РАСЧЕТ ПАРАМЕТРОВ ГРАВИТАЦИОННЫХ ТЕЧЕНИЙ В АТМОСФЕРЕ С ПОМОЩЬЮ МОДЕЛИ КОНЕЧНЫХ ЭЛЕМЕНТОВ

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Calculation of parameters of atmospheric gravity flows with a finite-element model

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In the present study, a numerical meteorological model is applied to the calculation of the pressure and speed of propagation of a cold front in the atmosphere over an artificial obstacle in the form of a hill, as well as along flat terrain.

The model is constructed on some basic principles developed by S. K. Godunov and E. I. Romenski :

Hyperbolicity

Fully divergent form of the governing equations

Consistency with the laws of thermodynamics

REFERENCES

Schultz, D. M., 2005 A Review of Cold Fronts with Prefrontal Troughs and Wind Shifts, Mon. Wea. Rev., 133 pp 2449-2472

Peshkov, I., Romenski, E., 2016 A hyperbolic model for viscous newtonian flows, *Continuum Mechanics and Thermodynamics,* **28** pp 85-104

Charba, J., 1974 Application of a gravity current model to analysis of squall-line gust fronts, *Mon. Wea. Rev.*, **102** pp 140-156

Yudin M S 2016 A numerical study of gravity waves in the atmosphere: smooth and steep orography effects IOP *Conference Series: Earth and Environmental Science* **48** DOI http://dx.doi.org/10.1088/1755-1315/48/1/012024

Marchuk G I 1974 *Numerical Solution of Atmospheric and Oceanic Problems* (Leningrad: Gidgometeoizdat) p 303

Gill A E 1982 Atmosphere-Ocean Dynamics (New York: Academic Press) p 319

Penenko V V, Aloyan A E 1985 *Models and Methods for Envronmental Protection Problems* (Novosibirsk: Nauka) p 256

Pielke, R.A., [Mesoscale Meteorological Modeling], Academic Press, Orlando, Fla (1984).

Bischoff-Gauss, I., Gross, G. and Wippermann, F., "Numerical studies on cold fronts. Part 2: Orographic effects on gravity flows," Meteorol. Atmos. Phys., 40, 159-169 (1989).

Schumann, U., "Influence of mesoscale orography on idealized cold fronts," J. Atmos. Sci., 44, 3423-3441 (1987).

Yudin, M.S. and Wilderotter, K., "Simulating atmospheric flows in the vicinity of a water basin," Computational Technologies, 11, 128-134 (2006).

Yudin, M.S., "Comparison of FDM and FEM models for a 2D gravity current in the atmosphere over a valley,"Bull. Novos. Comput. Center, 13, 95-101(2012).

Ikawa, M., "Comparison of some schemes for non-hydrostatic models with orography," J. Meteor. Soc. Japan, 66, 753-776 (1988).

Bischoff-Gauss, I., Gross, G., "Numerical studies on cold fronts. Part 1: Gravity flows in a neutral and stratified atmosphere," Meteorol. Atmos. Phys., 40, 150-158 (1989).

A general form of the basic equations .
With specially-chosen variables the system can be transformed to symmetric form .

$$\frac{\partial L_{q_i}}{\partial t} + \frac{\partial (u_k L)_{q_i}}{\partial x_k} = 0,$$

$$\frac{\partial L_{u_i}}{\partial t} + \frac{\partial[(u_k L)_{u_i} - r_{i\alpha} L_{r_{k\alpha}} - b_i L_{b_k} - d_i L_{d_k} + j_k L_{j_i} - \delta_{ik} j_\alpha L_{j_\alpha}]}{\partial x_k} = 0,$$

$$\frac{\partial L_{r_{il}}}{\partial t} + \frac{\partial[u_k L_{r_{il}} - u_i L_{r_{kl}}]}{\partial x_k} = 0,$$

$$\frac{\partial L_{d_i}}{\partial t} + \frac{\partial[u_k L_{d_i} - u_i L_{d_k} - e_{ikl} b_l]}{\partial x_k} = 0,$$

$$\frac{\partial L_{b_i}}{\partial t} + \frac{\partial[u_k L_{b_i} - u_i L_{b_k} + e_{ikl} d_l]}{\partial x_k} = 0,$$

$$\frac{\partial L_n}{\partial t} + \frac{\partial[u_k L_n + j_k]}{\partial x_k} = 0,$$

$$\frac{\partial L_{j_k}}{\partial t} + \frac{\partial[u_\alpha L_{j_\alpha} + n]}{\partial x_k} = 0$$

$$\frac{\partial}{\partial t}(q_i L_{q_i} + u_i L_{u_i} + r_{il} L_{r_{il}} + d_i L_{d_i} + b_i L_{b_i} + n L_n + j_k L_{j_k} - L) +$$

$$\frac{\partial}{\partial x_k}(u_k(q_i L_{q_i} + u_i L_{u_i} + r_{il} L_{r_{il}} + d_i L_{d_i} + b_i L_{b_i} + n L_n)) = 0$$

$$div\ L_r = 0,\ div\ L_b = 0,\ div\ L_d = 0,\ rot\ L_j = 0$$

Relation between Potential Temperature and Entropy

Air follows the ideal gas laws quite closely, and these are sufficiently accurate for most purposes.

For an ideal gas *cp is independent of pressure and temperature, so*

$$\eta = c_p \ln \theta + const.$$

(Adrian E. Gill Atmosphere-Ocean Dynamics *1982 Academic Press)*

$$\frac{dU}{dt} + \frac{\partial P}{\partial x} = f_1(V - V_g) - f_2 W + R_u,$$

$$\frac{dV}{dt} + \frac{\partial P}{\partial y} = -f_1(U - U_g) + R_\upsilon,$$

$$\frac{dW}{dt} + \frac{\partial P}{\partial z} + \frac{gP}{C_s^2} = f_2 U + g\,\frac{G^{1/2}\,\overline{\rho}\theta^{/}}{\theta} + R_\varpi$$

$$\frac{d\theta}{dt} = R_\theta,$$

$$\frac{ds}{dt} = R_s,$$

$$\frac{1}{C_s^2}\frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = \frac{\partial}{\partial t}\left(\frac{\overline{\rho}\theta^{/}}{\theta}\right)$$

$$U = \overline{\rho}u, V = \overline{\rho}\upsilon, P = \overline{\rho}p^{/},\ W = \overline{\rho}\omega$$

$$\delta\tau U + \frac{\partial}{\partial x}P + \frac{\partial}{\partial \xi}(G^{13}\,P) = -ADVU$$

$$\delta\tau V + \frac{\partial}{\partial y}P + \frac{\partial}{\partial \xi}(G^{13}\,P) = -ADVV$$

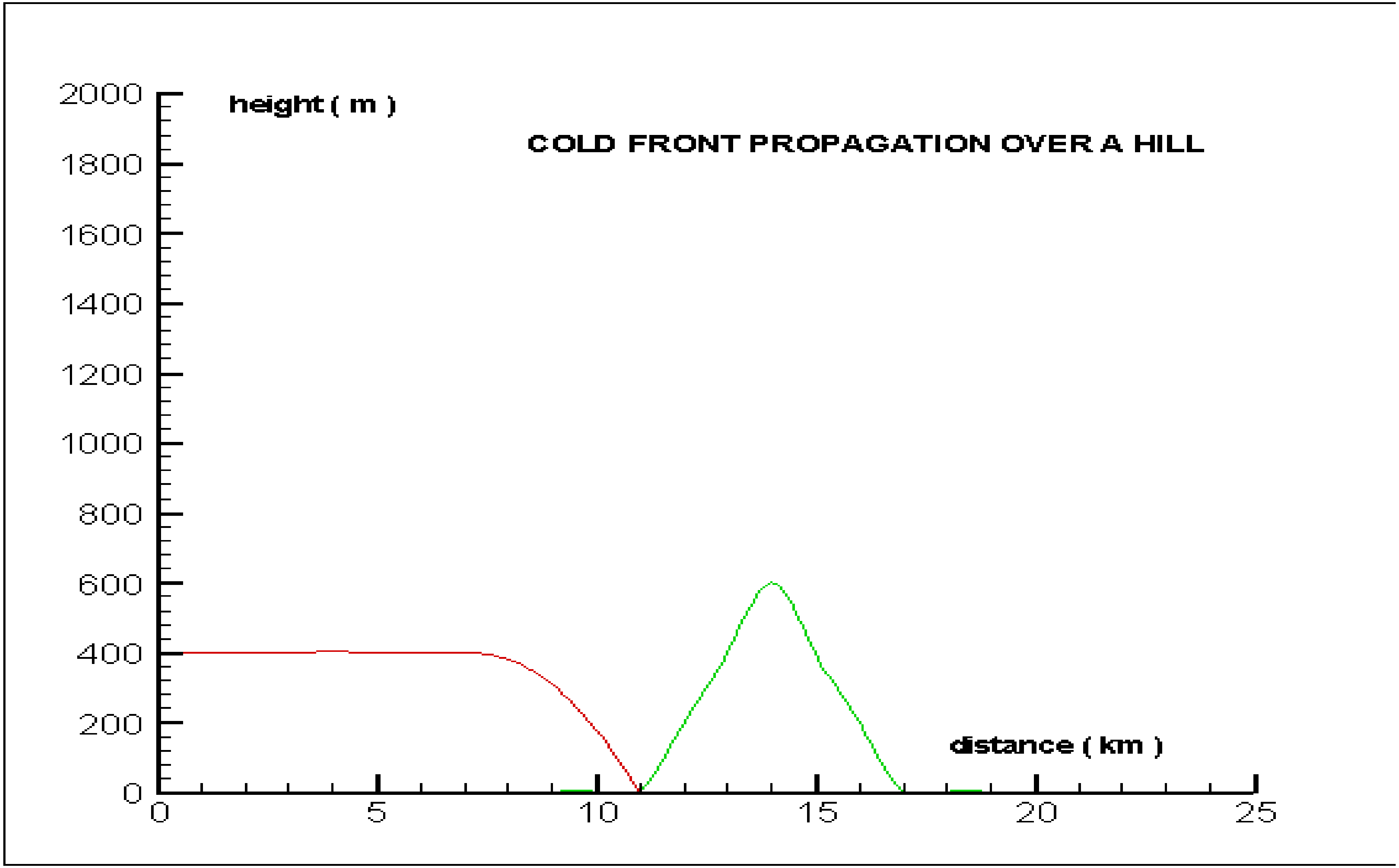
$$\delta\tau W + \frac{1}{G^{1/2}}\frac{\partial P^{\tau\beta}}{\partial \xi} + \frac{g P^{\tau\beta}}{Cs^2} = BUOY - ADVW$$

$$\frac{1}{Cs^2}\,\delta\tau P + \frac{\partial}{\partial x}\overline{U}^{\tau\gamma} + \frac{\partial}{\partial y}\overline{V}^{\tau\gamma} + \frac{\partial}{\partial \xi}(G^{13}\,\overline{U}^{\tau\gamma}) + \frac{\partial}{\partial \xi}(G^{13}\,\overline{V}^{\tau\gamma}) + \frac{1}{G^{1/2}}\frac{\partial \overline{W}^{\tau\beta}}{\partial \xi} = PFT$$

Changes in surface ozone concentration after atmospheric front propagation (372 fronts,1989-1993, Tomsk)

	Front type	Decrease %	Increase %	No change %
	Cold	70	24	6
	Warm	43	53	4
	Occlusion	35	48	17
	Surface cold	47	37	16
	Upper warm	12	56	32
	All types	49	40	11

(Belan B.D., Ozone in the troposphere., IAO SB RAS,Tomsk,2010.-488 pp.)



Cold front propagation over a hill. Stable stratification.

$$v_F = k(gh_{IFH}\Delta\theta / \theta)^{0.5}$$

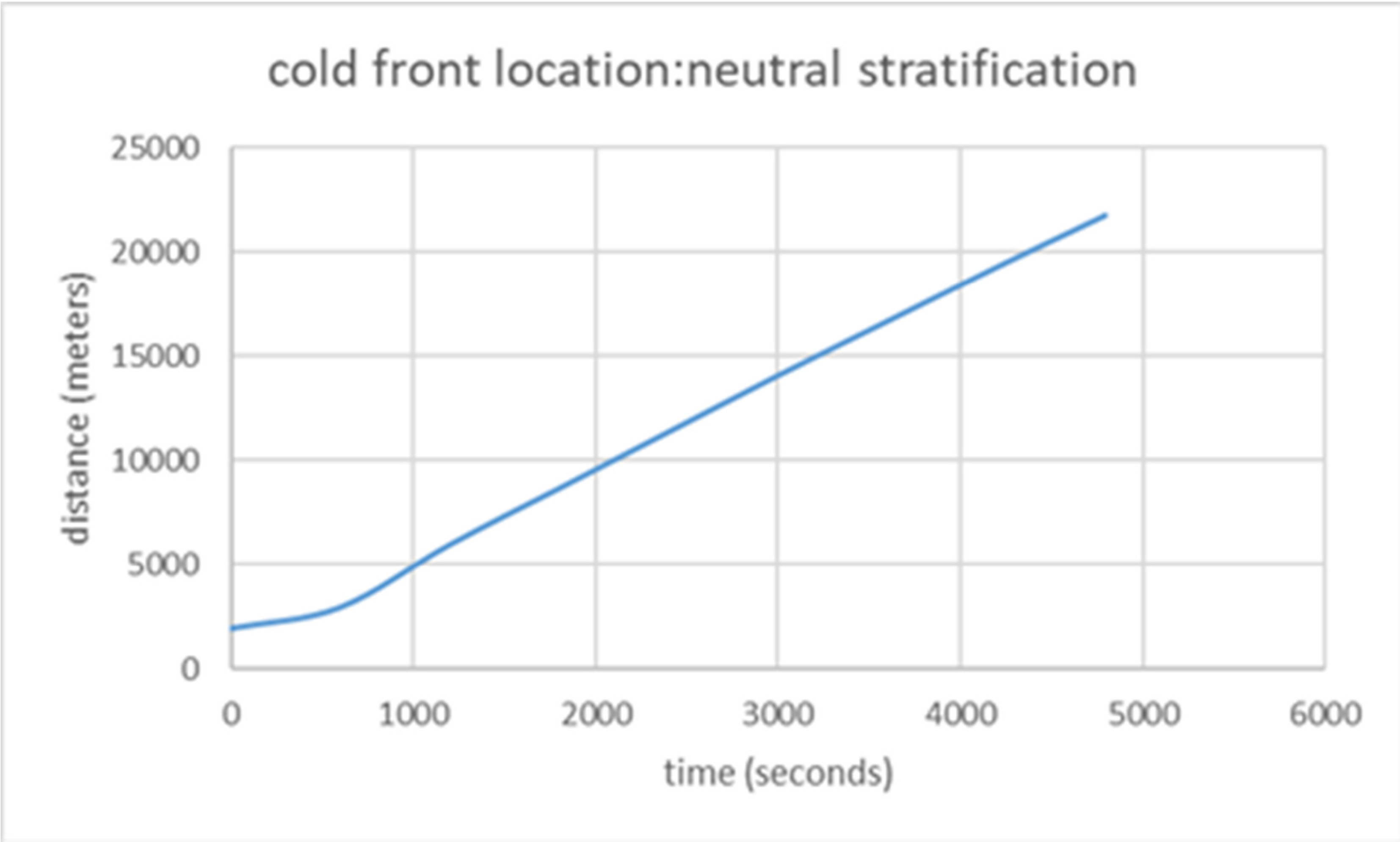
$$k = 0.81$$

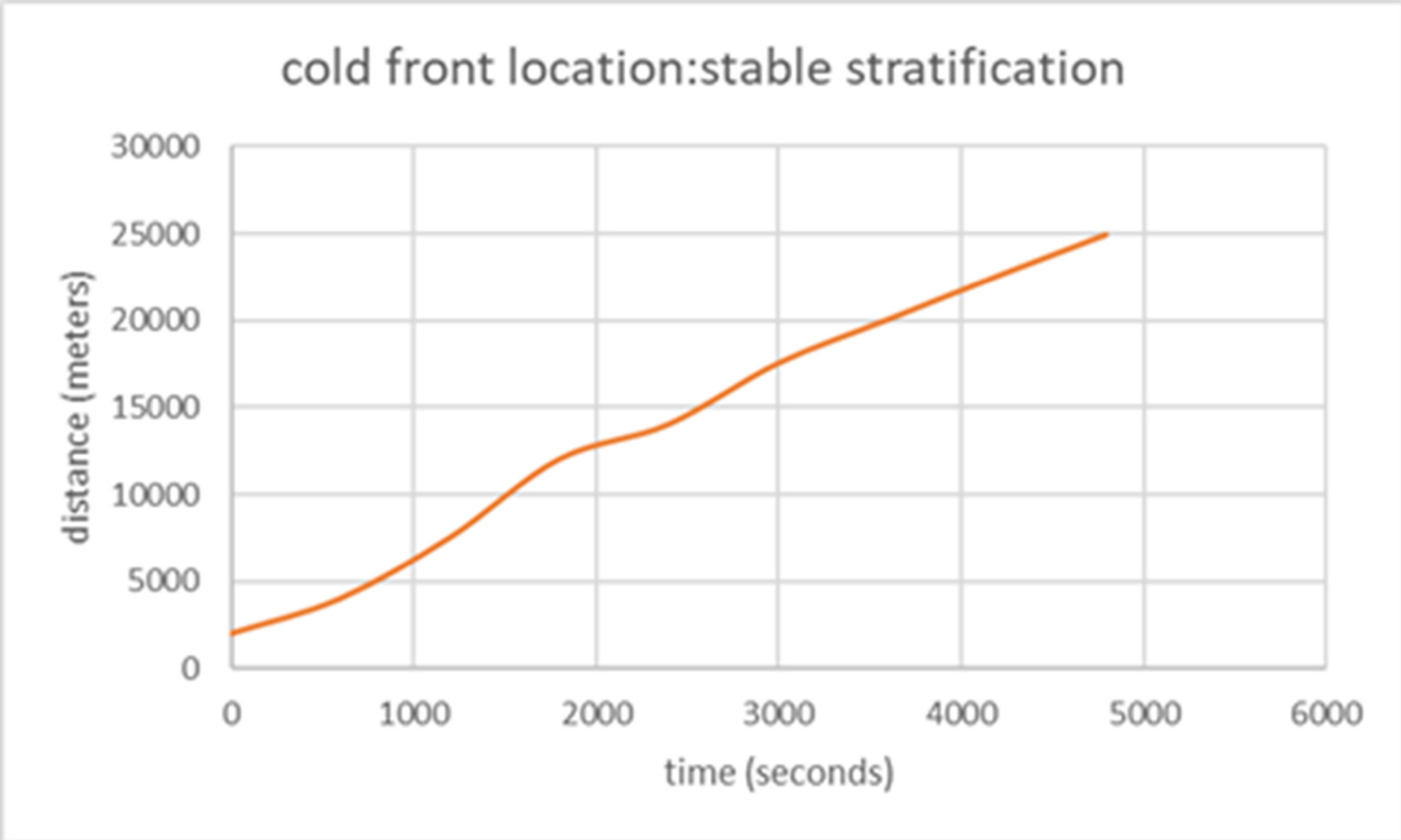
$$h_{IFH} = 400m$$

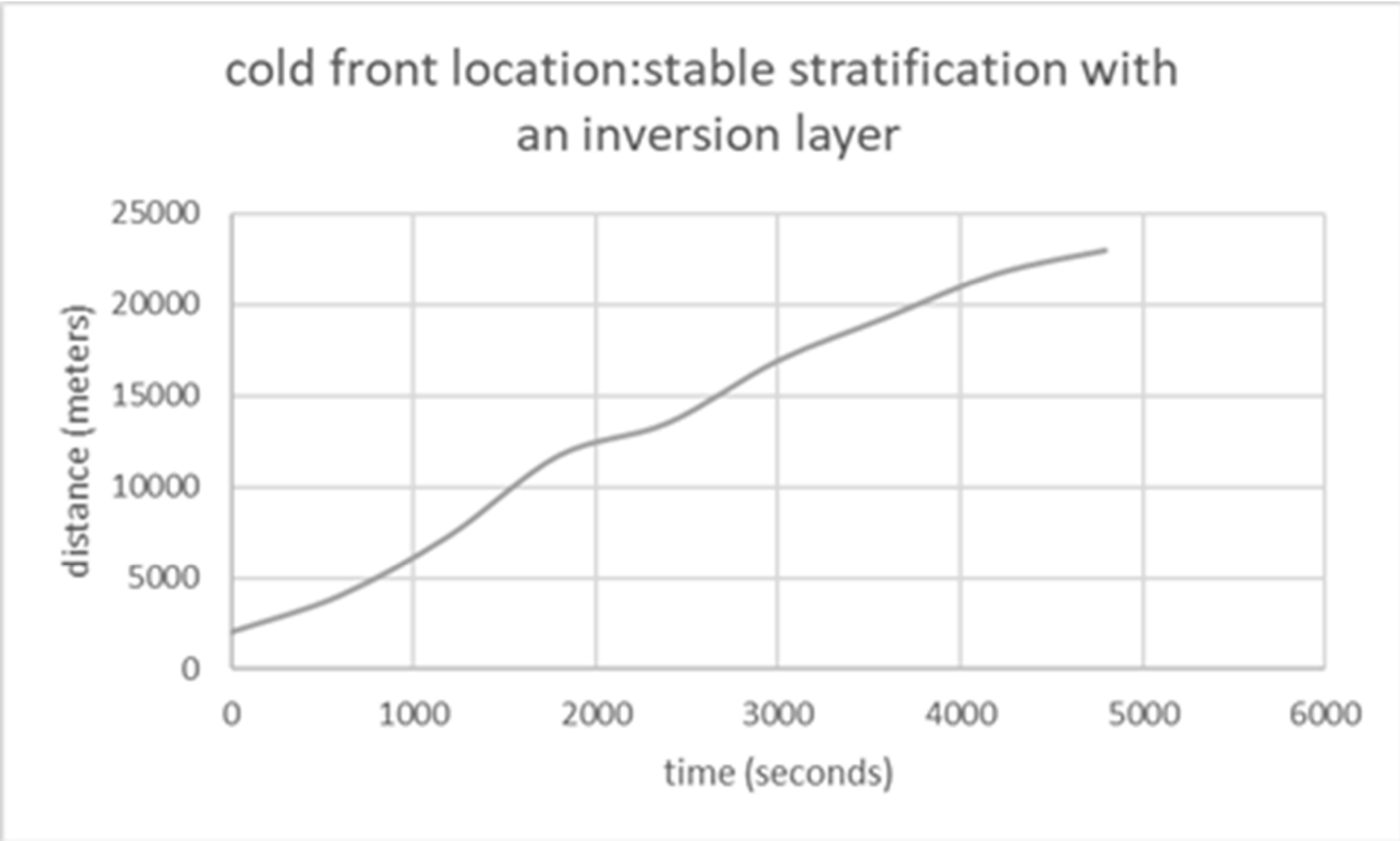
$$\theta = 288K$$

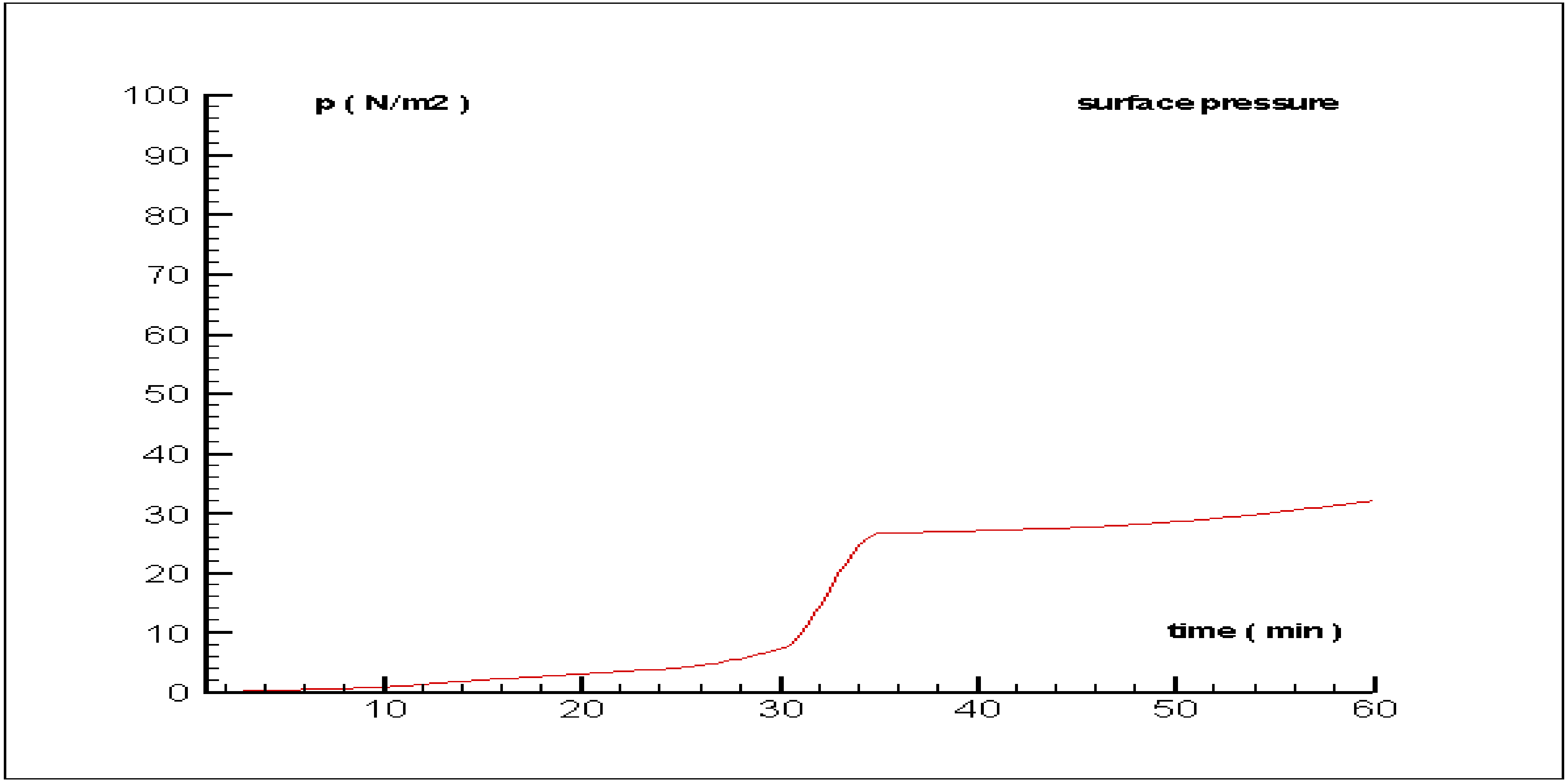
$$\Delta\theta = 2K$$

$$v_F = 4.2m / s$$

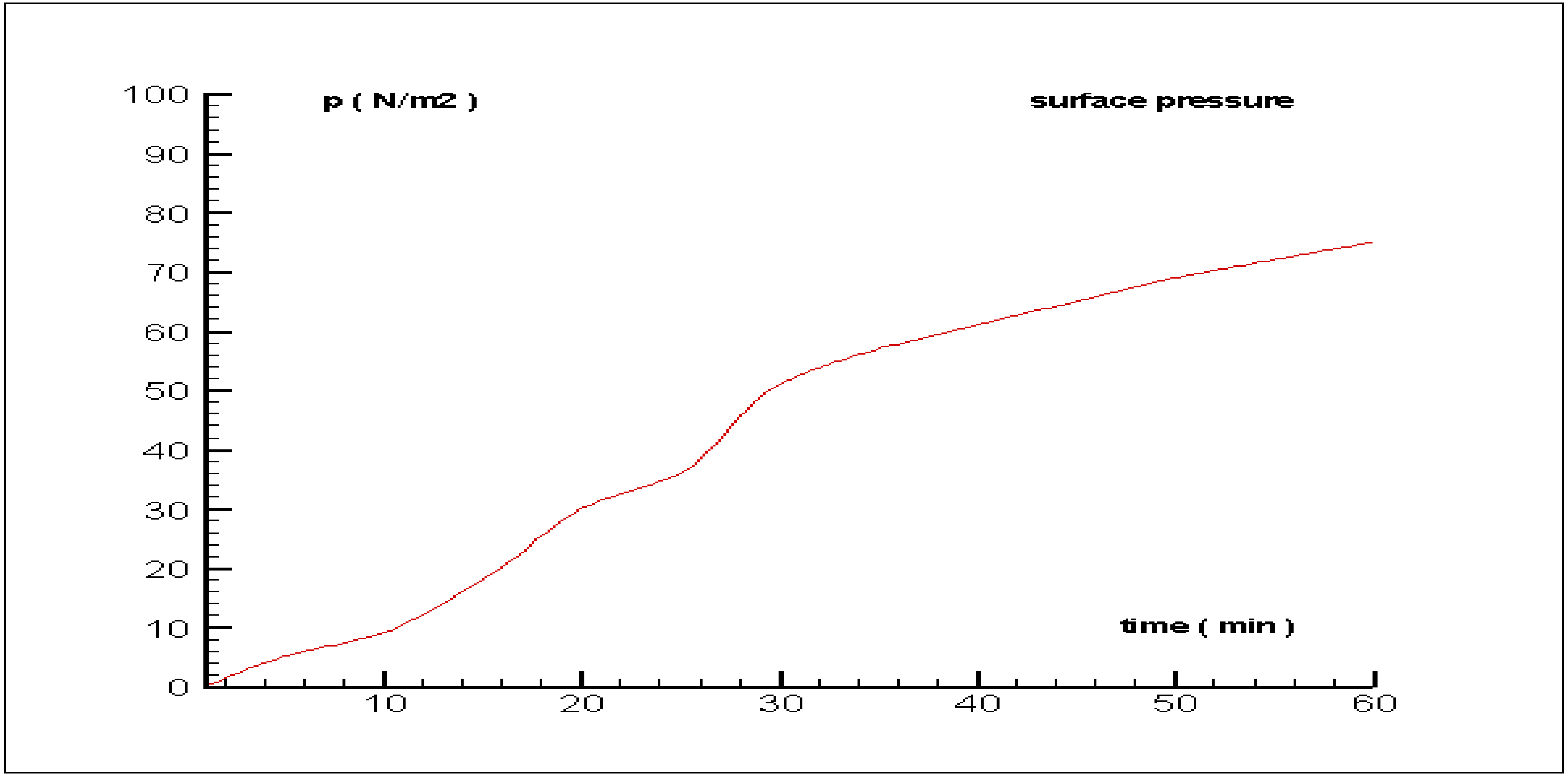

cold front location:neutral stratification
25000
20000
15000
10000
5000
0
distance (meters)
0
1000
2000
3000
4000
5000
6000
time (seconds)


cold front location:stable stratification
30000
25000
20000
15000
10000
5000
0
distance (meters)
0
1000
2000
3000
4000
5000
6000
time (seconds)

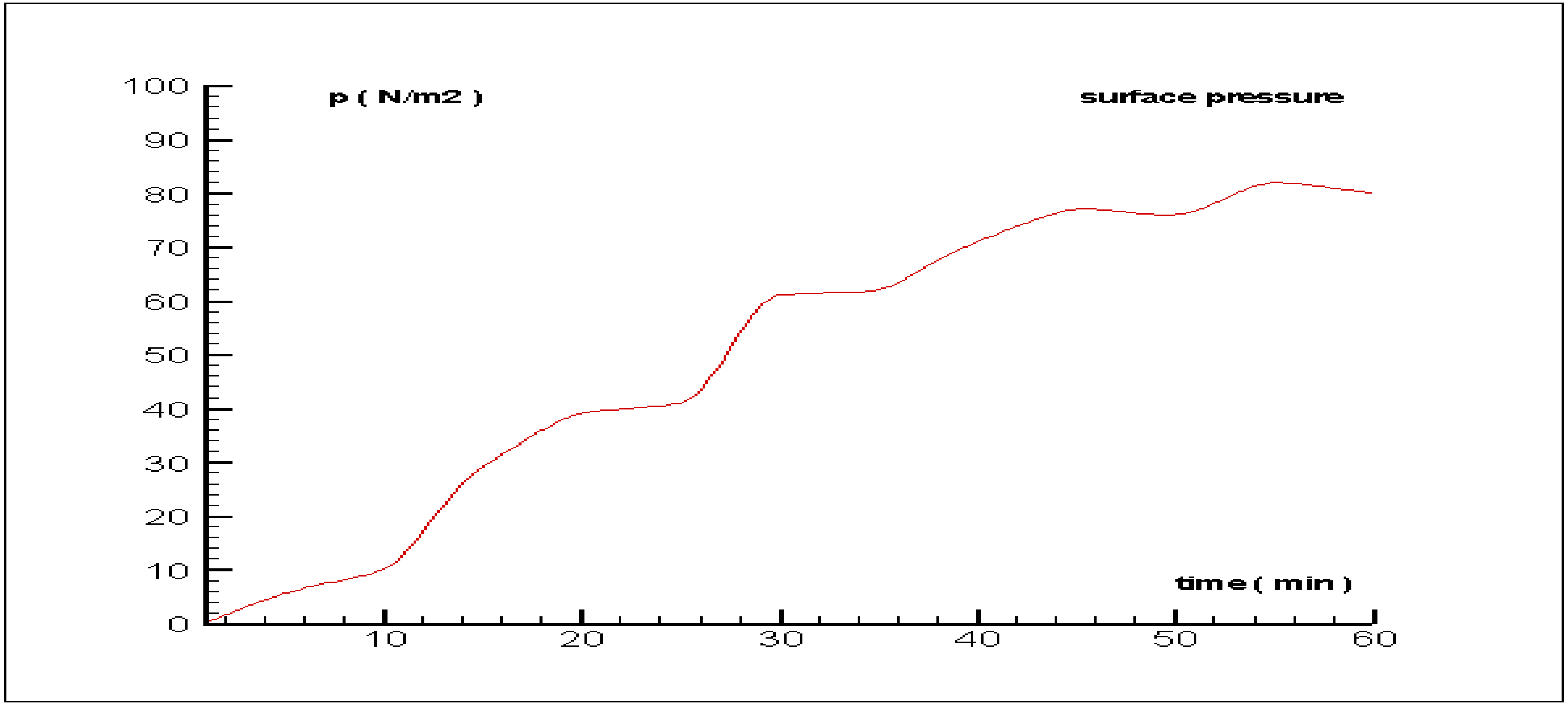

cold front location:stable stratification with an inversion layer
25000
20000
15000
10000
5000
0
0
1000
2000
3000
4000
5000
6000
distance (meters)
time (seconds)



Surface pressure at 12 km. Neutral stratification.



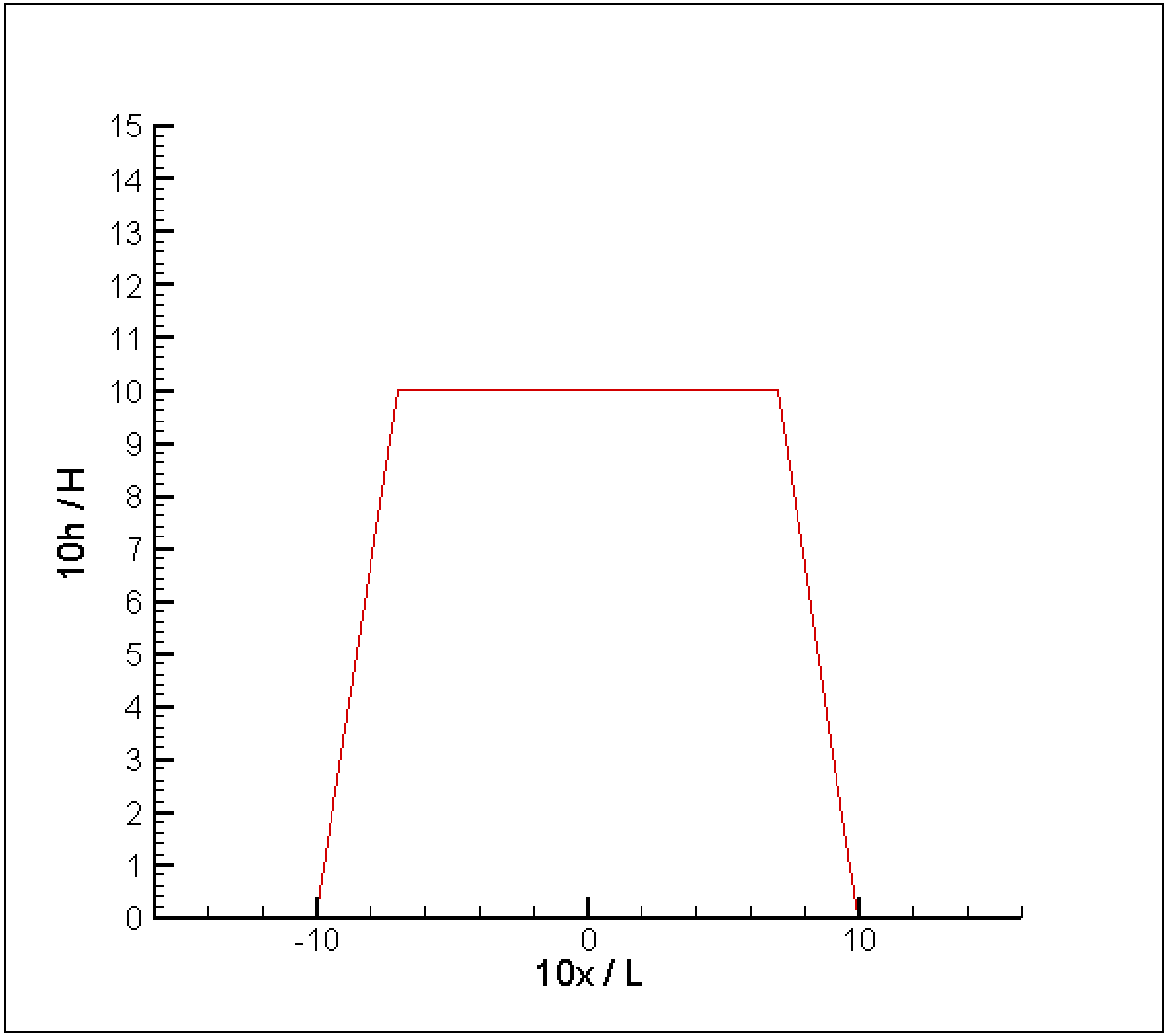
Surface pressure at 12 km. Stable stratification.



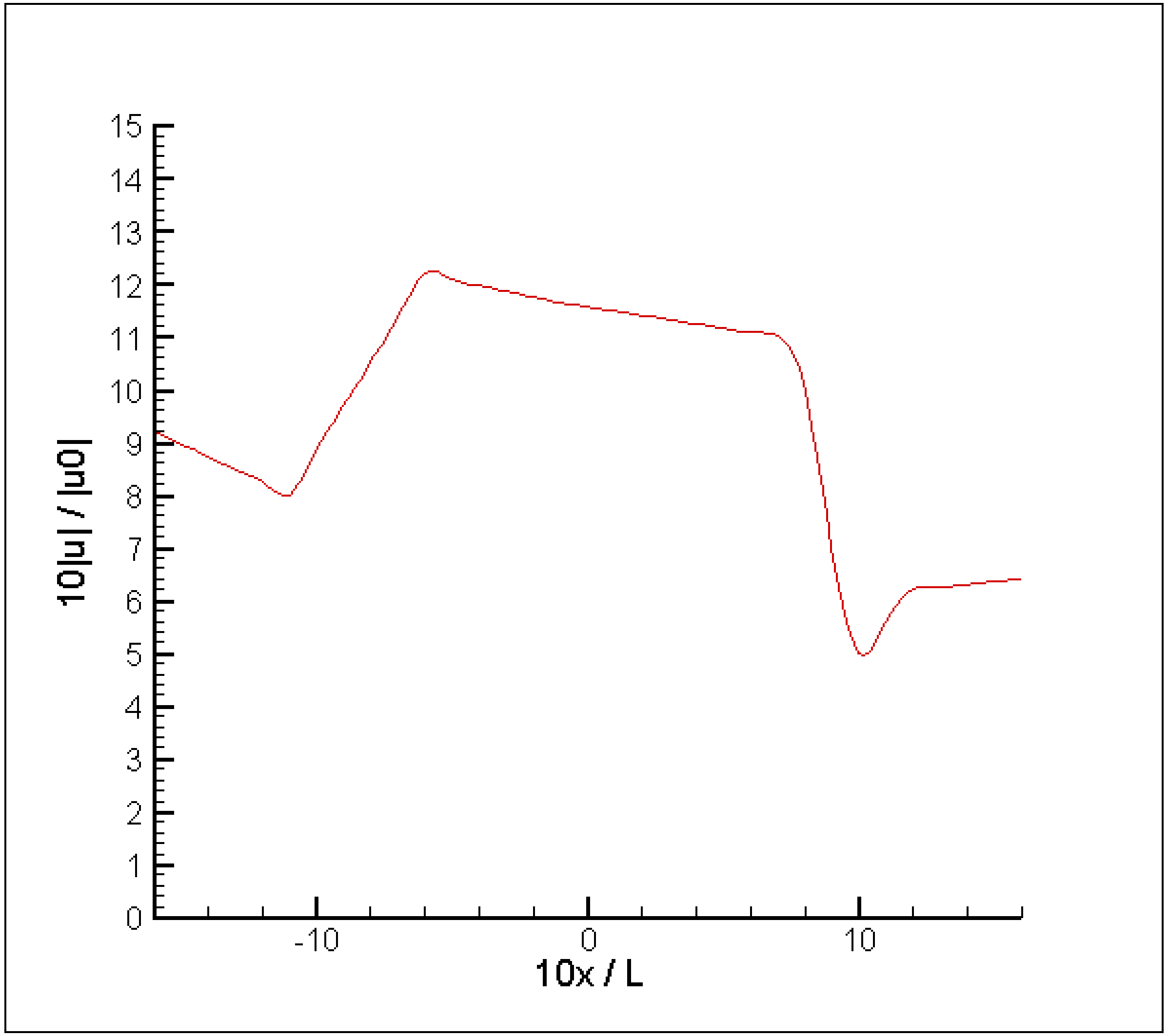
Surface pressure at 12 km. Stable stratification with inversion.

Cold front propagation over orographic obstacles of various shapes and stratifications

OBSTACLE HEIGHT (m)	INITIAL FRONT HEIGHT (м)	STRATIFICATION (K / 100m)	WINDWARD SPEED (m /sec)	LEEWARD SPEED (m /sec)
0	400	0.0	4.5	4.5
0	400	0.35	5.1	5.1
600	400	0.0	4.4	3.7
600	400	0.35	4.9	2.7
600	100	0.35	3.0	0.0
600	700	0.35	7.5	4.5
- 600	400	0.0	4.5	3.9



Trapezoidal obstacle: topography
Neutral stratification



Trapezoidal obstacle: wind speed
Neutral stratification

Conclusions

The results of the simulations of front speed in the propagation of a gravity current (cold front) in the atmosphere over flat terrain and over a hill under stable stratification were compared in this study with an empirical formula. A good agreement between the results of the calculations and the theoretical considerations has been shown.

A finite-element model based on triangular elements was used in the calculations. In this study an application of the model was made to simulating cold front propagation over an idealized hill-type obstacle in a stratified atmosphere with an inversion layer over an isolated hill. The study was performed under stable stratification in and beyond the inversion layer. It has been shown that the introduction of the inversion layer produces a significant decrease in the front speed both for the currents over the obstacle and those over flat orography.

The change in stratification from neutral to stable in the propagation of the cold atmospheric front has shown an increase in the front speed and a time evolution of the surface pressure that is in good agreement with the available observational data. Also, in contrast to the slow evolution of the surface pressure under neutral stratification, there is a considerable pressure jump in a stable atmosphere. This effect is increased by the introduction of the inversion layer. This phenomenon has been explained in theoretical considerations by Charba. The results of calculations of the present study are in good agreement with Charba's theory.