Numerical stochastic models of joint non-stationary and non-gaussian time series of weather elements for solving the statistical meteorology problems

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General simulation algorithms of gaussian sequences with a given correlation matrix

$$R_{(k)} = A_{(k)} A_{(k)}^{T}, \qquad \vec{\xi}_{(k)} = A_{(k)} \vec{\varphi}_{(k)}$$

$$R_{(k)} = P_{(k)} \Lambda_{(k)} P_{(k)}^{T}, \qquad \vec{\xi}_{(k)} = P_{(k)} \Lambda_{(k)}^{\frac{1}{2}} \vec{\varphi}_{(k)}$$

$$B_{(k)} \vec{\xi}_{(k)} = D_{(k)} \vec{\varphi}_{(k)},$$

$$B_{(k)} = \begin{Vmatrix} I & 0 & 0 & 0 \\ -\vec{B}^{T}[1] J_{(k)} & I & 0 & 0 \\ \dots & \dots & \dots & \dots \\ & -\vec{B}^{T}[k] J_{(k)} & & I \end{Vmatrix} \qquad D_{(k)} = \begin{Vmatrix} C_0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & C_{k-1} \end{Vmatrix}$$

$$R_{(k)} \vec{B}[k] = \vec{R}_k,$$

Models of gaussian joint time series

$$\vec{\xi}_{(n)} = \left(\vec{\xi}^{T}(t_1), \vec{\xi}^{T}(t_2), \ldots, \vec{\xi}^{T}(t_n)\right)^{T},$$

$$\vec{\eta}_{(n)} = \left(\vec{\eta}^{T}(t_1), \vec{\eta}^{T}(t_2), \ldots, \vec{\eta}^{T}(t_n)\right)^{T},$$

$$\vec{\xi}(t_i) = \vec{\xi}_i = (\xi_{1i}, \xi_{2i}, \ldots, \xi_{pi})^{T},$$

$$\vec{\eta}(t_i) = \vec{\eta}_i = (\eta_{1i}, \eta_{2i}, \ldots, \eta_{pi})^{T},$$

$$i = 1, \ldots, n$$

$$R_{(n)} = \begin{Vmatrix} R_0 & R_1 & \ldots & R_{n-1} \\ R_1^{T} & R_0 & \ldots & R_{n-2} \\ \ldots & \ldots & \ldots & \ldots \\ R_{n-1}^{T} & R_{n-2}^{T} & \ldots & R_0 \end{Vmatrix}, \qquad R_k = \begin{Vmatrix} R_{\vec{\xi}_i \vec{\xi}_{i+k}} & R_{\vec{\xi}_i \vec{\eta}_{i+k}} \\ R_{\vec{\eta}_i \vec{\xi}_{i+k}} & R_{\vec{\eta}_i \vec{\eta}_{i+k}} \end{Vmatrix},$$

where R_k, $k = 0, \ldots, n-1$ are $2p \times 2p$ matrixes

$$
R_{(n)} = \left\| \begin{matrix}
\begin{Vmatrix} R_{\vec\xi_i\vec\xi_i} & R_{\vec\xi_i\vec\eta_i} \\ R_{\vec\eta_i\vec\xi_i} & R_{\vec\eta_i\vec\eta_i} \end{Vmatrix} &
\begin{Vmatrix} R_{\vec\xi_i\vec\xi_{i+1}} & R_{\vec\xi_i\vec\eta_{i+1}} \\ R_{\vec\eta_i\vec\xi_{i+1}} & R_{\vec\eta_i\vec\eta_{i+1}} \end{Vmatrix} &
\cdots &
\begin{Vmatrix} R_{\vec\xi_i\vec\xi_{i+n-1}} & R_{\vec\xi_i\vec\eta_{i+n-1}} \\ R_{\vec\eta_i\vec\xi_{i+n-1}} & R_{\vec\eta_i\vec\eta_{i+n-1}} \end{Vmatrix} \\
\begin{Vmatrix} R_{\vec\xi_i\vec\xi_{i+1}} & R_{\vec\xi_i\vec\eta_{i+1}} \\ R_{\vec\eta_i\vec\xi_{i+1}} & R_{\vec\eta_i\vec\eta_{i+1}} \end{Vmatrix}^T &
\begin{Vmatrix} R_{\vec\xi_i\vec\xi_i} & R_{\vec\xi_i\vec\eta_i} \\ R_{\vec\eta_i\vec\xi_i} & R_{\vec\eta_i\vec\eta_i} \end{Vmatrix} &
\cdots &
\begin{Vmatrix} R_{\vec\xi_i\vec\xi_{i+n-2}} & R_{\vec\xi_i\vec\eta_{i+n-2}} \\ R_{\vec\eta_i\vec\xi_{i+n-2}} & R_{\vec\eta_i\vec\eta_{i+n-2}} \end{Vmatrix} \\
\cdots & \cdots & \cdots & \cdots \\
\begin{Vmatrix} R_{\vec\xi_i\vec\xi_{i+n-1}} & R_{\vec\xi_i\vec\eta_{i+n-1}} \\ R_{\vec\eta_i\vec\xi_{i+n-1}} & R_{\vec\eta_i\vec\eta_{i+n-1}} \end{Vmatrix}^T &
\begin{Vmatrix} R_{\vec\xi_i\vec\xi_{i+n-2}} & R_{\vec\xi_i\vec\eta_{i+n-2}} \\ R_{\vec\eta_i\vec\xi_{i+n-2}} & R_{\vec\eta_i\vec\eta_{i+n-2}} \end{Vmatrix}^T &
\cdots &
\begin{Vmatrix} R_{\vec\xi_i\vec\xi_i} & R_{\vec\xi_i\vec\eta_i} \\ R_{\vec\eta_i\vec\xi_i} & R_{\vec\eta_i\vec\eta_i} \end{Vmatrix}
\end{matrix} \right\| .
$$

Simulation algorithm of gaussian vector

$$\vec{\xi}_{(n)} = (\vec{\xi}_1^{\,T}, \vec{\xi}_2^{\,T}, \ldots, \vec{\xi}_n^{\,T})^T$$

Method of conditional distributions functions

$$\vec{\xi}_1 = C_0 \vec{\phi}_1,$$
$$\vec{\xi}_2 = \vec{B}^T[1] J_{(1)} \vec{\xi}_{(1)} + C_1 \vec{\phi}_2,$$
$$\ldots$$
$$\vec{\xi}_n = \vec{B}^T[n-1] J_{(n-1)} \vec{\xi}_{(n-1)} + C_{n-1} \vec{\phi}_n,$$

$\vec{\phi}_1, \vec{\phi}_2, \ldots, \vec{\phi}_n : E\vec{\phi}_k \vec{\phi}_k^{\,T} = I_p,\ E\vec{\phi}_k \vec{\phi}_l^{\,T} = 0,\ k \neq l,\ \ \vec{B}[k] = (B_1^T[k], \ldots, B_k^T[k])^T,\ \ k = 1, \ldots, n-1,$

$B_i\left[k\right]$ are $p \times p$ matrixes, C_i are lower triangular $p \times p$ matrixes, $C_i C_i^T = Q_i$.

$$J_{(k)} = \begin{Vmatrix} 0 & \ldots & I_p \\ \ldots & \ldots & \ldots \\ I_p & \ldots & 0 \end{Vmatrix}, \quad k = 1, \ldots, n-1$$

Calculation of matrix coefficients

$$\tilde{R}_{(k)}\vec{B}[k] = \vec{R}_k, \qquad Q_k = R_0 - \vec{B}^T[k]\tilde{R}_{(k)}\vec{B}[k],$$

$$\tilde{R}_{(k)} = J_{(k)}R_{(k)}J_{(k)},$$

Recurrent algorithm

$$B_1^T[1] = R_1^T R_0^{-1}, \quad \tilde{B}_1[1] = R_1 R_0^{-1},$$

$$Q_0 = R_0, \quad \tilde{Q}_0 = \tilde{R}_0,$$

$$(B_1^T[k+1],\ldots,B_k^T[k+1]) = \vec{B}^T[k] - B_{k+1}^T[k+1]\vec{\tilde{B}}[k]J_{(k)},$$

$$(\tilde{B}_1[k+1],\ldots,\tilde{B}_k[k+1]) = \vec{\tilde{B}}[k] - \tilde{B}_{k+1}[k+1]\vec{B}^T[k]J_{(k)},$$

$$B_{k+1}[k+1] = \tilde{Q}_k^{-1}(R_{k+1} - \vec{\tilde{R}}_k J_{(k)}\vec{B}[k]),$$

$$\tilde{B}_{k+1}[k+1] = \tilde{Q}_k^{-1}(R_{k+1}^T - \vec{R}_k^T J_{(k)}\vec{\tilde{B}}[k]),$$

$$Q_k = R_0 - \vec{R}_k^T\vec{B}[k], \quad \tilde{Q}_k = R_0 - \vec{\tilde{R}}_k\vec{\tilde{B}}[k],$$

$$C_k C_k^T = Q_k, \quad \vec{R}_k = (R_1^T,\ldots,R_k^T)^T, \quad \vec{\tilde{R}}_k = (R_1,\ldots,R_k)^T, \quad k = 1,\ldots,n-1.$$

Models of periodically collerated random processes

Scalar periodically collerated sequence

$$\xi_1, \xi_2, \ldots, \xi_p, \xi_{p+1}, \xi_{p+2}, \ldots, \xi_{2p}, \ldots, \xi_{(n-1)p+1}, \xi_{(n-1)p+2}, \ldots, \xi_{np}$$

Conditions for periodically colleration of random process $\vec{\xi}$:

$$E\xi(t_i + T) = E\xi(t_i), \quad D\xi(t_i + T) = D\xi(t_i), \; R_\xi(t_i + T, t_j + T) = R_\xi(t_i, t_j).$$

Numerical simulation of gaussian periodically collerated sequences

$$\vec{\xi}_i = (\xi_{1i}, \xi_{2i}, \ldots, \xi_{pi})^T, \quad i = 1, \ldots, n$$

$$\vec{\xi}_{(n)} = \left(\xi(t_1), \xi(t_2), \ldots, \xi(t_N)\right)^T,$$

$$T = p\Delta t, \quad T \le t_N, \quad \Delta t = t_{i+1} - t_i \quad (t_1 = 0, \Delta t = 1),$$

For infinite periodically collerated sequences $\vec{\xi}_1, \vec{\xi}_2, \ldots, \vec{\xi}_n, \ldots$

we can use a many-dimensional autoregression model of the order n-1:

$$\vec{\xi}_t = B_1^T[n-1]\vec{\xi}_{t-1} + \ldots + B_{n-1}^T[n-1]\vec{\xi}_{t-(n-1)} + C_{n-1}\vec{\varphi}_t.$$

Method of inverse distribution function

$$\xi_t = F_t^{-1}\left(\Phi\left(\eta_t\right)\right),$$

$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{x} e^{-u^2} du$ – standard normal distribution function ,

η_t – gaussian sequence, $M\eta_t = 0,\ D\eta_t = 1.\ M\eta_i\eta_j = g_{ij},$

$$r_{ij} = \frac{M\xi_i\xi_j - M\xi_i M\xi_j}{\sqrt{D\xi_i D\xi_j}} = R_{F_iF_j}(g_{ij}),$$

$$R_{F_iF_j}(g_{ij}) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} F_i^{-1}(\Phi(x))F_j^{-1}(\Phi(y))\varphi(x,y,g_{ij})dxdy,$$

$$\varphi(x,y,g_{ij}) = \left[2\pi\sqrt{1-g_{ij}^2}\exp\left(\frac{2g_{ij}xy - x^2 - y^2}{2(1-g_{ij}^2)}\right)\right]$$

$$g_{ij} = R_{F_iF_j}^{-1}(r_{ij})$$

Simulation method based on the normalization of a real time series

1. On the basis of the real data $\xi_1^*, \xi_2^*, \ldots, \xi_n^*$ the one-dimensional distribution $F_\xi(x)$ is estimated

2. The normalized sequence $\eta_1^*, \eta_2^*, \ldots, \eta_n^*$ is constructed :

$$\eta_i^* = \Phi^{-1}(F_\xi(\xi_i^*)),\ \ \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2} du$$

3. On the basis of the normalized sequence the covariance function $R_0^{N*}, R_1^{N*}, \ldots, R_n^{N*}$ is estimated.

4. Gaussian sequence $\eta_1, \eta_2, \ldots, \eta_n$ with the covariance function $R_0^{N*}, R_1^{N*}, \ldots, R_n^{N*}$ is constracted.

5. The sequence $\xi_1, \xi_2, \ldots, \xi_n$ with the help of transformation $\xi_i = F_\xi^{-1}(\Phi(\eta_i))$ of elements η_i of Gaussian sequence is constracted.

The correlation function is determined by used transformations

Aproximation of distributions

Temperature

Mixture of two gaussian distributions:

$$p_t(x) = \frac{\theta_t}{\sqrt{2\pi}\sigma_{1,t}} e^{-\frac{(x-\mu_{1,t})^2}{2\sigma_{1,t}^2}} + \frac{(1-\theta_t)}{\sqrt{2\pi}\sigma_{2,t}} e^{-\frac{(x-\mu_{2,t})^2}{2\sigma_{2,t}^2}}, \qquad 0 \le \theta_t \le 1,$$

$$\mu_t = m_t^{*(1)}, \quad \sigma_t^2 = \left(s_t^{*(1)}\right)^2. \qquad F\left(\theta_t, \mu_{1,t}, \mu_{2,t}, \sigma_{1,t}^2, \sigma_{2,t}^2\right) = \sum_{i=1}^{k} \frac{\left(p_{i,t}^{*} - p_{i,t}\right)^2}{p_{i,t}},$$

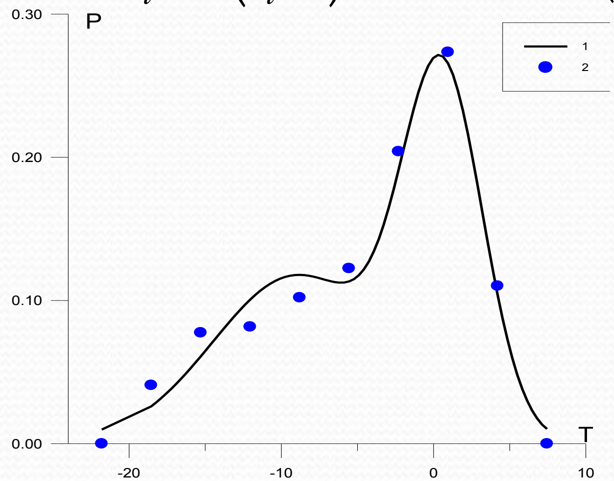


Fig 1. Empirical -1 и model -2 density.

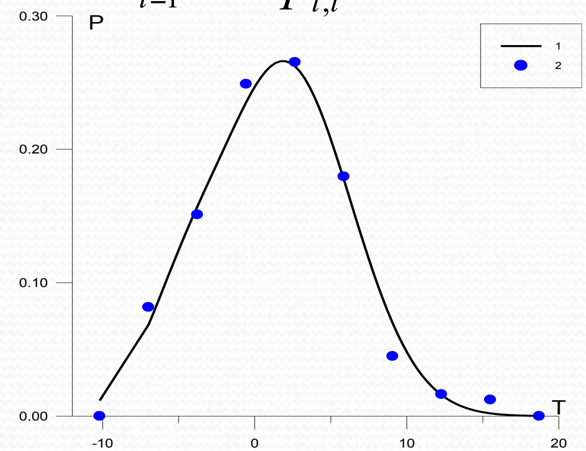


Fig 2. Empirical -1 и model -2 density.

$$A_t^{*(1)} = 0{,}64652,\ A_t^{(1)} = 0{,}64687, \left|A_t^{*(1)} - A_t^{(1)}\right| = 0{,}00035$$

$$F\left(\theta_t, \mu_{1,t}, \mu_{2,t}, \sigma_{1,t}^2, \sigma_{2,t}^2\right) = 0{,}15055$$

$$A_t^{*(1)} = 0{,}29382,\ A_t^{(1)} = 0{,}00000, \left|A_t^{*(1)} - A_t^{(1)}\right| = 0{,}29382$$

$$F\left(\theta_t, \mu_{1,t}, \mu_{2,t}, \sigma_{1,t}^2, \sigma_{2,t}^2\right) = 0{,}09424$$

А. С. Марченко, Л.А. Минакова. Вероятностная модель временных рядов температуры воздуха. // Метеорология и Гидрология, 1980, № 9, с. 39-47.

Aproximation of distributions

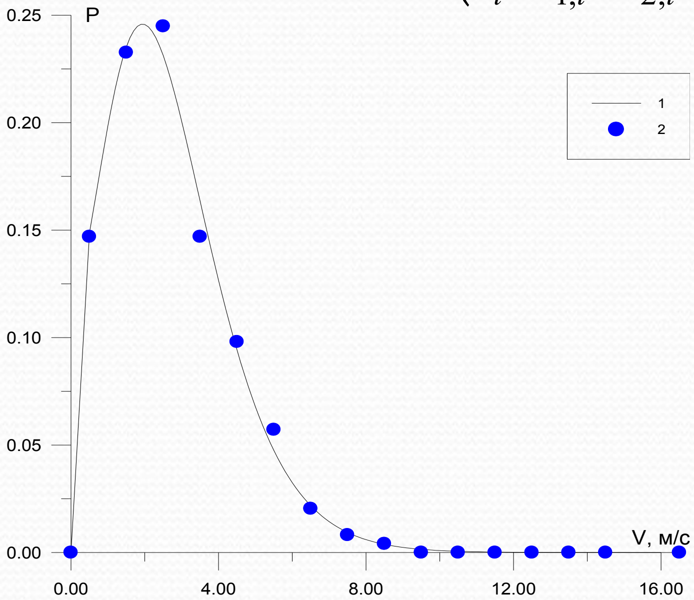
Module of wind speed

Mixture of two gamma distributions:

$$p_t(x) = \theta_t \frac{x^{\nu_{1,t}-1} e^{-\frac{x}{\lambda_{1,t}}}}{\lambda_{1,t}^{\nu_{1,t}} \Gamma(\nu_{1,t})} + (1-\theta_t) \frac{x^{\nu_{2,t}-1} e^{-\frac{x}{\lambda_{2,t}}}}{\lambda_{2,t}^{\nu_{2,t}} \Gamma(\nu_{2,t})}, \qquad 0 \le \theta_t \le 1,$$

$$\mu_t = m_t^{*(2)}, \qquad \sigma_t^2 = \left(s_t^{*(2)}\right)^2.$$

$$F(\theta_t, \nu_{1,t}, \nu_{2,t}, \lambda_{1,t}, \lambda_{2,t}) = \sum_{i=1}^{k} \frac{\left(p_{i,t}^* - p_{i,t}\right)^2}{p_{i,t}},$$



$$A_t^{*(1)} = 0{,}64652,\ A_t^{(1)} = 0{,}64687, \left|A_t^{*(1)} - A_t^{(1)}\right| = 0{,}00035$$

$$F(\theta_t, \mu_{1,t}, \mu_{2,t}, \sigma_{1,t}^2, \sigma_{2,t}^2) = 0{,}15055$$

Fig 3. Empirical -1 и model -2 density.

А. С. Марченко, А. Г. Семочкин. Модели одномерных и совместных распределений неотрицательных случайных величин. // Метеорология и Гидрология, 1982, № 3, с. 50-56.

Approximation of one-dimensional distribution of relative humidity by mixture of two beta distributions

$$f_H(x) = \frac{\theta}{B(\nu_1,\mu_1)} \cdot x^{\nu_1 - 1} \cdot (1-x)^{\mu_1 - 1} + \frac{1-\theta}{B(\nu_2,\mu_2)} \cdot x^{\nu_2 - 1} \cdot (1-x)^{\mu_2 - 1}$$

$$0 \le \theta \le 1, \qquad \theta + \theta' = 1$$

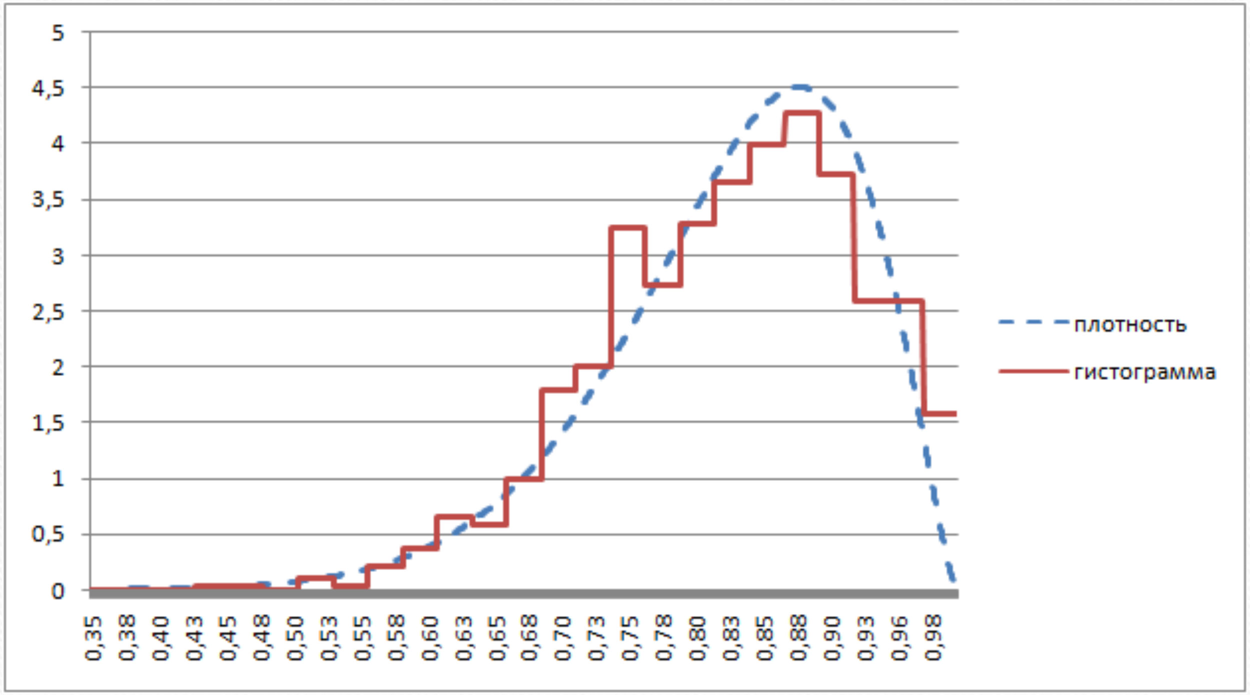


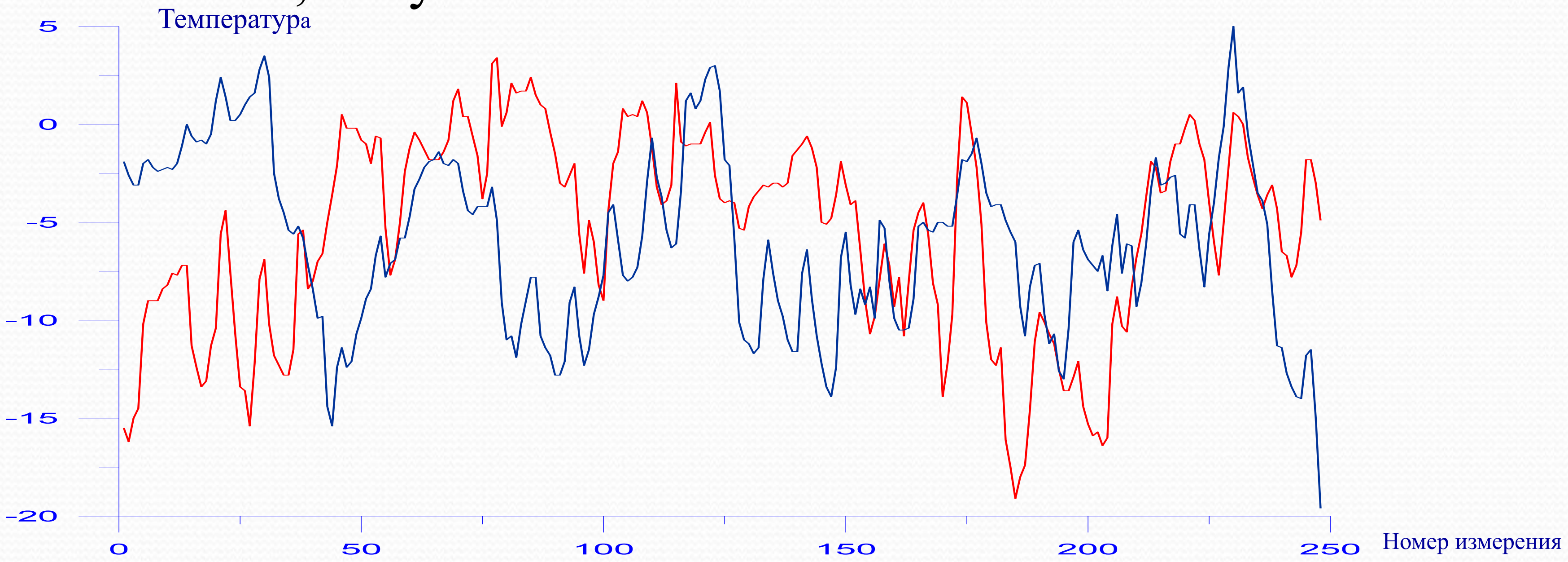
Fig. Histogram of daily average values of relative humidity and the density of the mixture of two beta distributions .

Estimation of the matrix covariation function

$$\bar{R}_{\zeta_i\zeta_{i+k}} = \frac{1}{m}\sum_{j=1}^{m}\left(\frac{1}{n}\sum_{i=1}^{n-k}(\vec{\xi}_i^{\,j} - E\vec{\xi}_{n,m})(\vec{\xi}_{i+k}^{\,j} - E\vec{\xi}_{n,m})^T\right),$$

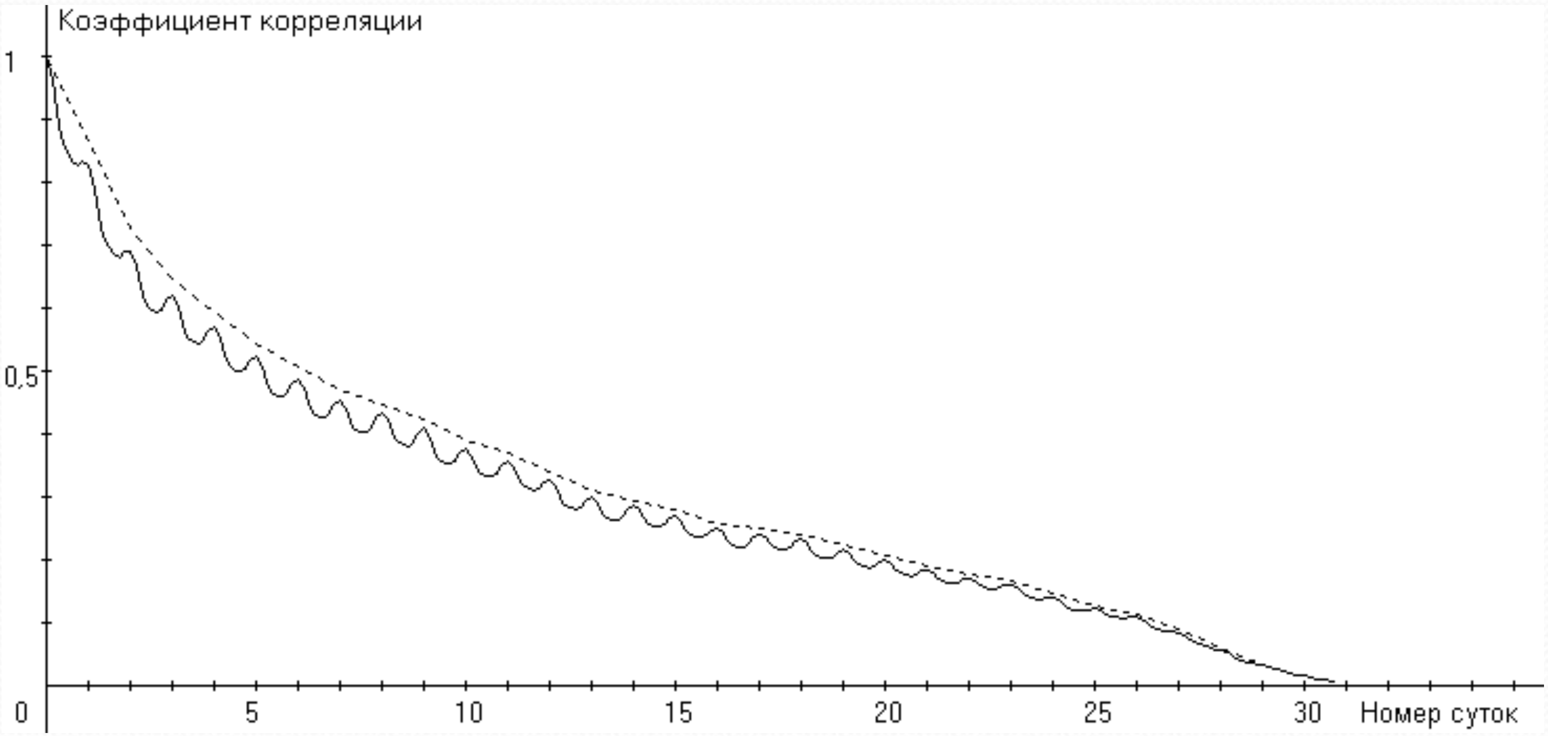
$$E\vec{\xi}_{n,m} = \frac{1}{m}\sum_{j=1}^{m}\left(\frac{1}{n}\sum_{i=1}^{n}\vec{\xi}_i^{\,j}\right).$$

Real data: 8 times per day measurements of air temperature, Astrakhan, 34 years.



Air temperature, January 1967 (red line) и 1987 (blue line) years.

Correlation function of time series of air temperature (the stationary approximation)



Correlation function of daily average temperature (- - -) and the correlation function of the temperature with a step equal 3 hour (----), calculated in the assumption of stationarity of the process, January (Astrakhan)

Piecewise-linear envelope correlation function of periodically correlated process

Let $\xi_1, \xi_2, K\ , \xi_p, \xi_{p+1}, \xi_{p+2}, K\ , \xi_{2p}, K\ , \xi_{(n-1)p+1}, \xi_{(n-1)p+2}, K\ , \xi_{np}$ is periodically collerated sequence with zero mean and block Toeplitz covariance matrix

$$R_{(n)} = \begin{pmatrix} R_0 & R_1 & \dots & R_{n-1} \\ R_1^T & R_0 & \dots & R_{n-2} \\ \dots & \dots & \dots & \dots \\ R_{n-1}^T & R_{n-2}^T & \dots & R_0 \end{pmatrix}, \; R_k = \begin{pmatrix} r_k^0 & r_k^1 & L & r_k^{p-1} \\ r_k^{-1} & r_k^0 & L & r_k^{p-2} \\ L & L & L & L \\ r_k^{-(p-1)} & r_k^{-(p-2)} & K & r_k^0 \end{pmatrix}, \; R_k \neq R_k^T,$$

ρ^*_{lp+t} is value of linear function at the point $(lp+t)$ passing through the points $\left(lp, \frac{(n-l)pr_l^0}{np}\right), \left((l+1)p, \frac{(n-l-1)pr_{l+1}^0}{np}\right)$ $l = \overline{0, n-2},\ t = \overline{0, p-1},$ and ρ_h weighted average of the elements of h-th diagonal jf the matrix $R_{(n)}$ with weights $\frac{n-h}{np}$.

Statement

If for all i, k $\quad r_k^0 \geq r_k^i,$ then $\rho_{lp+t} \leq \rho^*_{lp+t}, \quad n \to \infty.$

Consequent

Let $\rho_h = \frac{1}{np} \sum_{t=1}^{np-h} \xi_t \xi_{t+h}$. At $n \to \infty$ for all i, k the inequality $\rho_{lp+t} \leq \rho^*_{lp+t}$ takes place.

Example

Let $\xi_1, \xi_2, \ldots, \xi_p, \xi_{p+1}, \xi_{p+2}, \ldots, \xi_{2p}, \ldots, \xi_{(n-1)p+1}, \xi_{(n-1)p+2}, \ldots, \xi_{np}$

be periodically correlated sequence with block -Toeplitz covariation matrix

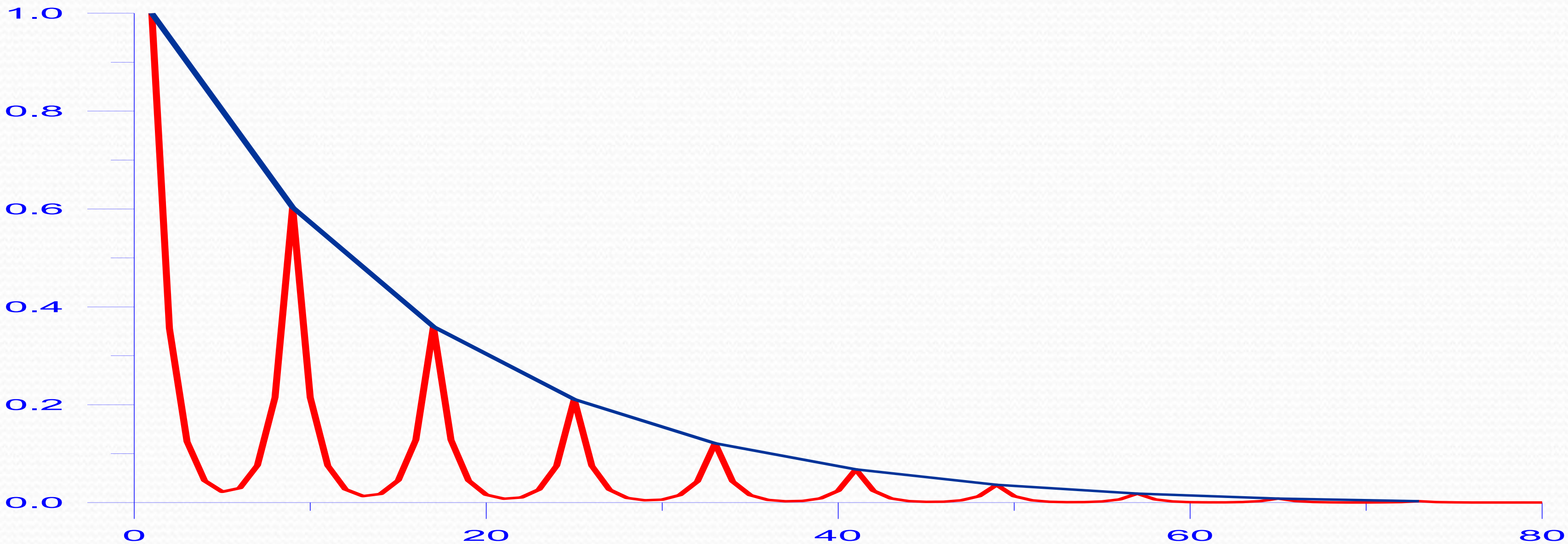
$$R_{(n)} = G_p \otimes R_n,$$

where

$$G_p = (g_p^{ij}),\ g_p^{ij} = \exp(-\alpha|i-j|), \quad i, j = 0, \ldots, p-1,$$

$$R_n = (r_n^{kl}),\ r_n^{kl} = \exp(-\beta|k-l|), \quad k, l = 0, \ldots, n-1,$$

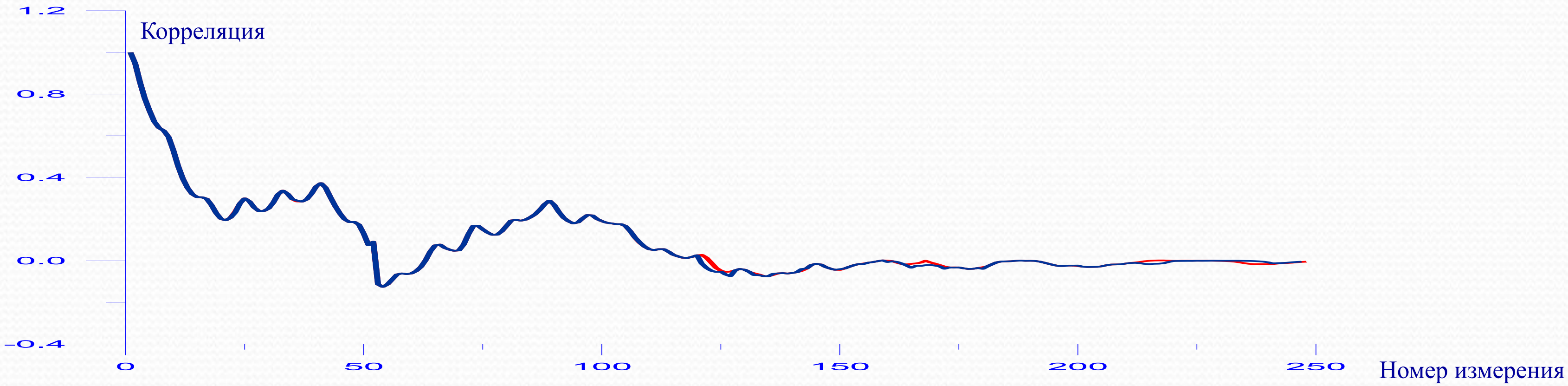
$$\alpha = 0.9, \quad \beta = 0.4, \quad p = 8.$$



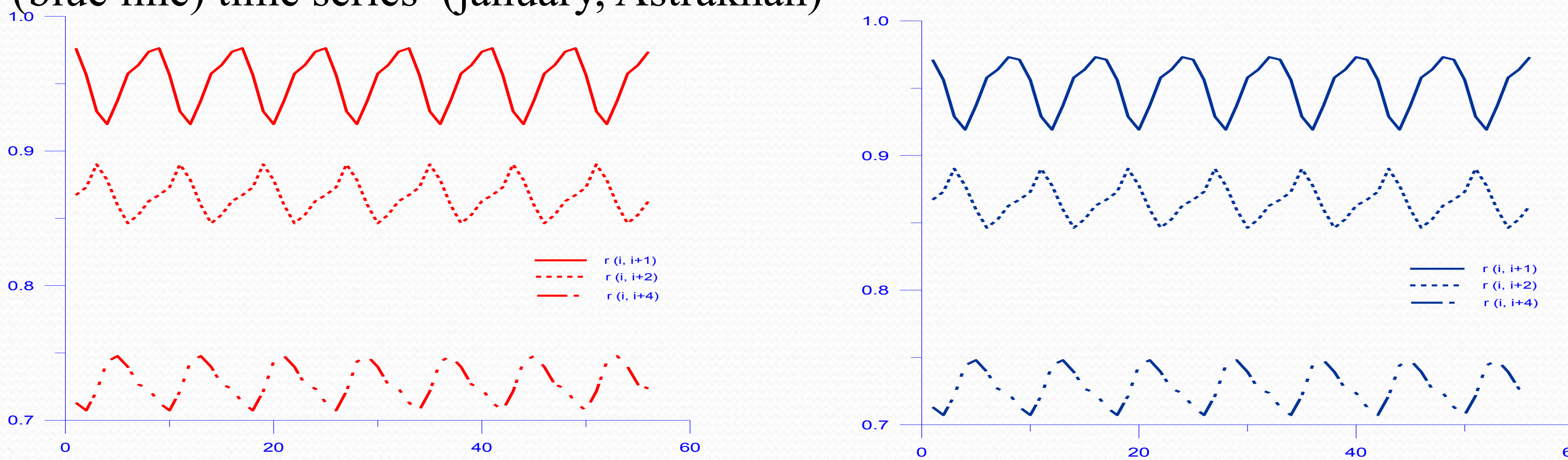
Correlation function of sequence calculated with the help of the model samples in the stationary approximation (red curve) and piecewise-linear envelope curve (blue curve).

Numerical simulation of air temperature time series

1. Stationary model with correlation function estimated by real data
2. Stationary model with piecewise-linear correlation function
3. Periodically correlated model



Correlation functions of actual (red line) and simulated in stationary approximation (blue line) time series (january, Astrakhan)



Correlation functions of real (left part) and model (right part) time series.(Periodically correlated, january, Astrakhan)

Statistical characteristics of real and simulated time series

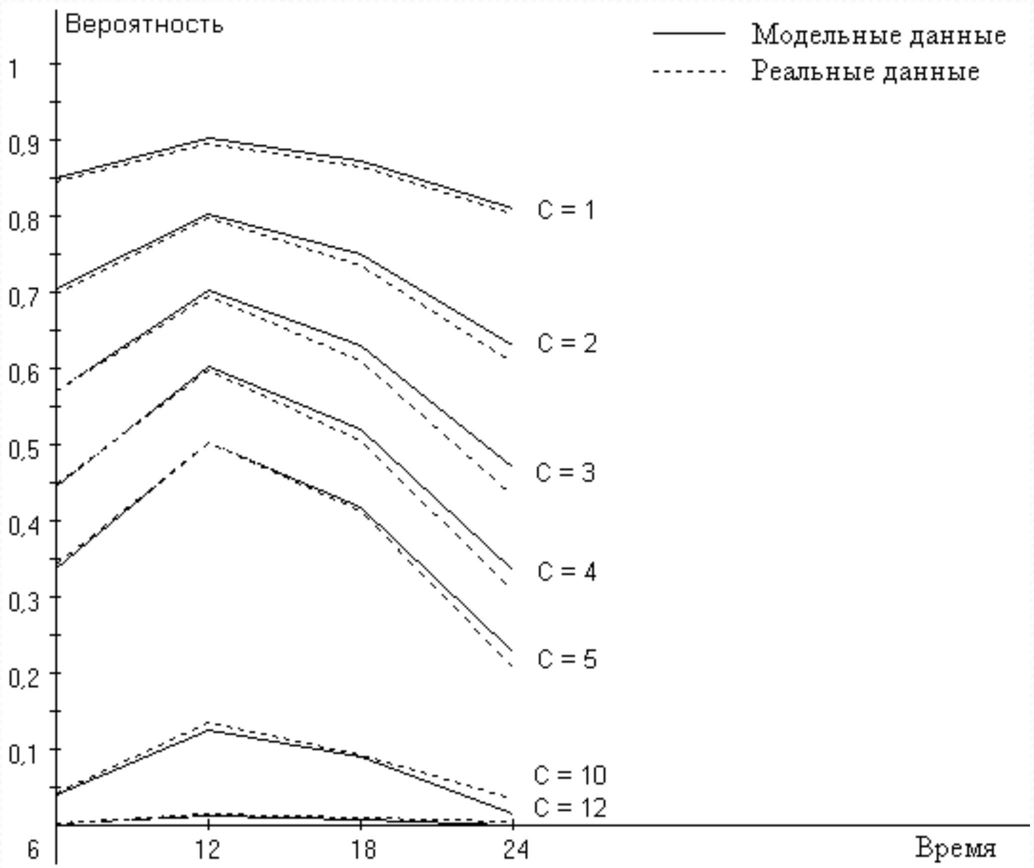
LºC	3 hours				6 hours			
	РД	СМ	СМКЛ	ПКМ	РД	СМ	СМКЛ	ПКМ
-5	0.6509	0.6512	0.6516	0.6508	**0.5412**	**0.5415**	**0.5419**	**0.5410**
-10	0.0364	0.0362	0.0371	0.0365	0.0347	0.0342	0.0350	0.0348
-20	**0.0312**	**0.0315**	**0.0319**	**0.0311**	0.0301	0.0307	0.0311	0.0302
-25	0.0282	0.0283	0.0284	0.0280	0.0225	0.0229	0.0232	0.0224
-30	0.0000	0.0001	0.0001	0.0001	0.0000	0.0001	0.0001	0.0003

Table 1. Probabilities of the events: the air temperature decreases below of the certain level during the given period

t ºC	6 hours				9 hours			
	РД	СМ	СМКЛ	ПКМ	РД	СМ	СМКЛ	ПКМ
5	0.1310	0.1520	0.1582	0.1339	0.2325	0.2397	0.0241	0.2334
10	0.0112	0.0201	0.0012	0.0109	0.0330	0.0313	0.0311	0.0330
15	**0.0020**	**0.0009**	**0.0009**	**0.0018**	0.0040	0.0037	0.0036	0.0041
20	0.0004	0.0002	0.0002	0.0003	**0.0007**	**0.0011**	**0.0012**	**0.0007**
25	0.0002	0.0001	0.0001	0.0003	0.0002	0.0002	0.0001	0.0002

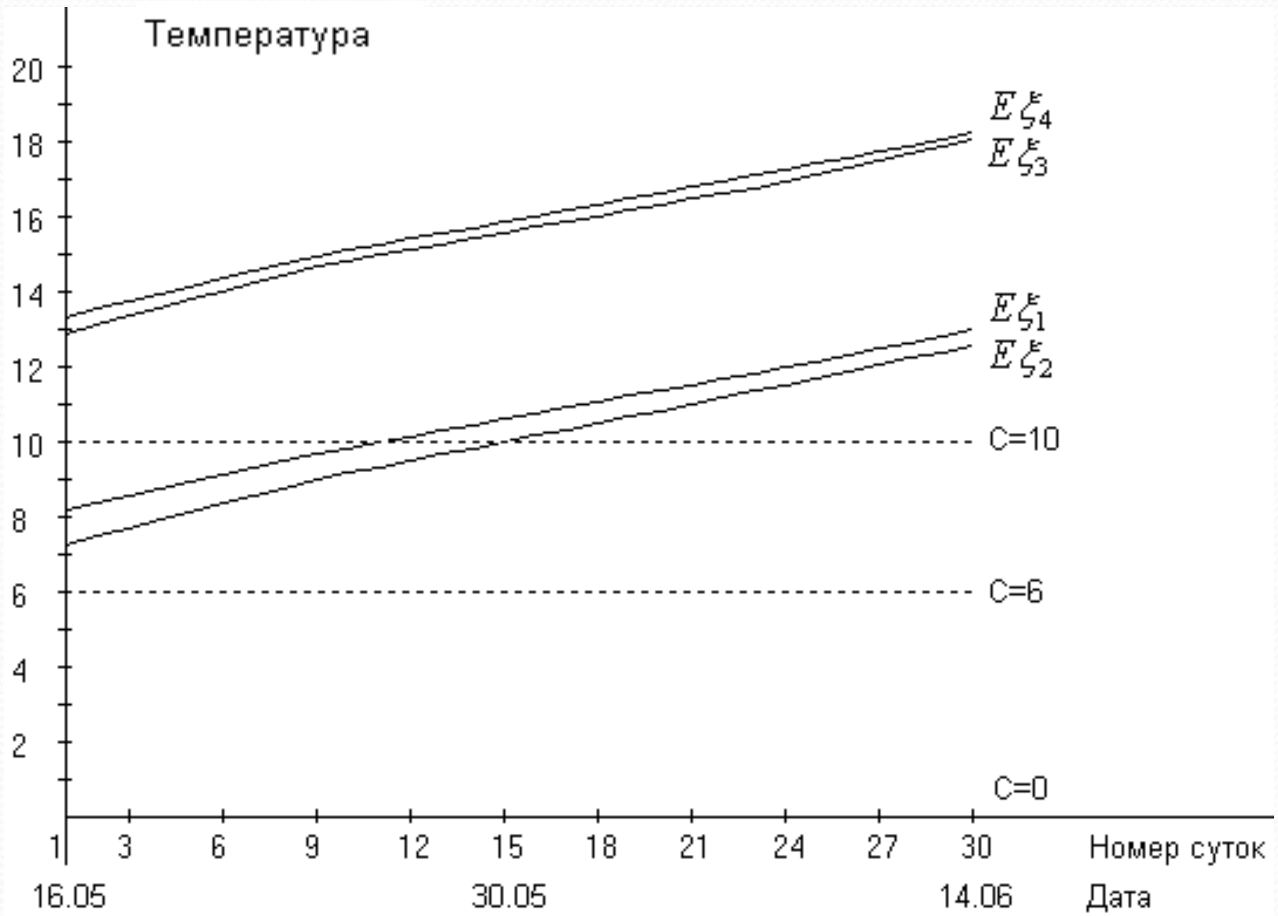
Table 2. Probabilities of the events: temperature changes on the certain value during the given period

Statistical characteristics of real and model time series
(periodically correlated model)



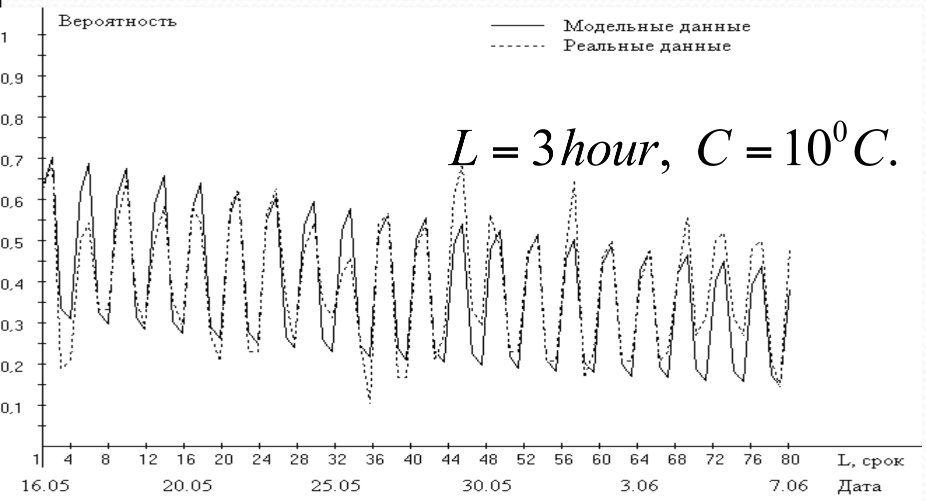
Probability $P(\left|\xi_{ip+l} - \xi_{jp+m}\right| \geq C)$ that the temperature changes by C = 1,2,3,4,5,10,12 (°C) during 6,12,18,24 hours. Model data - (-----) and real data - (- - -) (May-June, Sverdlovsk)

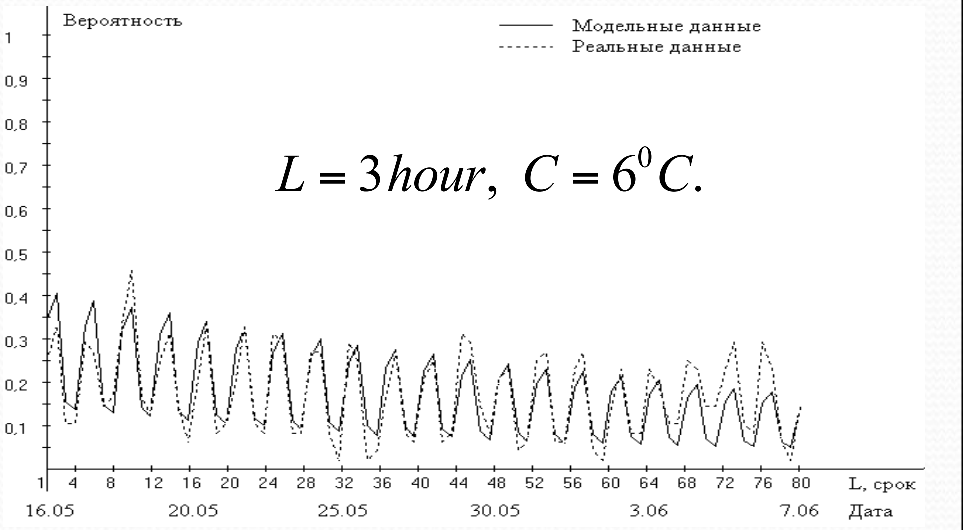
Periodically correlated model with linear trend

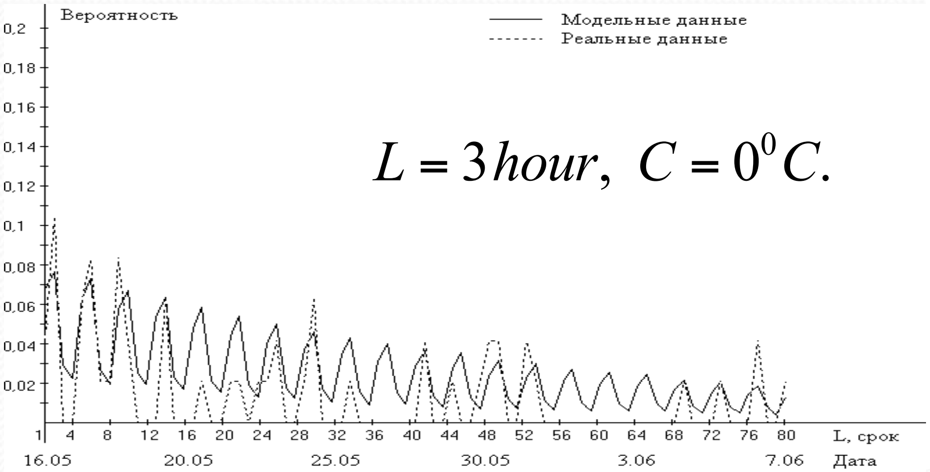


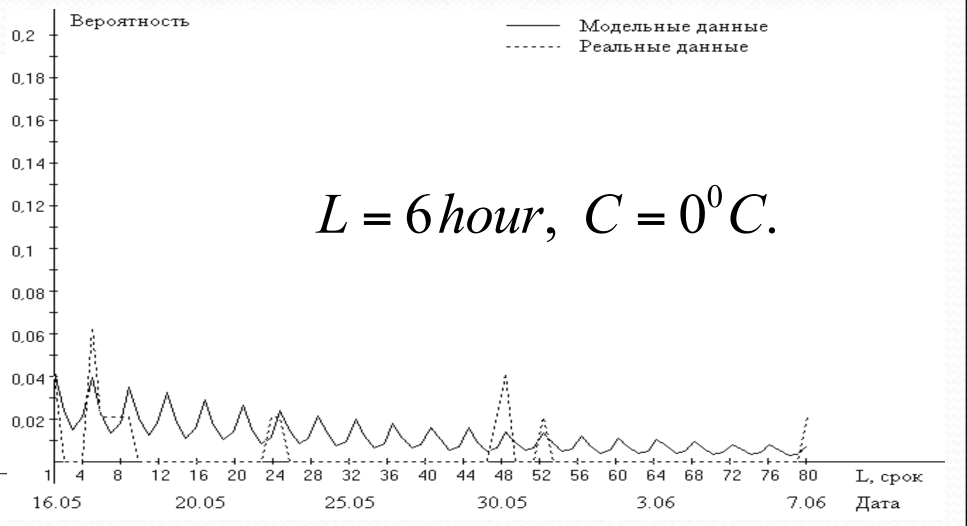
Average values $E\xi_i,\ i = 1,\ldots,p$ of air temterature and levels C = 0, 6, 10 (°C) by day (Sverdlovsk).

Probability $P_{i,l}(c, L)$ of temperature decreasing below the level C during the given period

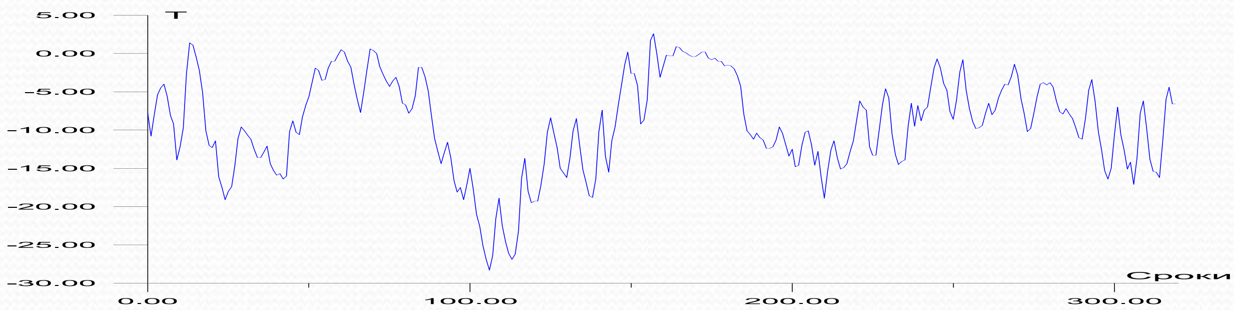


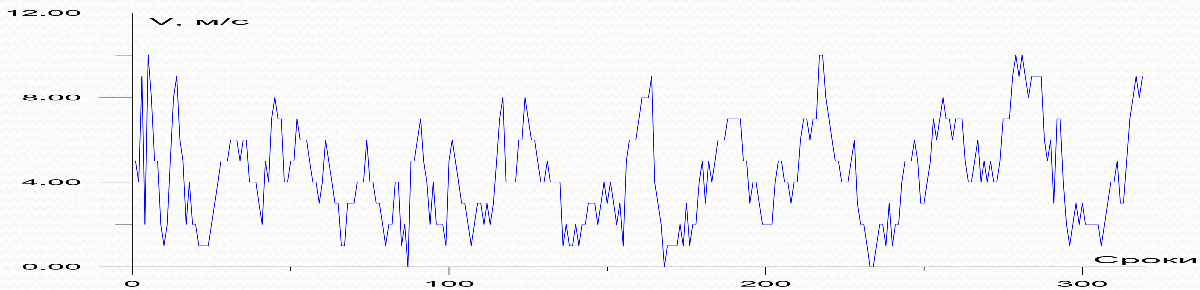






Numerical models of non-stationary time series of air temperature and wind (Astrakhan)





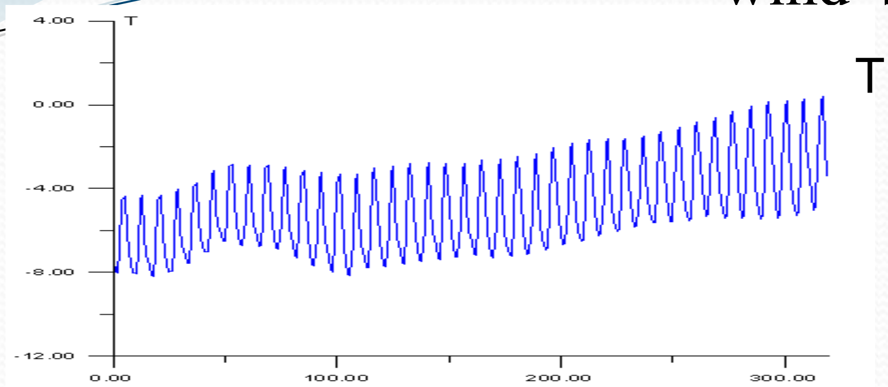
$$R_{\vec{\xi}\vec{\xi}} = M\left(\vec{\xi} - M\vec{\xi}\right)\left(\vec{\xi} - M\vec{\xi}\right)^T = \begin{pmatrix} R_{\vec{\xi}^{(1)}\vec{\xi}^{(1)}} & R_{\vec{\xi}^{(1)}\vec{\xi}^{(2)}} \\ R_{\vec{\xi}^{(2)}\vec{\xi}^{(1)}} & R_{\vec{\xi}^{(2)}\vec{\xi}^{(2)}} \end{pmatrix},$$

$$R_{ks}^{*(ij)} = \frac{1}{(2L+1)N-1}\sum_{l=-L}^{L}\sum_{n=1}^{N}\left(\vec{\xi}_{k+l}^{*(i)}[n] - \vec{m}_k^{*(i)}\right)\left(\vec{\xi}_{s+l}^{*(j)}[n] - \vec{m}_s^{*(j)}\right)^T.$$

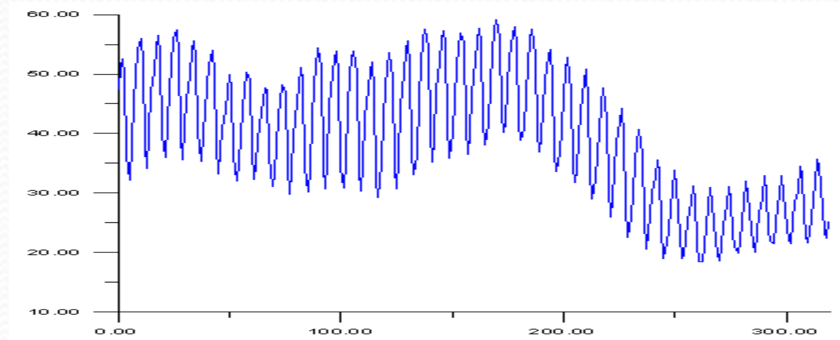
N - years, L – smoothing step, $k,s = 1,\ldots,K$, K, - number of days

$i,j = 1,2$ - number of element (1 – temperature, 2 – module of wind speed)

Empirical characteristics of air temperature and wind speed



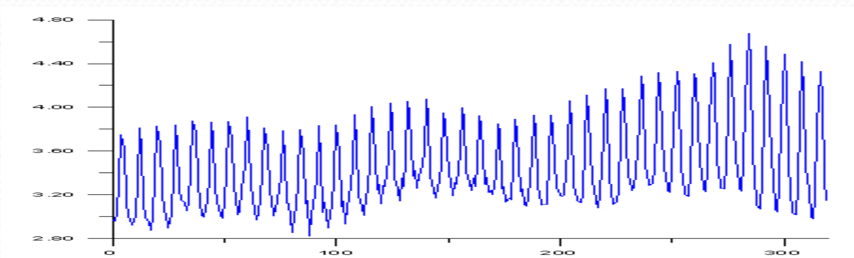
T

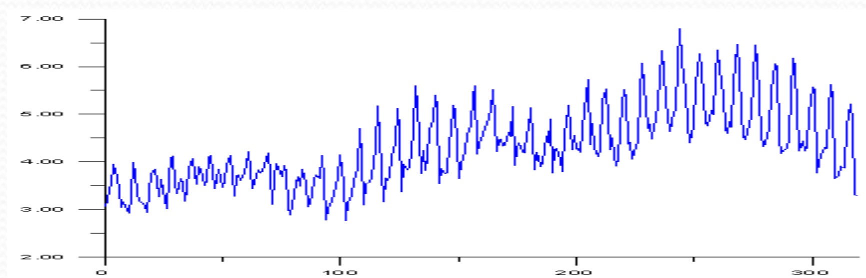


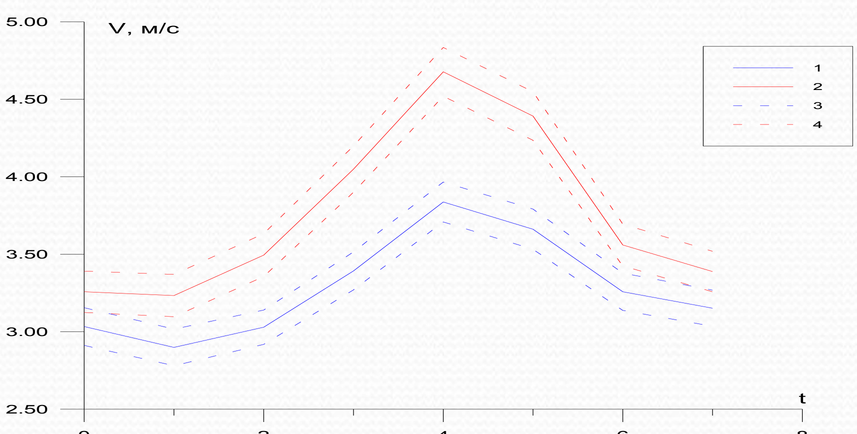
Mean

V

Variance



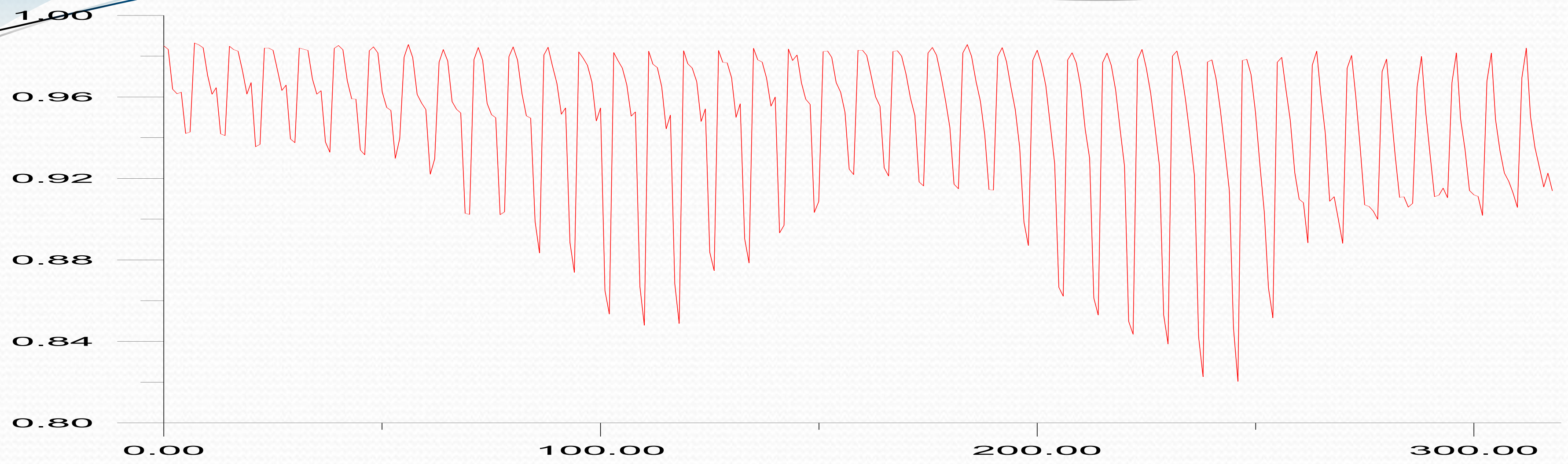




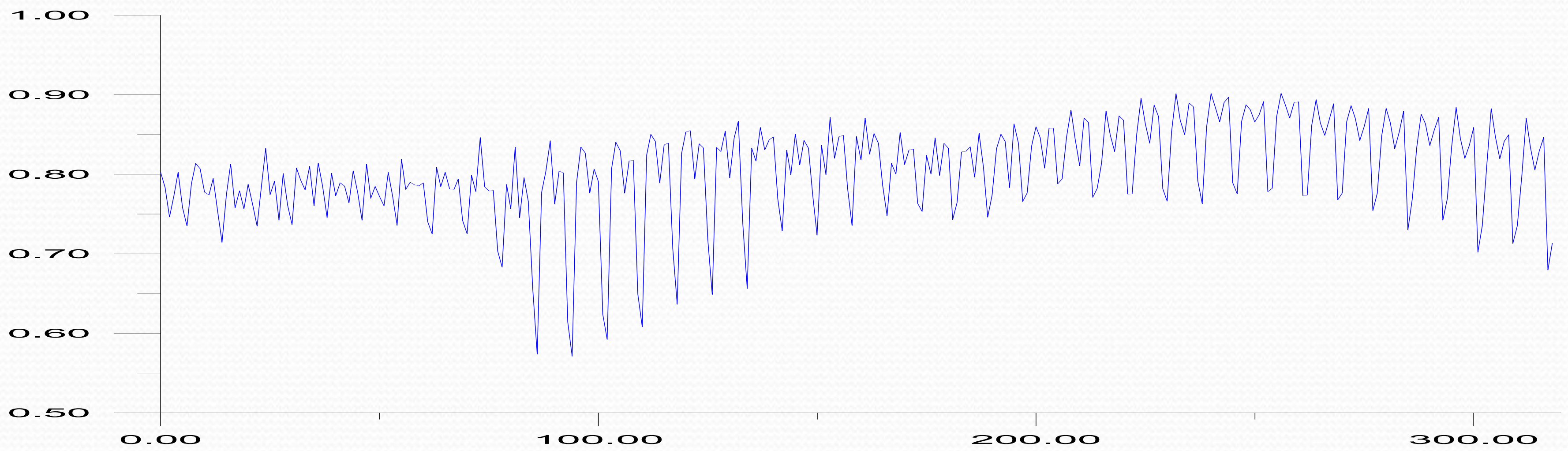
Asymmetry

Means of module of wind speed for 2 days:
1 – mean for 23 of january;
2 – mean for 24 of february;
3, 4 – limits of statistical estimation error 1 и 2.

Structure of correlation matrix



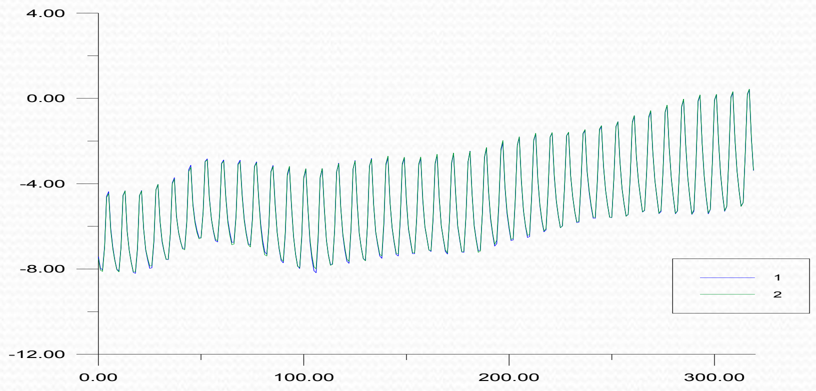
The first collateral diagonal of the correlation matrix of air temperature



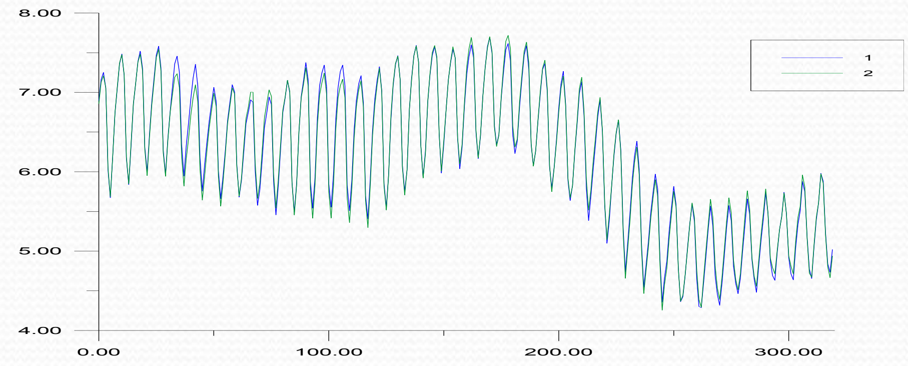
The first collateral diagonal of the correlation matrix of module of wind speed

Precision of reproduction of the entrance parameters of models

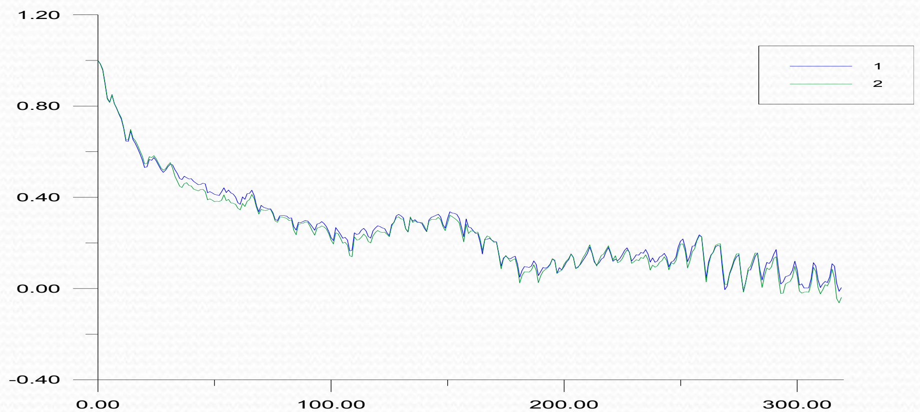
Mean



Standart deviation

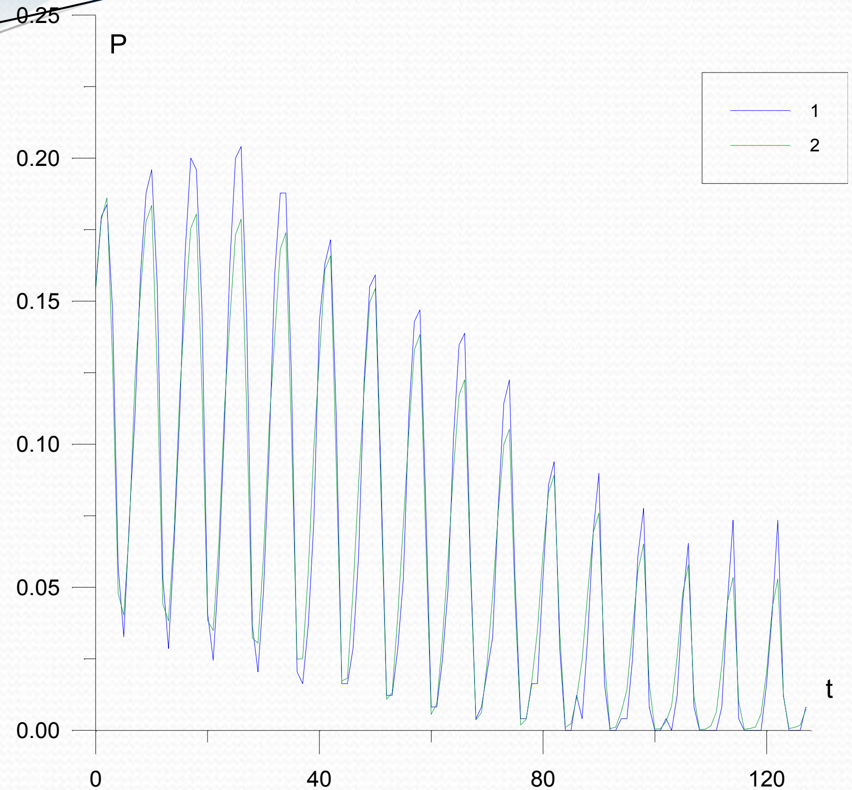


Line of correlation matrix

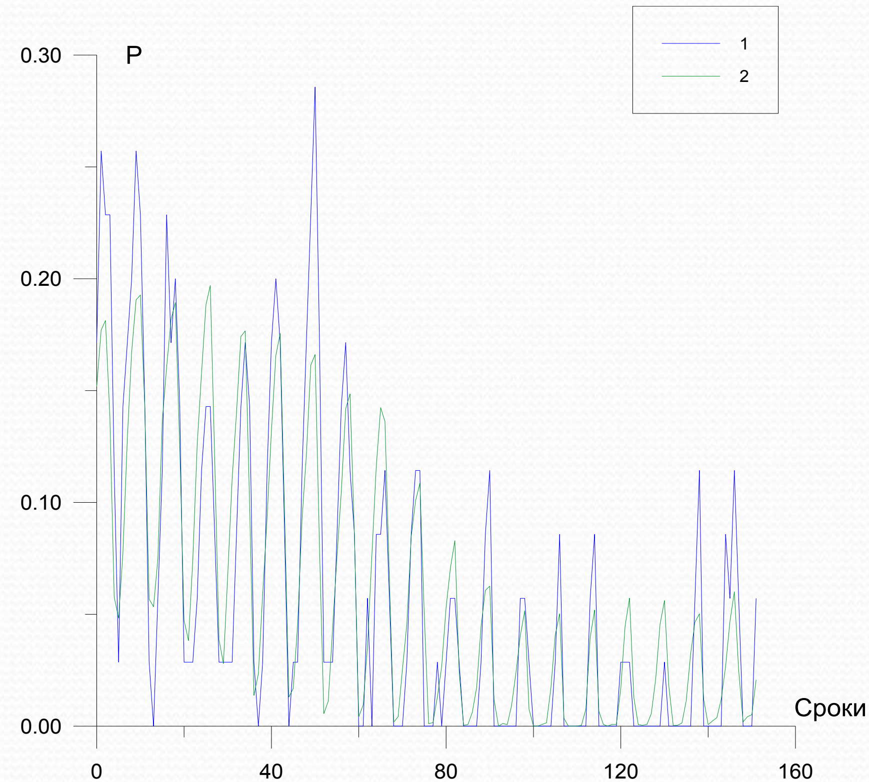


1 – real data, 2 – model data

Results of joint modeling of air temperature and module of wind speed



Wind speed 2,5 m/s - 4,5 m/s

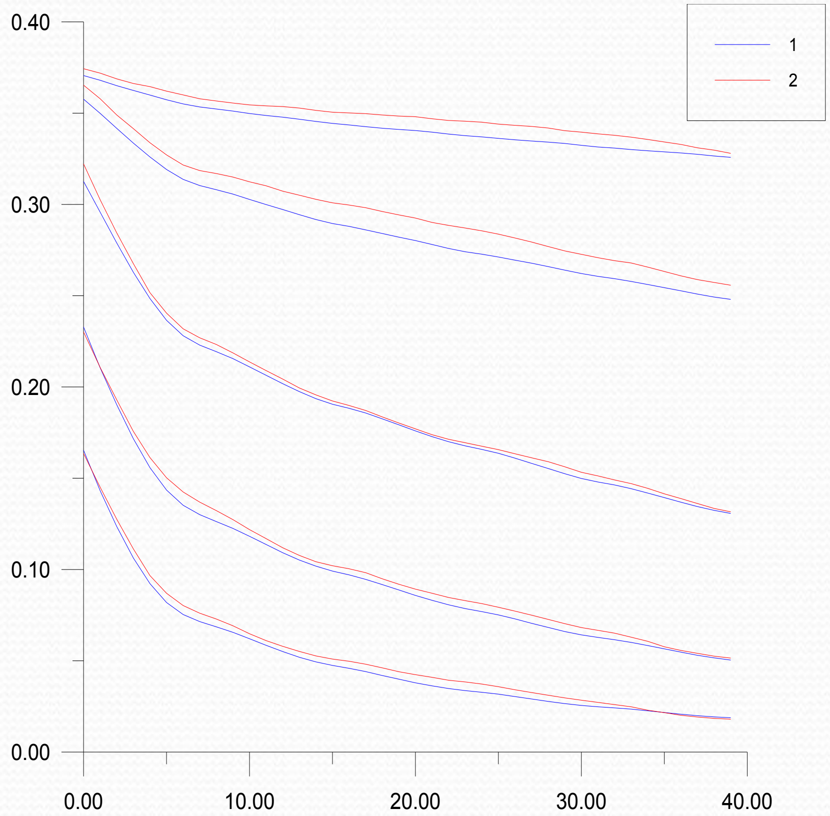


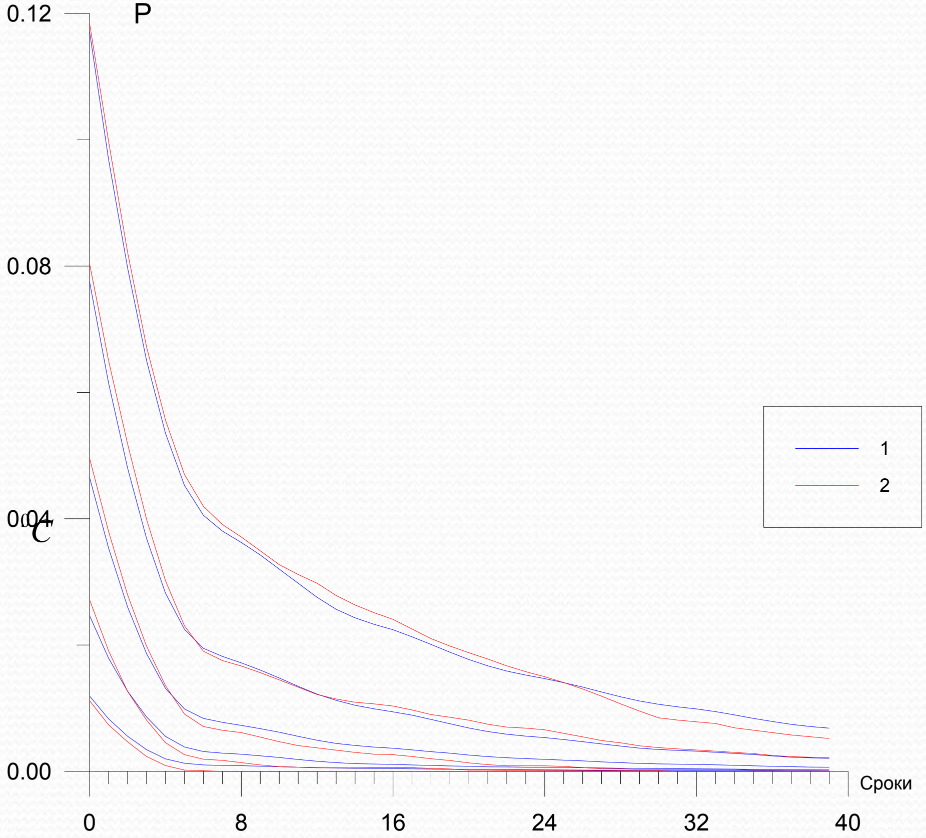
Wind speed 4,5 m/s - 6,5 m/s

Probabilities of joint realization of events: $T < -15^0 C, \quad V_1 \leq |V| < V_2$

1 – model data, 2 – real data.

Results of joint modeling of air temperature and module of wind speed



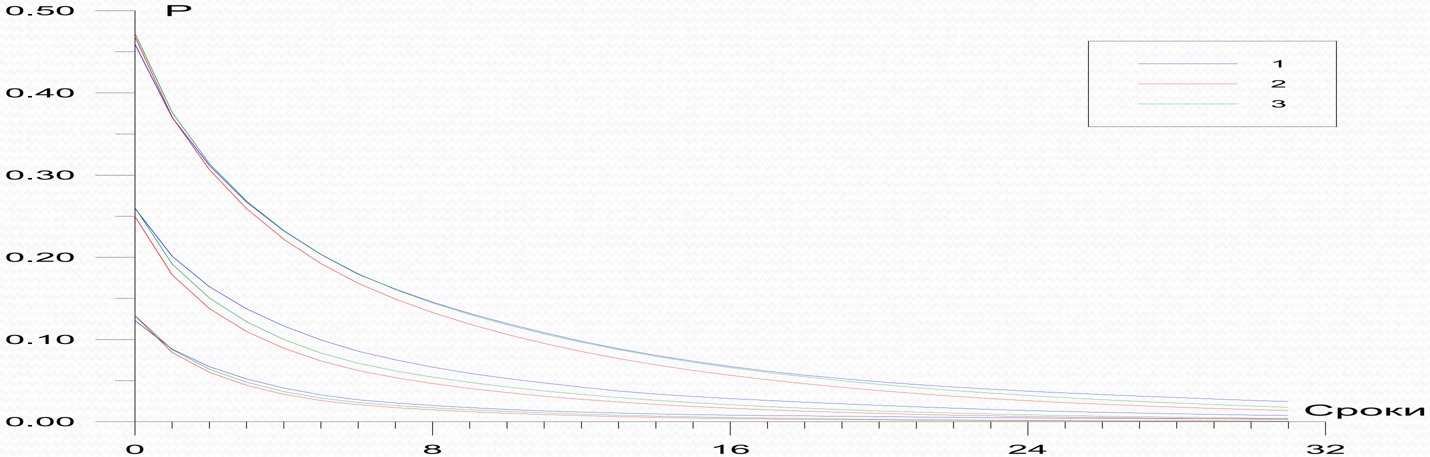


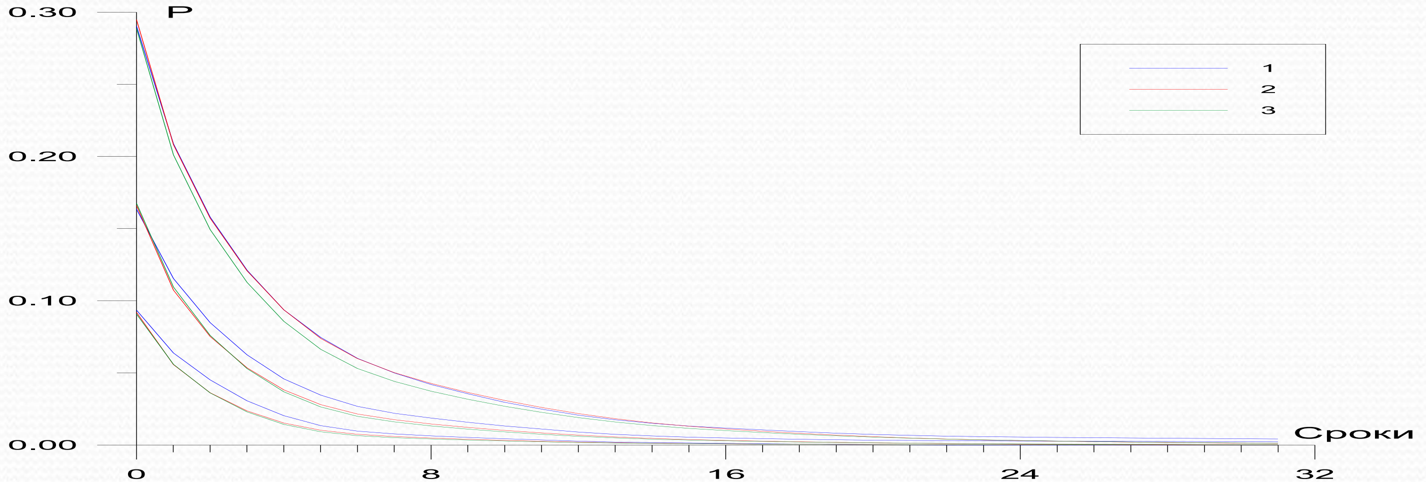
Probabilities of joint realization of events: $|V| - 2.5\ m/s, \quad T < T_i,$

$T = +7, +4, +1, 0, -2, -8, -11, -14, -17, -20\ {}^0C,$ 1 – model data, 2 – real data.

(Astrakhan)

Results of joint modeling of air temperature and module of wind speed





Probabilities of joint realization of events: $|V| - 2.5\ m/s^0C, \quad T < T_i,$

$T = +7, +4, +1, 0, -2, -8, -11, -14, -17, -20\ {}^0C,$ 1 – model data, 2 – real data.

(Astrakhan)

Conclusion

The numerical stochastic parametrical models of joint time series of various weather elements (air temperature , speed of a wind, relative humidity etc.), taking into account one-dimensional distributions and matrix correlation functions of real processes are constructed. The approximation of periodically correlated process is used. According to this approximation the daily periodic character of parameters of one-dimensional distributions and correlation functions is taken into account.

On the basis of these models the statistical properties of the adverse meteorological phenomena (for example, long adverse temperature phenomena, adverse combinations of meteorological elements etc.) are investigated.

The study of statistical characteristics of atmospheric processes involving adverse weather conditions, such as significant long-term decreases of air temperature, droughts, long-term stormy weather when a strong wind is accompanied by heavy precipitation, etc. is of great scientific and practical importance. For example, this is important in problems of agroclimatology, navigation, planning of heating systems, and many other applications.

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Thanks for your attention!