Modeling and simulation the stable stratified boundary layer with low-level jet: comparison with the wind tunnel data.

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Experimental arrangement for SBL with low-level jet

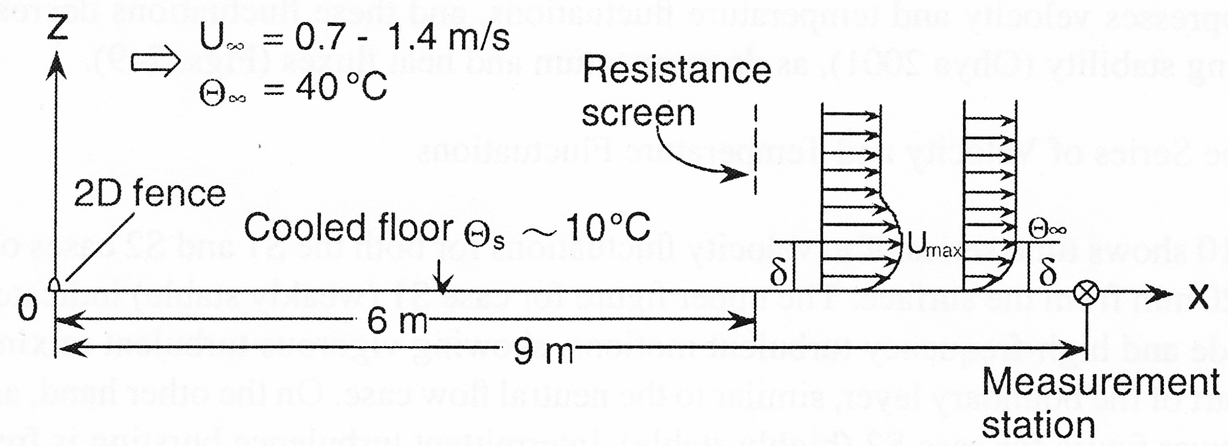


Fig. 1 Experimental arrangement for SBL with low-level jet

Modeling of turbulent stresses and turbulent heat fluxes

Turbulence equations

1) *Traceless Reynolds stress tensor*

$$b_{ij} = \langle u_i u_j \rangle - (2/3)\, E \delta_{ij}$$

$$\frac{D}{Dt} b_{ij} + D_{ij} = -\frac{4}{3} E S_{ij} - \Sigma_{ij} - Z_{ij} + B_{ij} - \Pi_{ij}$$

2) *Turbulent kinetic energy*

$$E = \langle u_i^2 \rangle / 2$$

$$\frac{DE}{Dt} + \frac{1}{2} D_{ii} = -\tau_{ij} \frac{\partial U_i}{\partial x_j} + \beta_i h_i - \varepsilon$$

Modeling of turbulent stresses and turbulent heat fluxes

3) *Turbulent heat fluxes*

$$h_i = \langle \theta u_i \rangle$$

$$\frac{D}{Dt} h_i + D_i^h = -h_j \frac{\partial U_i}{\partial x_j} - \tau_{ij} \frac{\partial \Theta}{\partial x_j} + \beta_i \langle \theta^2 \rangle - \Pi_i^\theta$$

4) *Temperature variance* $\langle \Theta^2 \rangle$

$$\frac{D}{Dt} \langle \theta^2 \rangle + D_\theta = -2 h_i \frac{\partial \Theta}{\partial x_i} - 2 \varepsilon_\theta$$

The other tensors are defined as follows:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

$$R_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right)$$

$$\Sigma_{ij} = b_{ik}S_{kj} + S_{ik}b_{kj} - \frac{2}{3}\delta_{ij}b_{km}S_{mk}$$

$$Z_{ij} = R_{ik}b_{kj} - b_{ik}R_{kj}$$

$$B_{ij} = \beta_i h_j + \beta_j h_i - \frac{2}{3}\delta_{ij}\beta_k h_k$$

$$\Pi_{ij} \equiv \langle u_i \frac{\partial p}{\partial x_j}\rangle + \langle u_j \frac{\partial p}{\partial x_i}\rangle - \frac{2}{3}\delta_{ij}\langle p u_k \rangle$$

$$D_{ij} \equiv \frac{\partial}{\partial x_k}\langle \left(u_i u_j - (1/3) u_i u_i \delta_{ij}\right) u_k \rangle$$

The pressure-shear /scalar correlations
The parameterization of 'slow' terms

$$\Pi_{ij} = \langle u_i p,_j \rangle + \langle u_j p,_i \rangle - \frac{2}{3}\delta_{ij}\langle u_k p,_k \rangle$$

$$\Pi_i^{\theta} = \langle \theta p,_i \rangle$$

$$\Pi_{ij}^{(1)} : \; b_{ij} / \tau, \qquad \Pi_i^{\theta(1)} : \; h_i / \tau_{p\theta}$$

$$\tau = E / \varepsilon$$

$$\Pi_i^{\theta(1)} \equiv \langle p \frac{\partial \theta}{\partial x_i} \rangle \cong -\frac{c_{1\theta}}{\tau_{p\theta}} h_i$$

$$\tau_{p\theta} : \; \tau$$

New dependence for the pressure correlation $\Pi_i^\theta = \overline{\theta p_{,i}}$ in the stably stratified turbulence

Relaxation linear model for the slow term: $$\Pi_i^\theta = \overline{\theta p_{,i}} \approx \frac{\overline{u_i \theta}}{\tau_{p\theta}}$$

'Standard' the SOC models usually assume, that

$$\tau_{p\theta} \approx \tau = \frac{2E}{\varepsilon}$$

<u>Such closure may not necessarily apply to the stably stratified flows!</u>
Because we use the original theoretical work of Weinstock (1989), pointed out that the time scale $\tau_{p\theta}$ must include a buoyancy damping factor

$$\tau_{p\theta} = \frac{\tau}{1 + a\tau^2 N^2}$$

'Weinstock's damping factor'

RANS-approach for turbulent stratified flows

$$\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0,$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + W\frac{\partial U}{\partial z} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} - \frac{\partial\langle u^2\rangle}{\partial x} - \frac{\partial\langle uw\rangle}{\partial z} + D_u,$$

$$\frac{\partial W}{\partial t} + U\frac{\partial W}{\partial x} + W\frac{\partial W}{\partial z} = -\frac{1}{\rho_0}\frac{\partial P}{\partial z} - \frac{\partial\langle uw\rangle}{\partial x} - \frac{\partial\langle w^2\rangle}{\partial z} + \beta\Theta g,$$

$$\frac{\partial \Theta}{\partial t} + U\frac{\partial \Theta}{\partial x} + W\frac{\partial \Theta}{\partial z} = -\frac{\partial\langle u\theta\rangle}{\partial x} - \frac{\partial\langle w\theta\rangle}{\partial z}.$$

Three parameter turbulence model

$$\frac{\partial E}{\partial t}+U_k\frac{\partial E}{\partial x_k}=\frac{\partial}{\partial x_k}\left(c_E\frac{E}{\varepsilon}\langle u_k u_k\rangle\frac{\partial E}{\partial x_k}\right)-\langle u_i u_k\rangle\frac{\partial U_i}{\partial x_k}+\beta g\delta_{i3}\langle u_i\theta\rangle-\varepsilon$$

$$\frac{\partial \varepsilon}{\partial t}+U_k\frac{\partial \varepsilon}{\partial x_k}=\frac{\partial}{\partial x_k}\left(c_\varepsilon\frac{E}{\varepsilon}\langle u_k u_k\rangle\frac{\partial \varepsilon}{\partial x_k}\right)+c_{\varepsilon 1}\frac{\varepsilon}{E}\left(-\langle u_i u_k\rangle\frac{\partial U_i}{\partial x_k}+\beta g\delta_{i3}\langle u_i\theta\rangle\right)-c_{\varepsilon 2}\frac{\varepsilon^2}{E}$$

$$\frac{\partial\langle\theta^2\rangle}{\partial t}+U_k\frac{\partial\langle\theta^2\rangle}{\partial x_k}=\frac{\partial}{\partial x_k}\left(c_{\theta 2}\frac{E}{\varepsilon}\langle u_k u_k\rangle\frac{\partial\langle\theta^2\rangle}{\partial x_k}\right)-\langle u_k\theta\rangle\frac{\partial\Theta}{\partial x_k}-\frac{1}{R}\frac{\varepsilon}{E}\langle\theta^2\rangle$$

$$\left(c_E=0{,}22,\ c_\varepsilon=0{,}18,\ c_{\varepsilon 1}=1{,}40,\ c_{\varepsilon 2}=1{,}90,\ c_{\theta 2}=0{,}22,\ R=0{,}6\right)$$

Improved Full Explicit Algebraic Models for Reynolds Stresses and Scalar Fluxes : 2D case

$$\left(<uw>, <vw>\right) = -K_M\left(\frac{\partial U}{\partial z}, \frac{\partial V}{\partial z}\right)$$

$$K_M = E\tau S_M$$

$$K_H = E\tau S_H$$

$$\tau = \frac{E}{\varepsilon}$$

$$<w\theta> = -K_H\frac{\partial \Theta}{\partial z} + \gamma_c$$

$$\gamma_c = \frac{1}{D}\left\{1 + \frac{2}{3}\alpha_2^2 G_M + s_6 G_H\right\}\alpha_5(\tau\beta g)\langle\theta^2\rangle$$

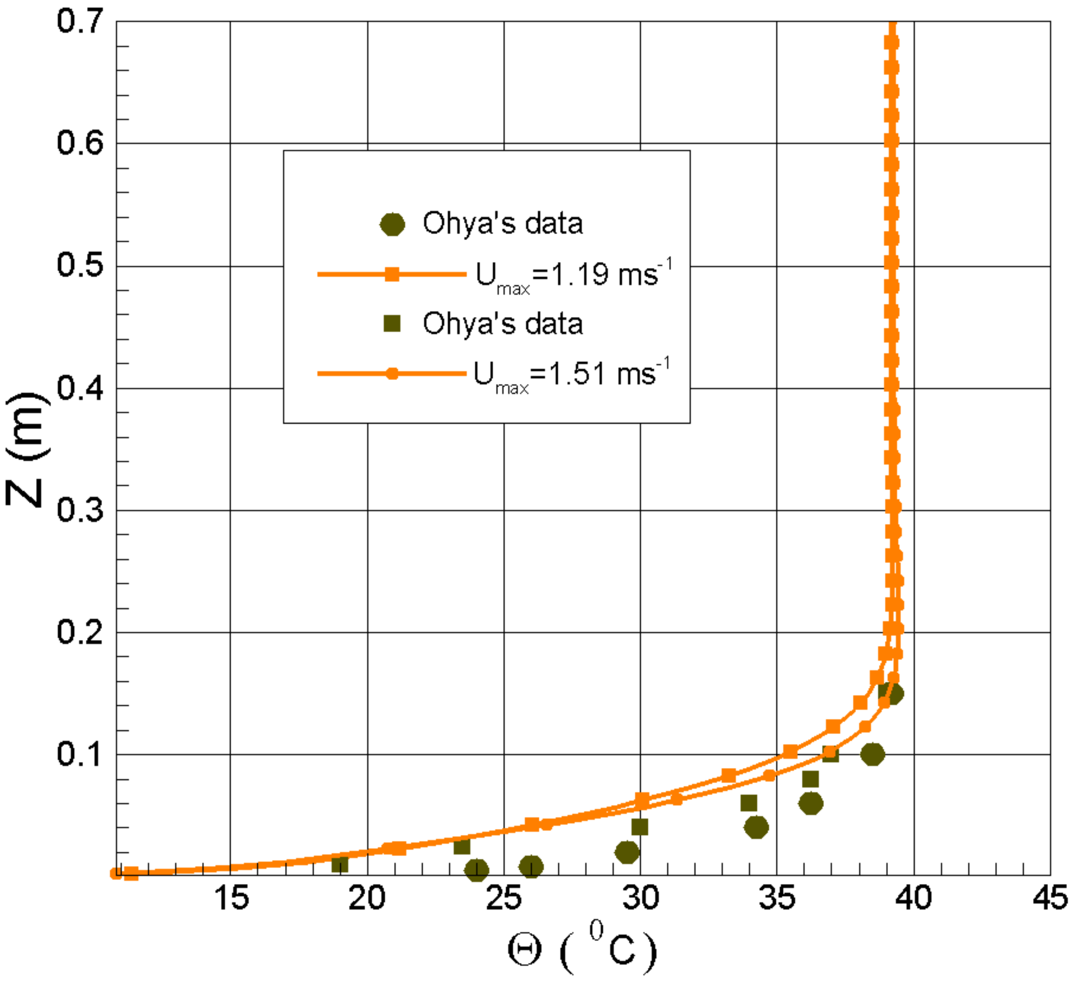
$$G_H \equiv (\tau N)^2 \qquad G_M \equiv (\tau S)^2 \qquad N^2 = \beta g\frac{\partial \Theta}{\partial z} \qquad S^2 \equiv \left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2$$

$$S_M = \frac{1}{D}\left\{\begin{matrix} s_0\left[1 + s_1 G_H\left(s_2 - s_3 G_H\right)\right] + s_4 s_5 \times \\ \times\left(1 + s_6 G_H\right)\tau\beta g\left(\langle\theta^2\rangle / E\right)\end{matrix}\right\}$$

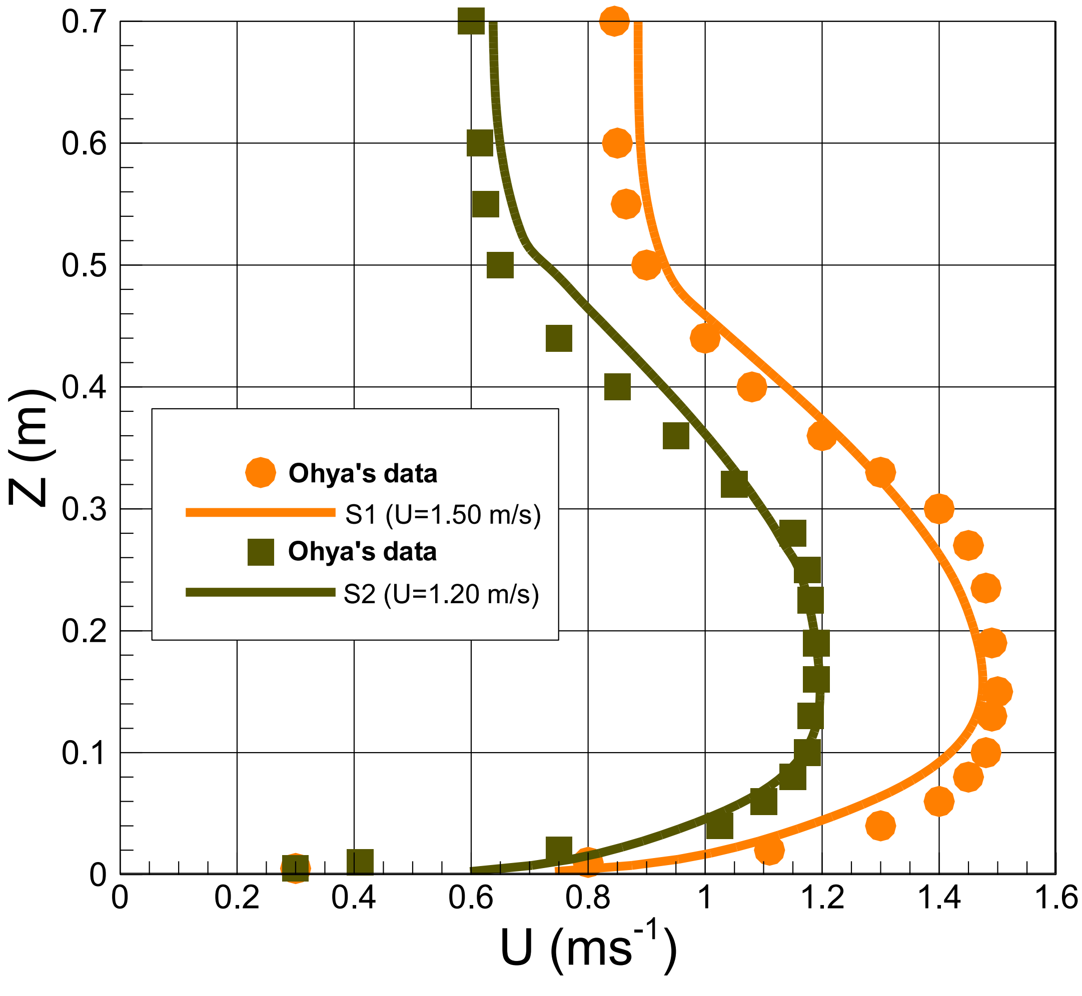
$$S_H = \frac{1}{D}\left\{\frac{2}{3}\frac{1}{c_{1\theta}^*}\left(1 + s_6 G_H\right)\right\}$$

$$D = 1 + d_1 G_M + d_2 G_H + d_3 G_M G_H + d_4 G_H^2 + (d_5 G_H^2 - d_6 G_M G_H) G_H$$

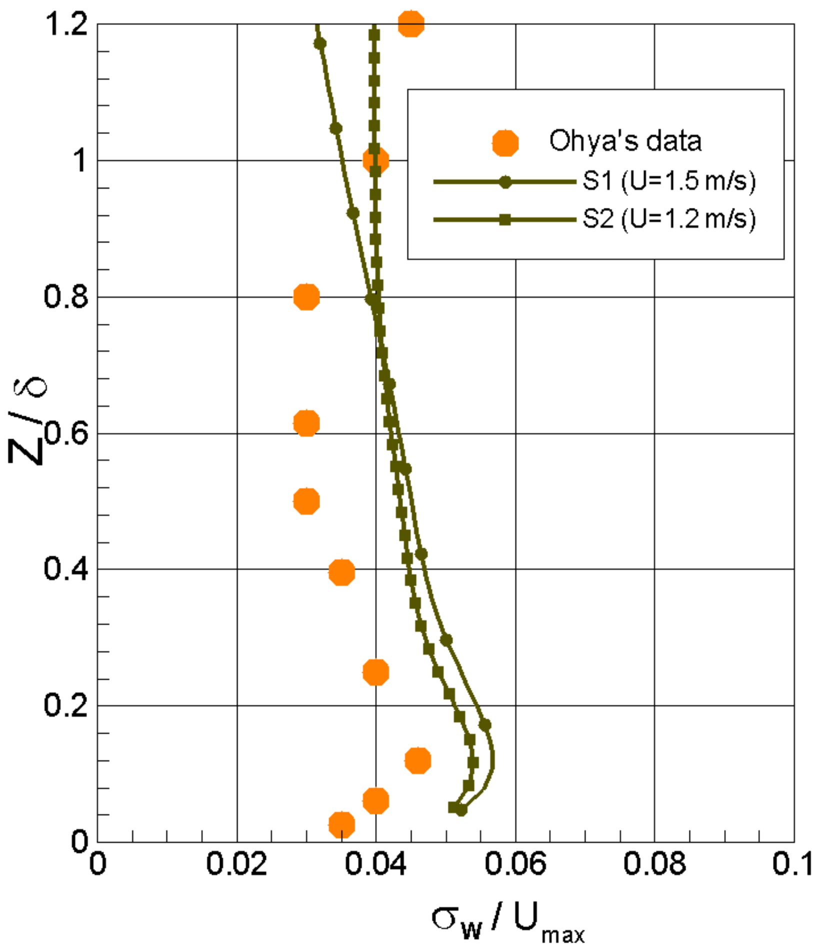
Vertical profiles of mean temperature

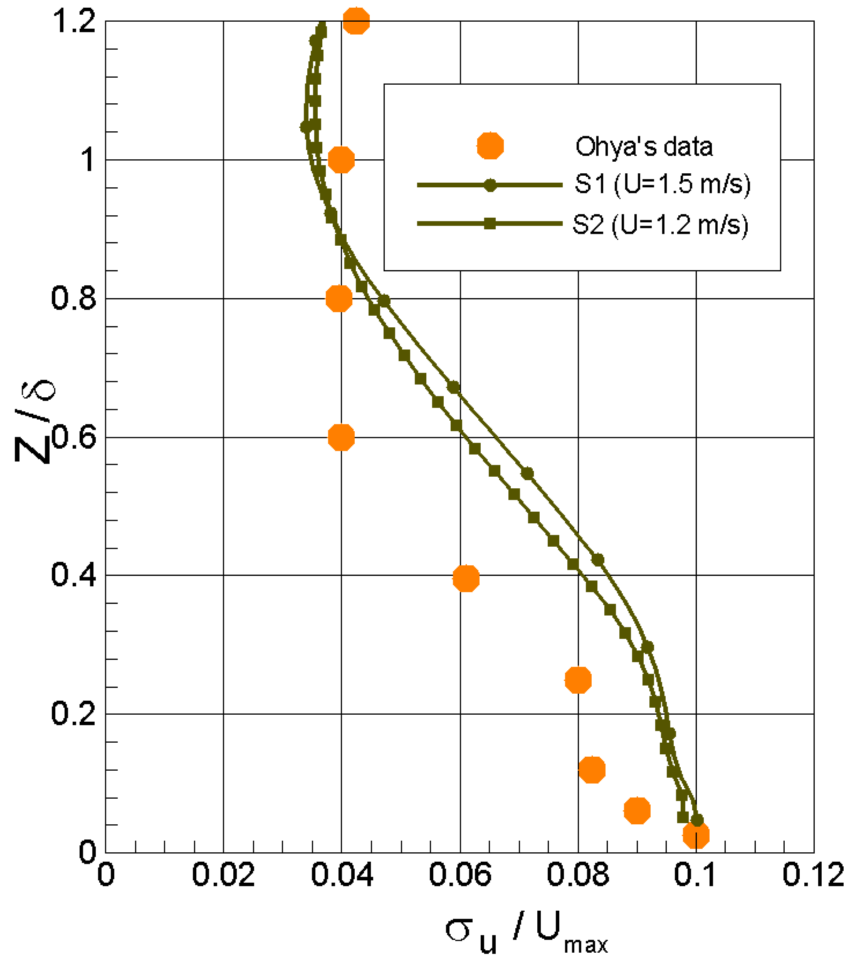


Vertical profiles of mean U velocity

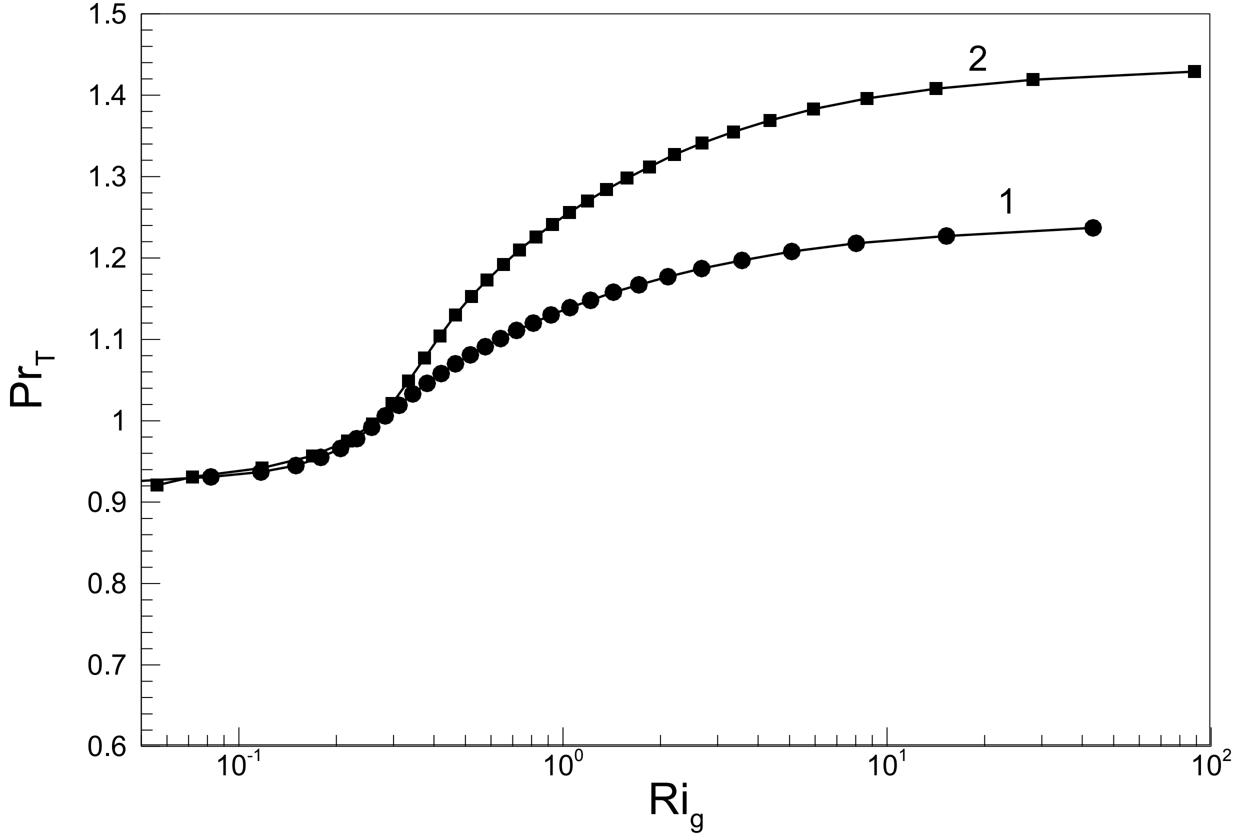


Vertical profiles of velocity fluctuations

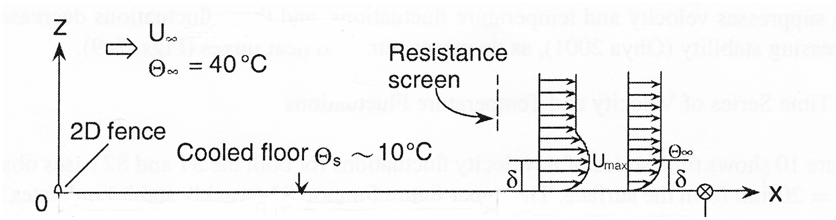




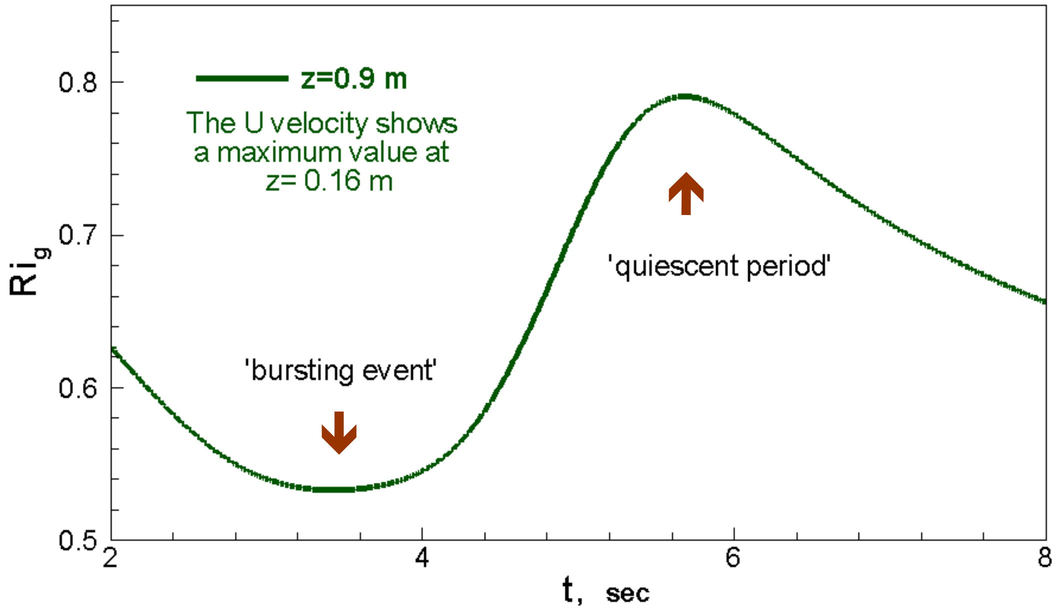
Turbulent Prandtl number as function of Richardson number



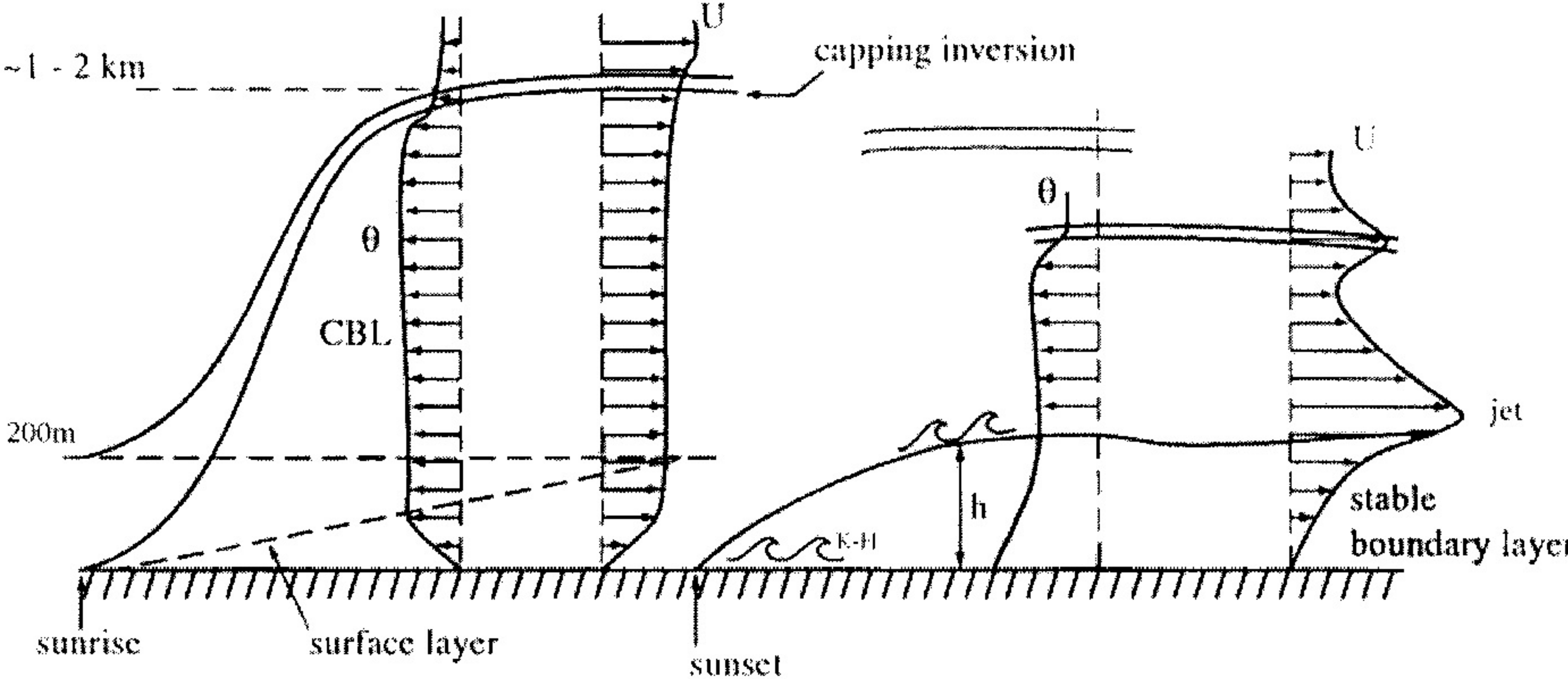
Time history of gradient Richardson number



$$Ri_g = \frac{\beta g \dfrac{\partial \Theta}{\partial z}}{\left(\dfrac{\partial U}{\partial z}\right)^2}$$

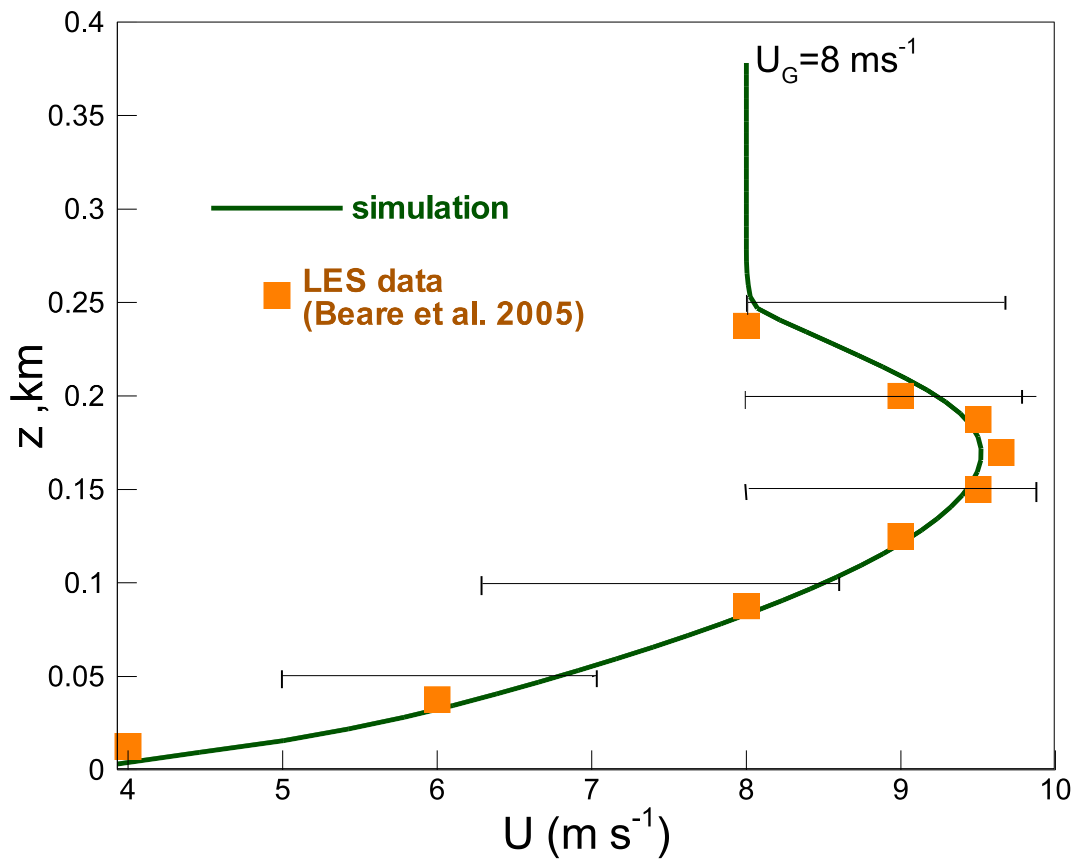


Thermal Stratified Boundary Layer over Flat Terrain

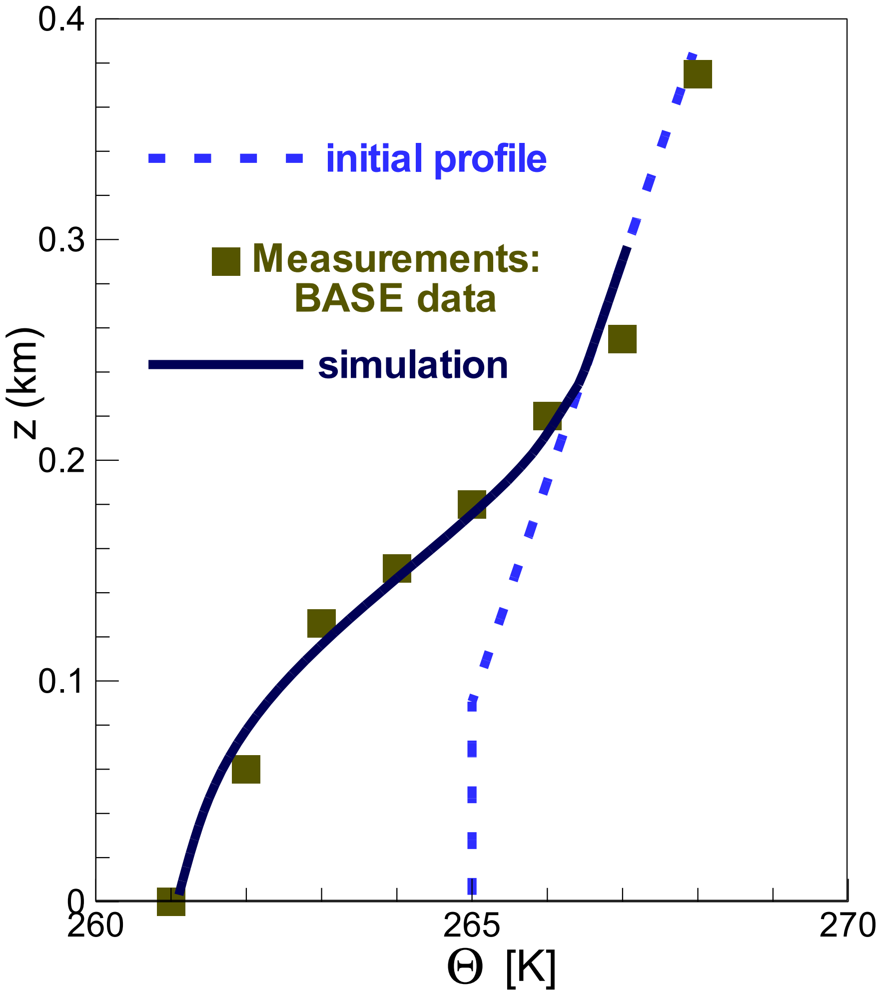


The potential temperature θ and velocity U are shown for the convective and stable boundary layers.

Velocity profile in SBL with Low-Level Jet



The potential temperature in the SSBL



The surface temperature (265 K initially) decreasing at a constant rate of 0.05 K/h. Such a profile developed into the observed profile (square symbols at the left on a figure) after 8 h of simulation.

The elevated inversion layer within the SBL, similar to the ones here, have been found by Kosovic and Carry (2000) on the Arctic sea in their LES simulations.

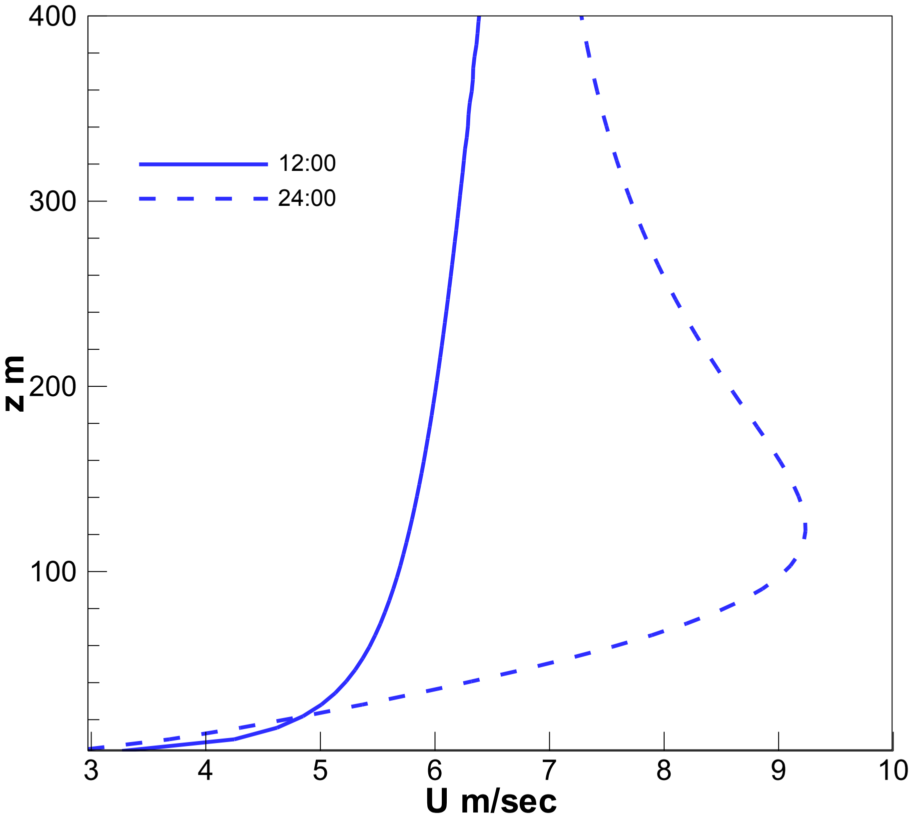
Model results for total horizontal wind speed

Time variation of the total horizontal wind speed .

★**The ground temperature was specified as**

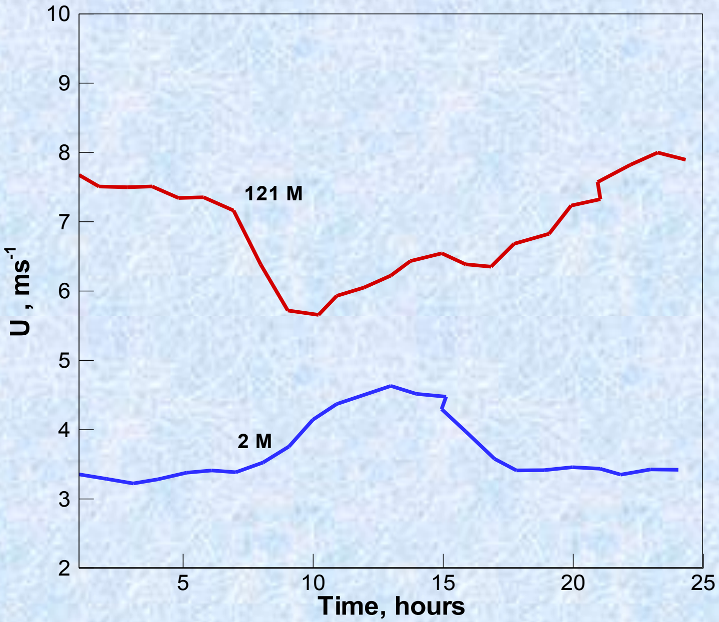
$$\Theta(x,0,t) = 6 \cdot \sin(\pi t / 43200)$$

This is the only nonstationary boundary condition of the problem, which models the 12-hour cycle of solar heating of the Earth's surface with decreasing at a constant rate of 0.6 K/h.

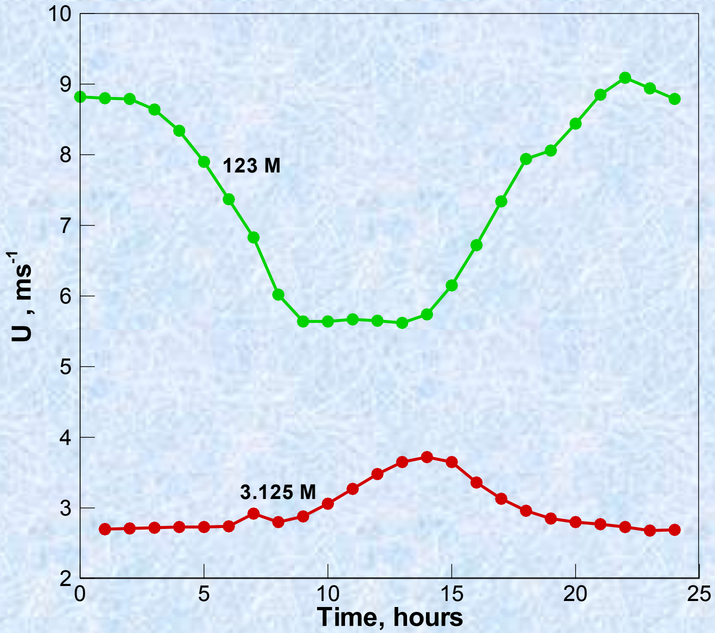


Time variation of total horizontal wind speed

- **Observations data**



- **Modeling results**



THANK YOU!