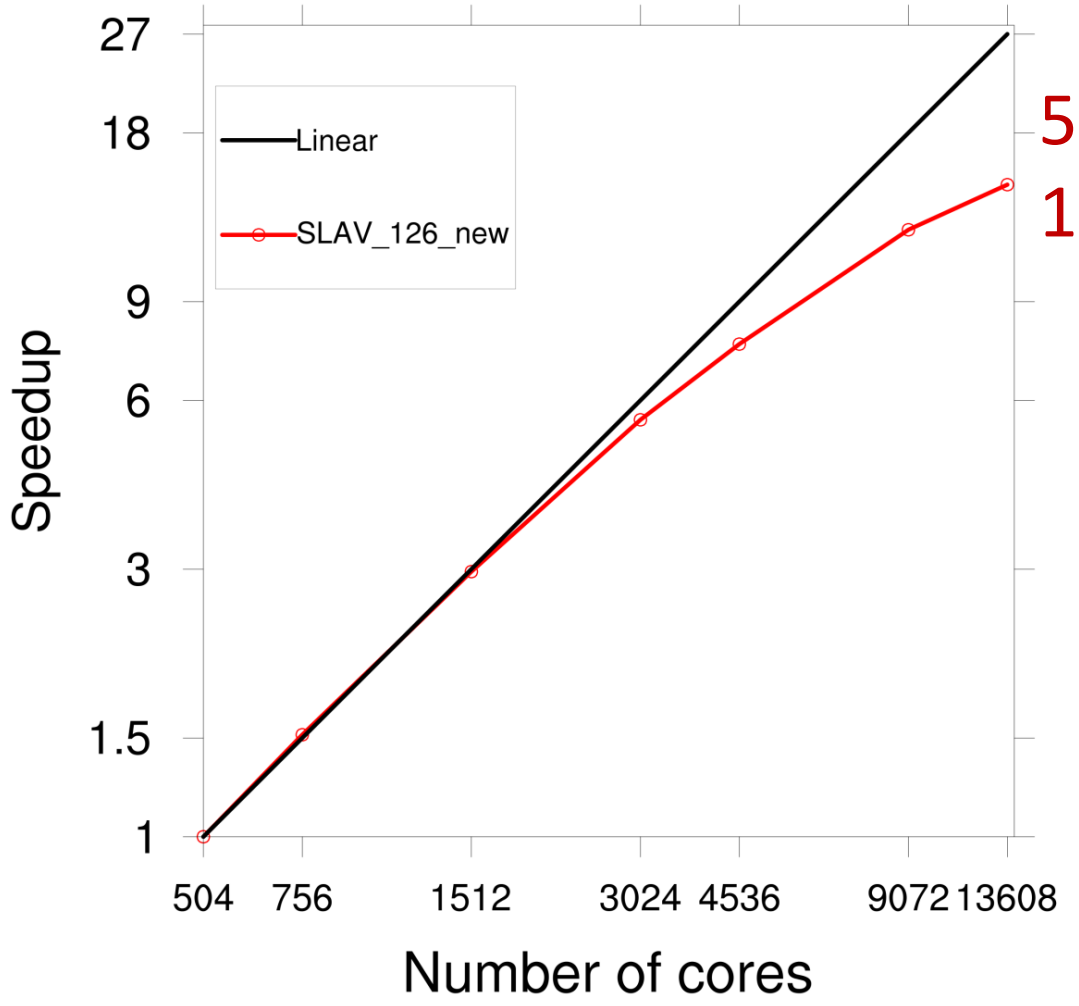
Current version: SL-AV20*

- SISL time-stepping
- **Regular** lat-lon grid, variable latitude resolution
- Vor-div formulation Z-grid
- **Fourier space in longitude**, FD in latitude
- Implicit 4^{th} order hyper-diffusion
- **1d MPI decomposition**, OpenMP

*Tolstykh M. et al. Vorticity-divergence semi-Lagrangian global atmospheric model SL-AV20: dynamical core //Geoscientific Model Development. – 2017. – Т. 10. – №. 5. – С. 1961-1983.

SL-AV20 strong scaling

- SL-AV20 model with the grid resolution 3024x1513x126
- Cray XC40-LC supercomputer



53% PE at 13608 cores

Global semi-Lagrangian atmosphere model SL-AV

Current version: SL-AV20*

- SISL time-stepping
- **Regular** lat-lon grid, variable latitude resolution
- Vor-div formulation Z-grid
- **Fourier space in longitude**, FD in latitude
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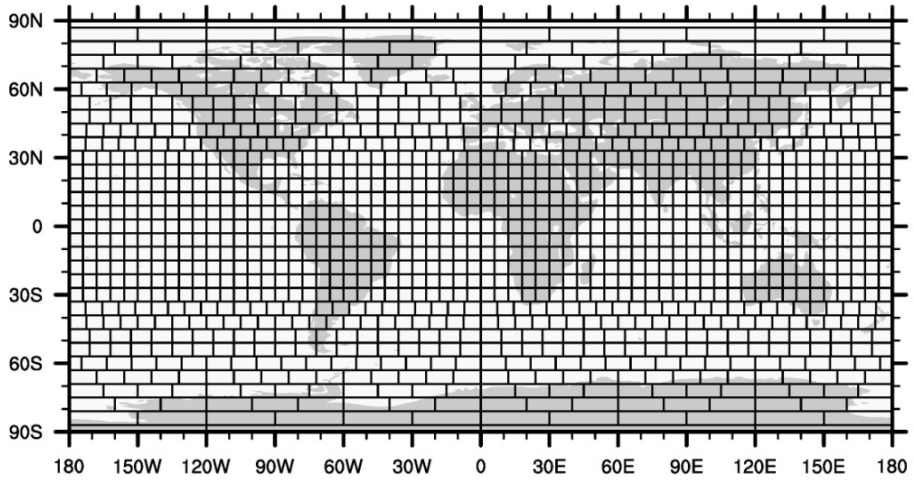
- **1d MPI decomposition**, OpenMP

New version: SL-AV10

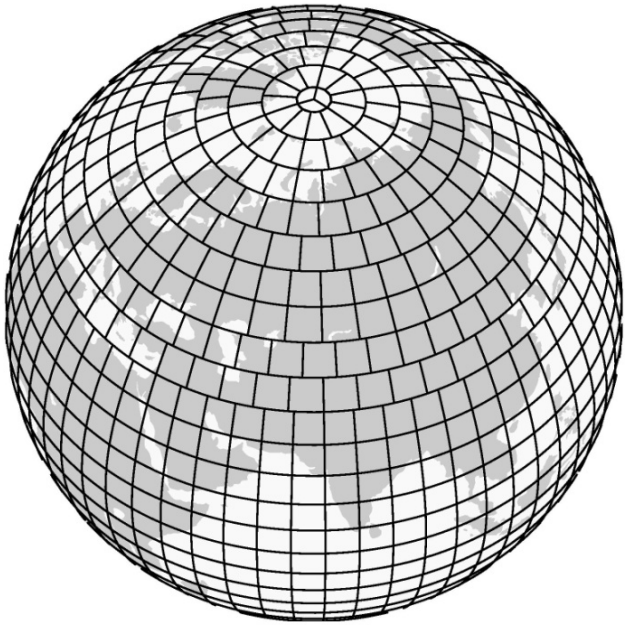
- SISL time-stepping
- **Reduced** lat-lon grid, variable latitude resolution
- Vor-div formulation Z-grid
- **Fully grid-point**
- Implicit 4^{th} order hyper-diffusion
- **2d MPI decomposition**, OpenMP

*Tolstykh M. et al. Vorticity-divergence semi-Lagrangian global atmospheric model SL-AV20: dynamical core //Geoscientific Model Development. – 2017. – Т. 10. – №. 5. – С. 1961-1983.

Reduced latitude-longitude grid



- Semi-structured (easy to code high-order methods)
- Easy to implement in the existing regular lat-lon model code
- Weaker pole singularity



Z-grid (vorticity-divergence, non-staggered) – works well, but need to solve two Poisson problems (Tolstykh et al. GMD, 2017)

Solution of the Poisson equations may be a bottleneck in the high-resolution version of the model

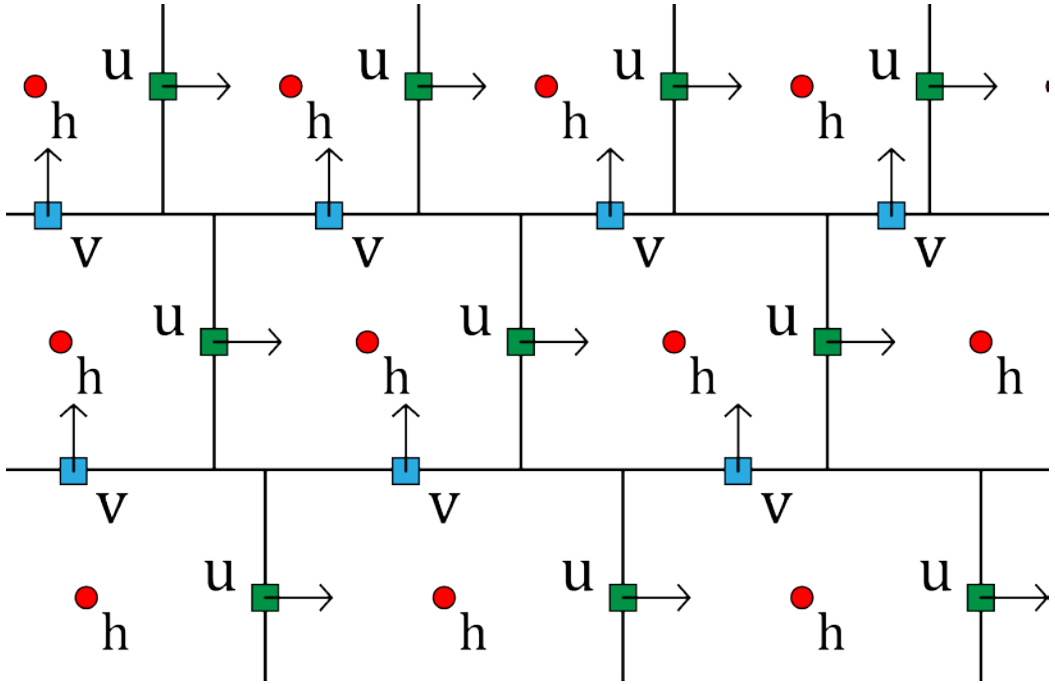
- Arakawa C-grid – is it possible?

Shallow-water model

$$\begin{cases} \dfrac{D\boldsymbol{V}}{Dt} = -2\boldsymbol{\Omega} \times \boldsymbol{V} - \nabla(\Phi + \Phi_s) \\ \dfrac{D(\Phi + \Phi_s)}{Dt} = -\Phi(\nabla \cdot \boldsymbol{V}) + \boldsymbol{V} \cdot \nabla\Phi_s \end{cases}$$

- Semi-implicit semi-Lagrangian time-stepping
- Grid-point approximation of differential operators with local stencils

Reduced lat-lon grid C-staggering



- h-points are located latitudes $\boldsymbol{\varphi_j}$ with lon-spacing $\boldsymbol{\Delta\lambda_j}$
- u-points half-step shifted in lon-direction from the h-points
- v-points are located at the latitudes $\boldsymbol{\varphi_{j+1/2}}$ with lon-spacing $\boldsymbol{\Delta\lambda_{j+1/2}}$

$$\mathrm{N}_{\lambda}^{j+1/2} = (N_{\lambda}^{j} + N_{\lambda}^{j+1})/2$$

Differential operators discretization
Zonal part

- Standart 2nd order formula similar to the regular C-grid case:

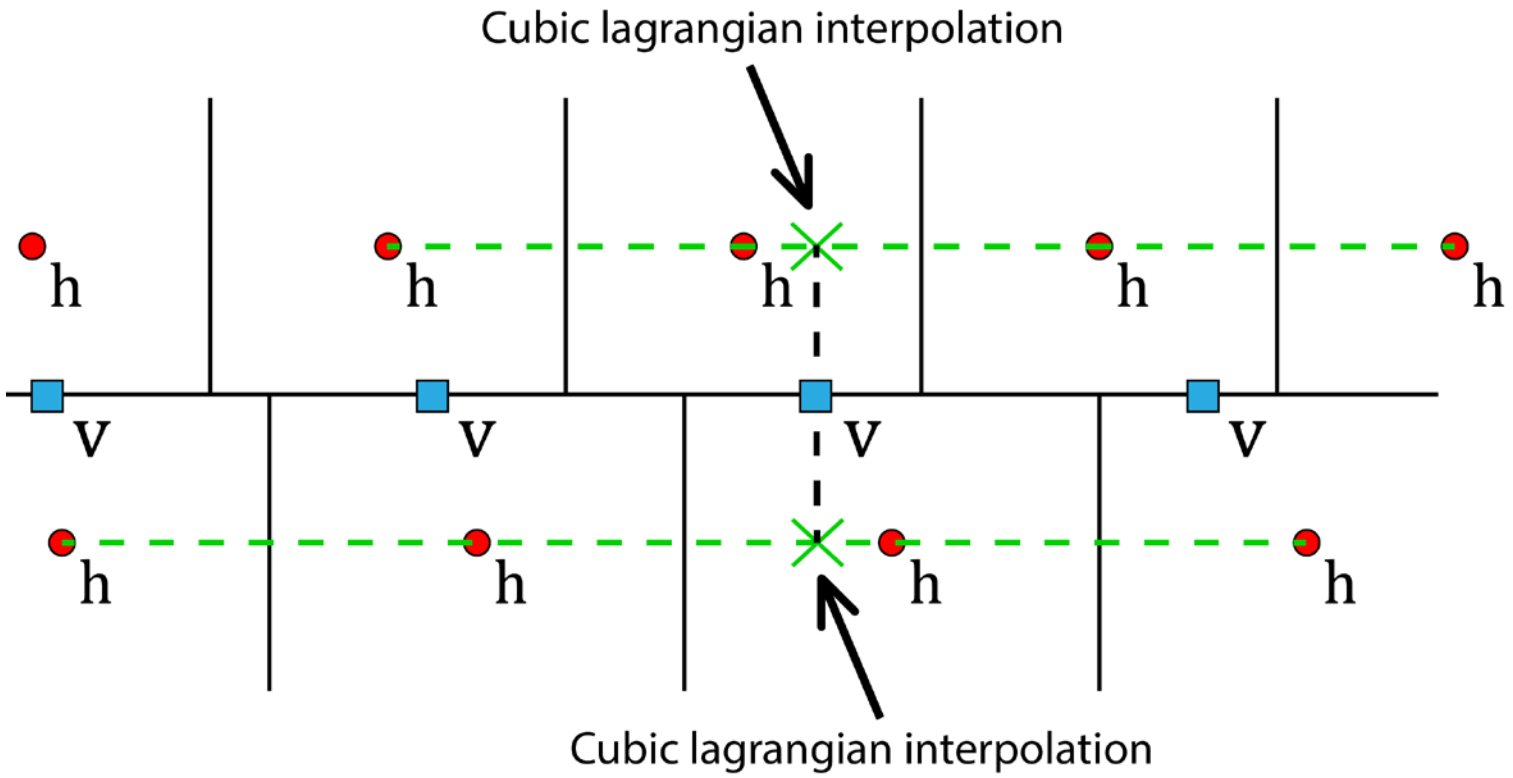
$$\left(\frac{1}{a\cos\varphi}\frac{\partial f}{\partial \lambda}\right)_{i+\frac{1}{2},j} \approx \frac{1}{a\cos\varphi_j}\frac{f_{i+1,j}-f_{i,j}}{\Delta\lambda_j}$$

Differential operators discretization
Meridional part

Gradient operator

$$\left(\frac{1}{a}\frac{\partial f}{\partial \varphi}\right)_{i,j+\frac{1}{2}} \approx \frac{1}{a}\frac{\tilde{f}_{i,j+1} - \tilde{f}_{i,j}}{\Delta\varphi_{j+\frac{1}{2}}}$$

$\tilde{f}_{i,j}, \tilde{f}_{i,j+1}$ - interpolated values

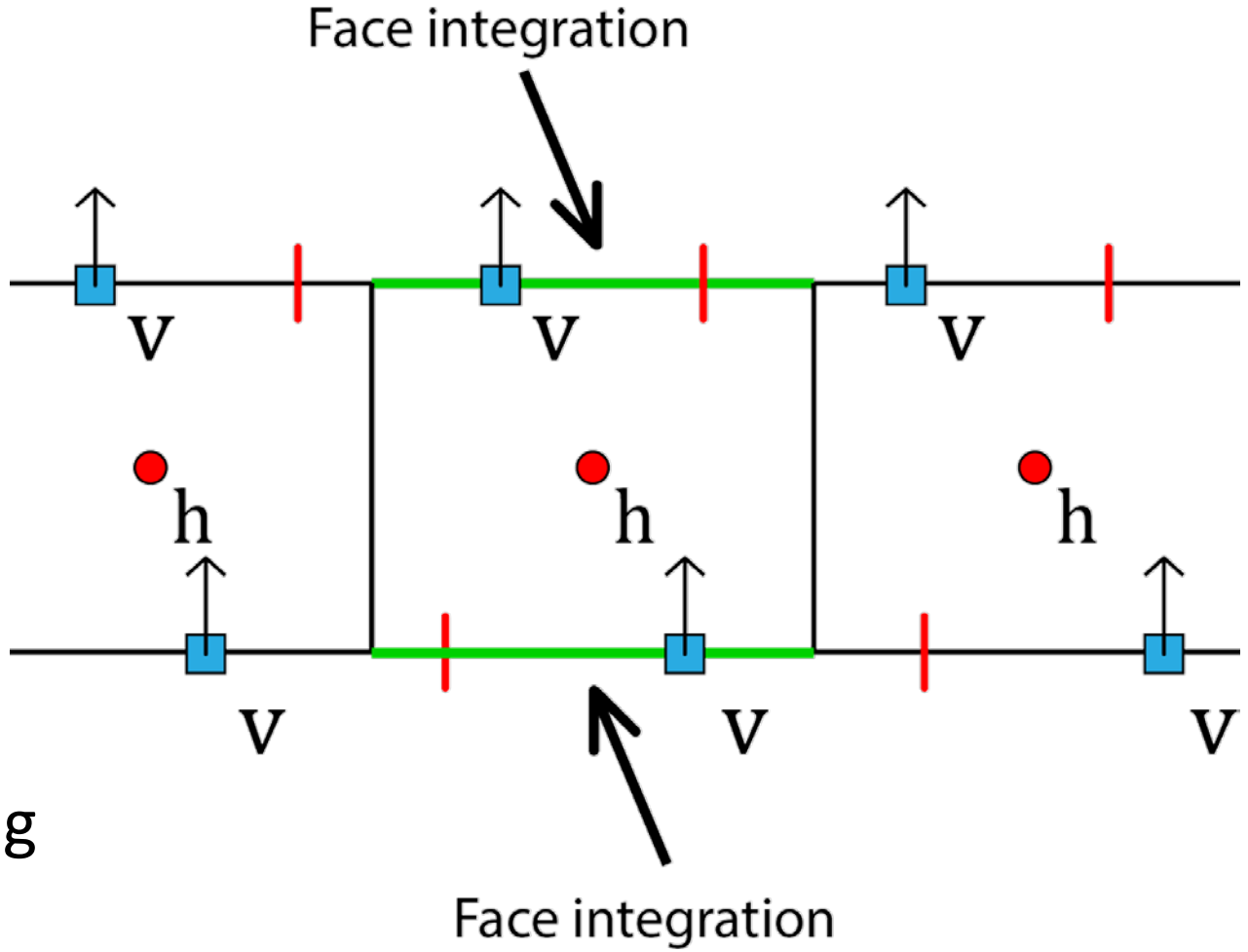


Differential operators discretization
Meridional part

Divergence operator

$$\mathrm{div}_{\varphi} = \frac{1}{\mathrm{a}\cos\varphi_j}\frac{F_{i,j+1/2} - F_{i,j-1/2}}{\Delta\varphi_j}$$

$F_{i,j\pm1/2}$ - flux, assuming linear subgrid distribution of $\boldsymbol{V}$ along cell-face



Coriolis force term

$$\begin{cases} \dfrac{Du}{Dt} - fv + \cdots = 0 \\ \dfrac{Dv}{Dt} + fu + \cdots = 0 \end{cases}$$

Bilinear interpolation is used in order to get v, u components at u, v grid points

Numerical experiments grids setup

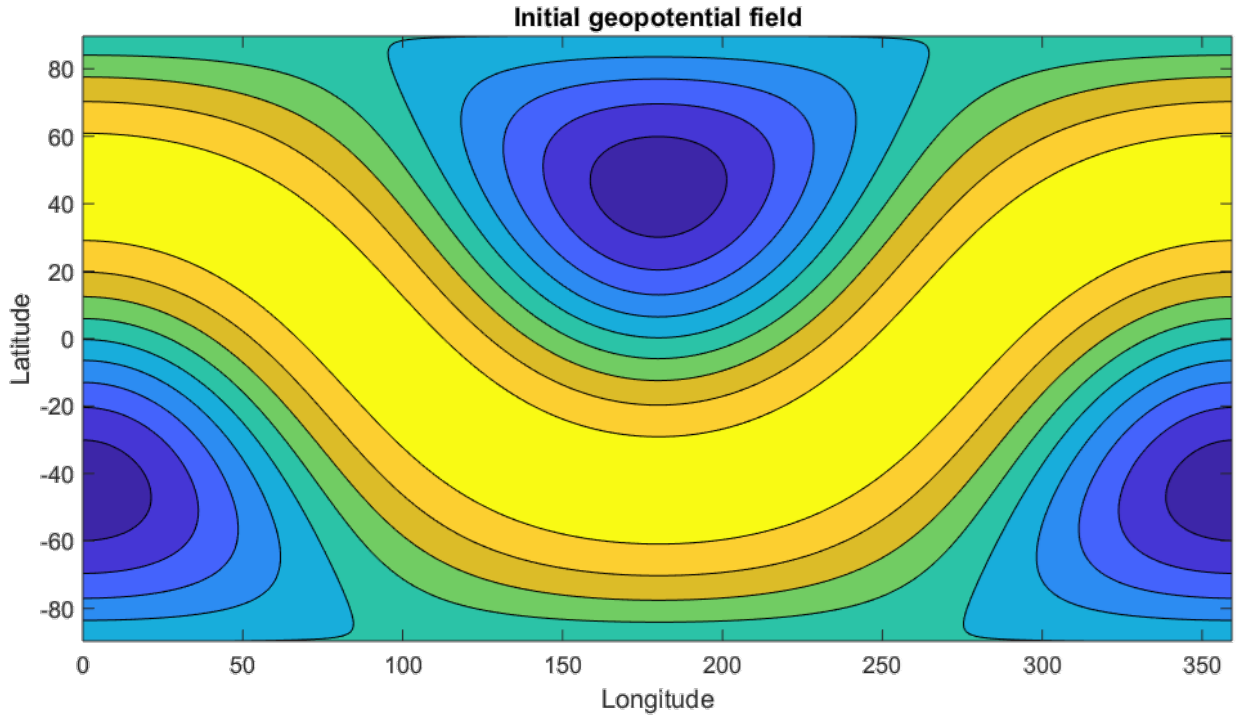
- Grids with initial resolution $N_\lambda \times N_\varphi$:

  256x128, 384x192, 512x256

- Linear reduction towards poles from N_λ to αN_λ, $\alpha \in \{1, 0.7, 0.5\}$

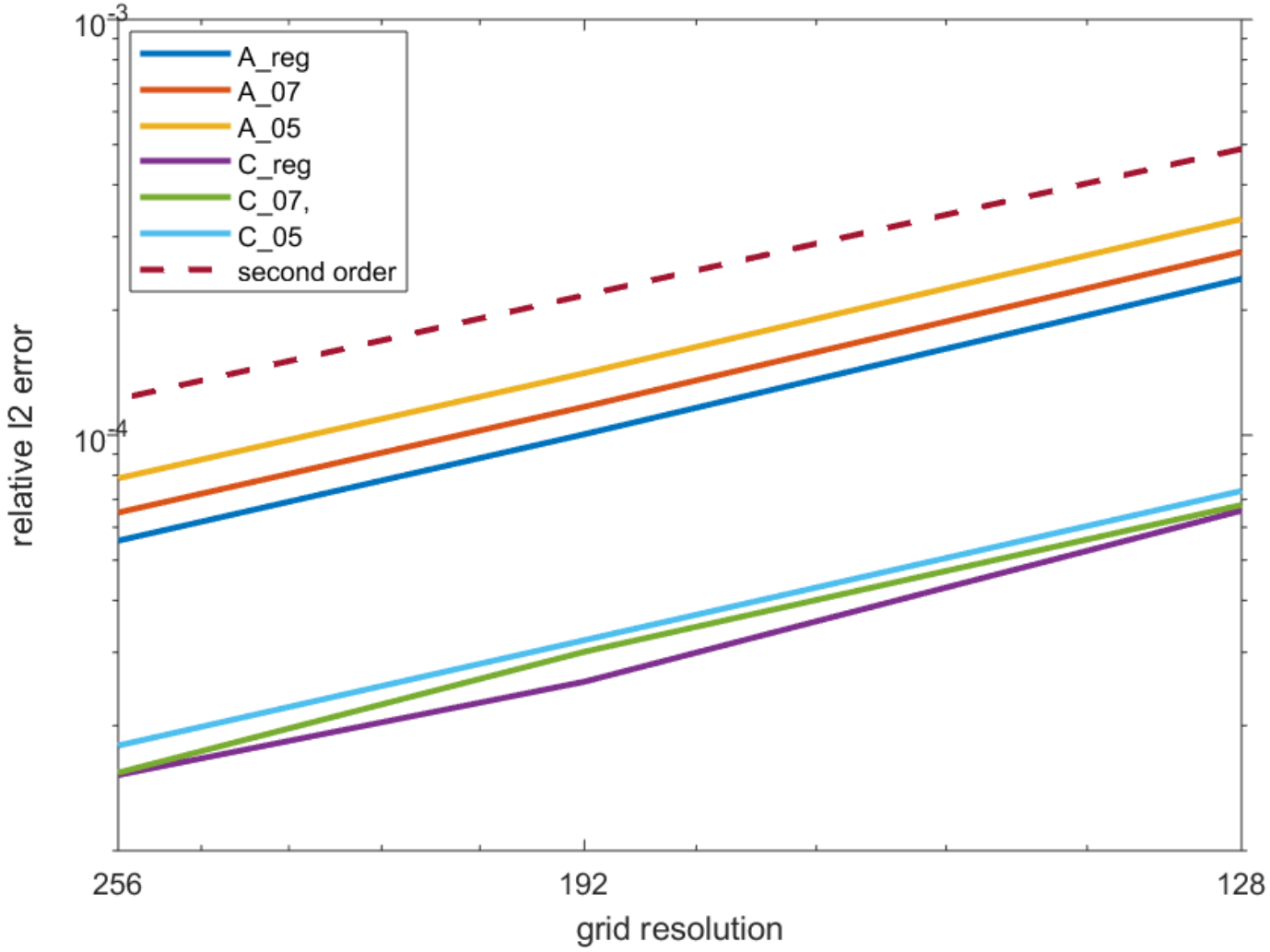
Steady state geostrophic flow test case

- large scale geostrophically balanced flow in the rotated spherical coordinate system
- Both the wind and height fields are expected to remain unchanged
- Compare geopotential field after 5 days of simulation using model with different resolutions, reduction rates and variables placement



Steady state geostrophic flow test case

- Second order error reduction for all cases
- Relative error norms of the model with staggered grid is about 4 times less



Relative l2 error of the geopotential field at day 5

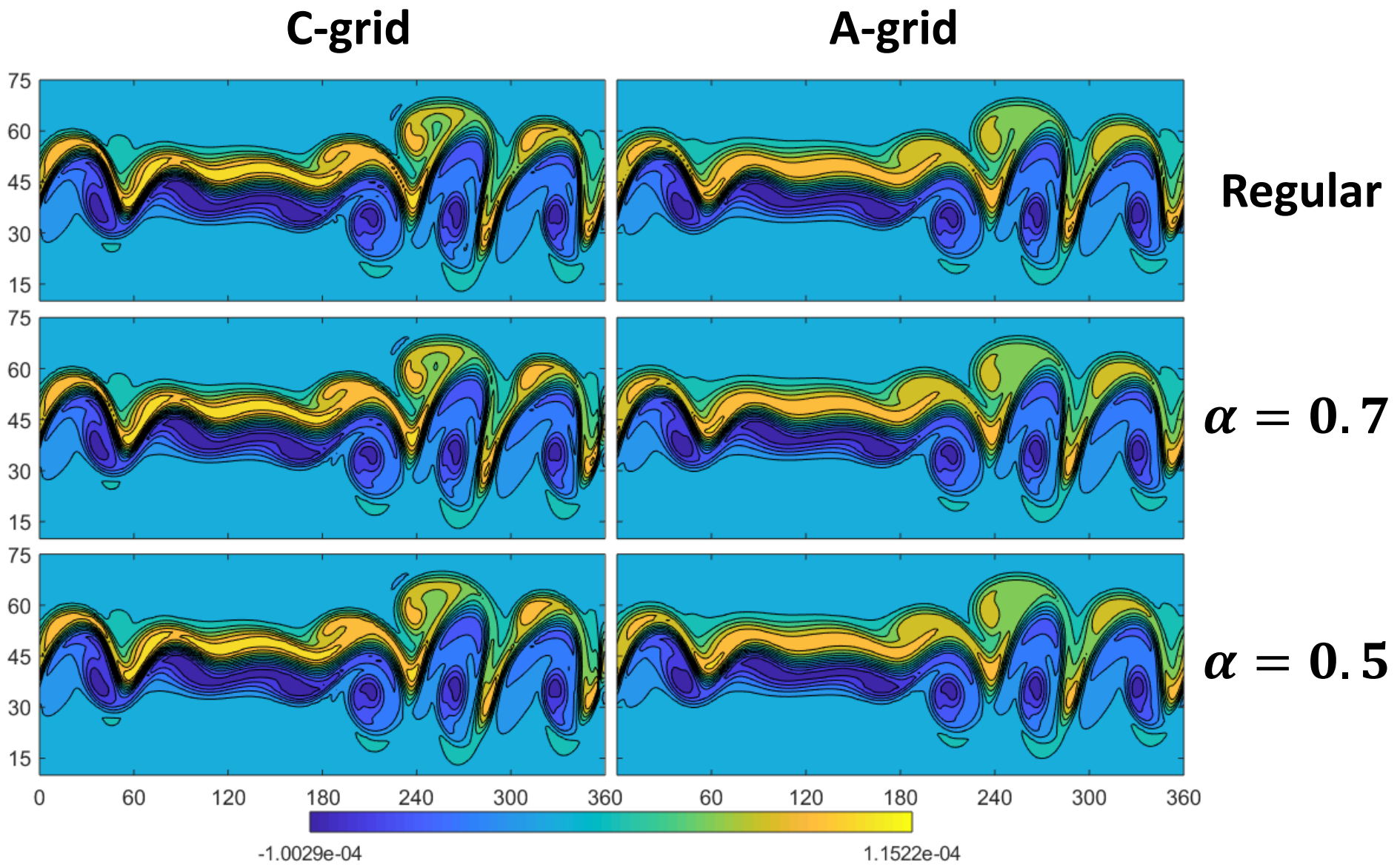
Barotropic instability test case

- balanced mid-latitude zonal flow with Gaussian perturbation in the height field

- Compare relative vorticity field after 6 days of simulation using model with different resolutions, reduction rates and variables placement

Barotropic instability test case

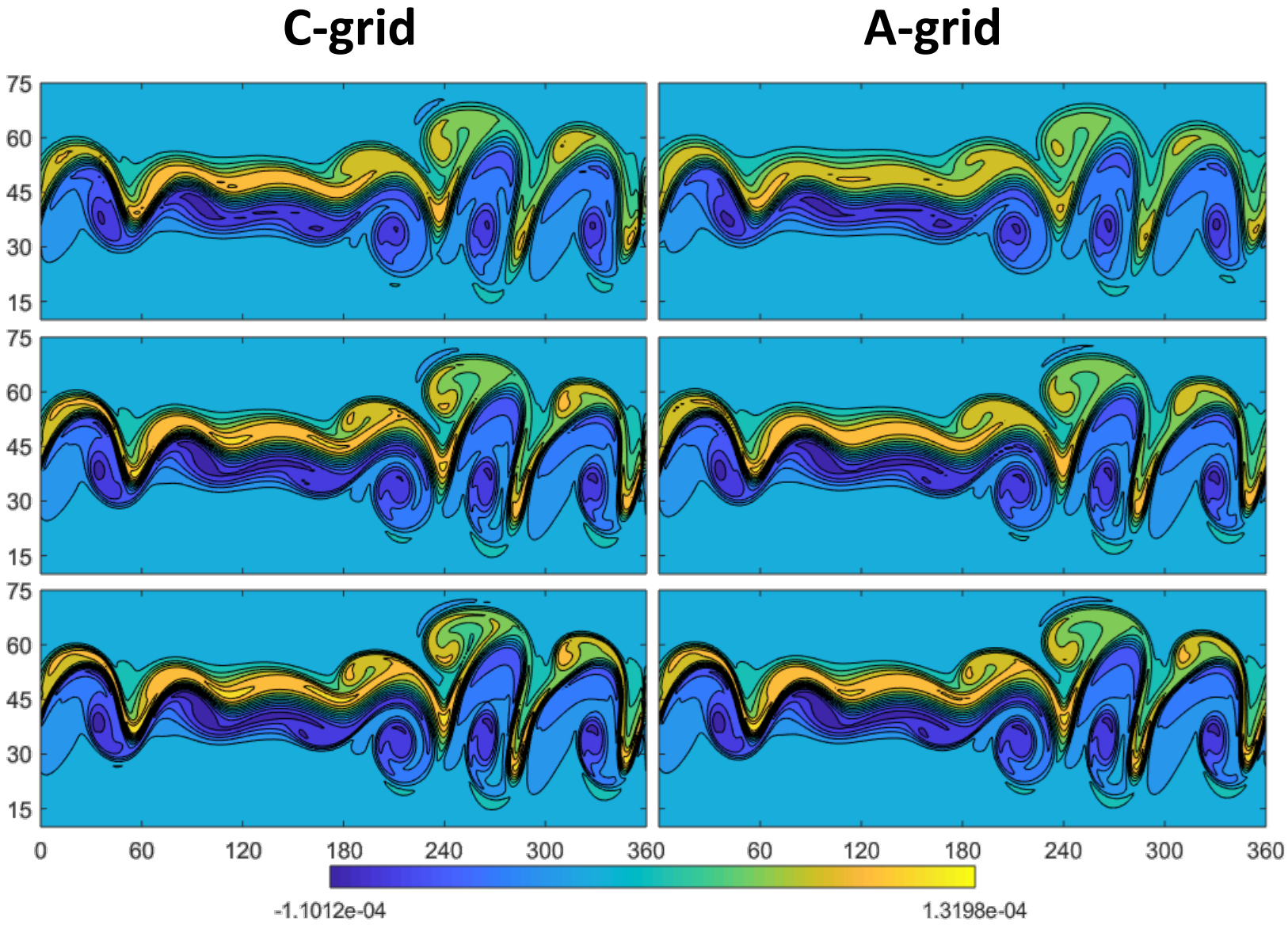
Initial grid resolution 256x128



Barotropic instability test case

Solution convergence

Reduction rate
$\alpha = 0.5$ in all cases



Summary

- Semi-implicit semi-Lagrangian shallow water model at the staggered reduced lat-lon grid is developed
- FD/FV 2nd order spatial discretization is proposed
- The use of variables staggering allows to obtain more accurate solutions in comparison to unstaggered case within “steady state geostrophic flow” and “barotropic instability” idealized test cases
- Staggered reduced lat-lon grid can be considered as a possible alternative to the Z-grid