Sensitivity of the surface air temperature to small perturbations of the SST on seasonal/annual time intervals from fluctuation-dissipation theorem

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“Atmospheric” system

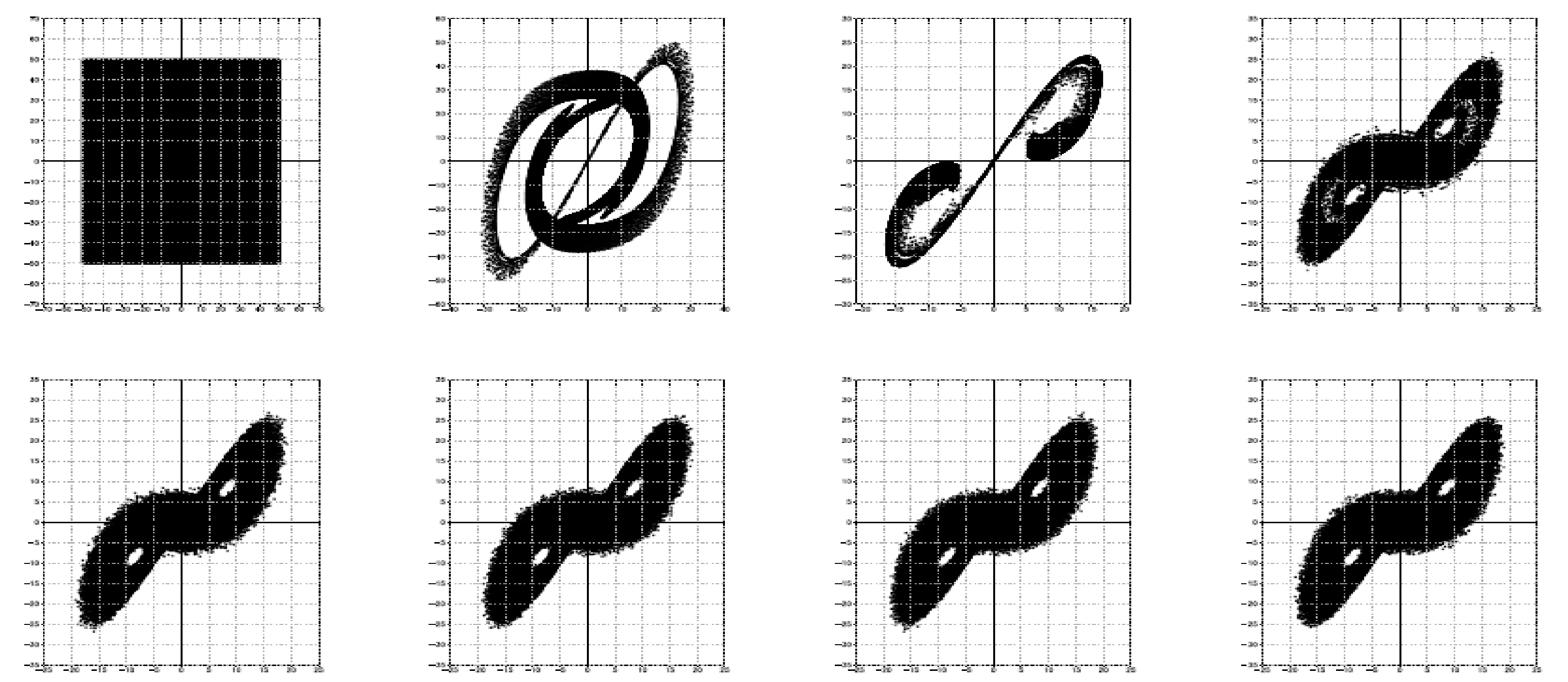
$$\frac{du}{dt} = F(u) + f.$$

$$u(0) = u_0, \quad F : R^N \to R^N \quad f = const(t).$$

- **Lorenz’63 “toy” atmospheric model**
- **Barotropic model (2D NS system) &CO (2level QG, Marshall-Molteni etc.)**
- **Primitive equation systems (Atmospheric GCMs) with no annual cycle**

$$\frac{du}{dt} = F(u) + f, u(0) = u_0 \quad \rightarrow \quad u(t) = S(t,0)u_0$$

In many important cases almost all trajectories fall into small compact invariant set (system attractor) and stay on it forever



evolution of 1 million point ensemble for Lorenz'63, t=0, 0.05, 0.2, 1., 2.5, 5., 15., 30 (x,y) plane

ATTRACTOR

invariant, attracting and compact set inside system phase space

All simple atmospheric models (barotropic vorticity equation, 2level QG system, 2D Navier-Stokes (all inf. dimensional)) have finite dimensional attractors

Bogolyubov theorem: There is (at least one) invariant measure (equilibrium PDF) on the attractor. In typical cases there is "physical" (or Sinai-Ruelle-Bowen or SRB) measure on the attractor.

CLIMATE = "SET OF SYSTEM STATES FOR LONG EVOLUTION TIMES" = ATTRACTOR

Let us take some functional of interest *W(u)* and calculate its average value along trajectory *u(t)*

$$\frac{du}{dt} = F(u) + f, u(0) = u_0 \to u(t) = S(t)u_0$$

$$\langle W(u) \rangle (u(0)) = \lim_{T\to\infty} \frac{1}{T}\int_0^T W(u(t))dt.$$

(limit exists because of Birkhoff theorem)

Time-averaging for ergodic systems *does not depend* on initial condition for almost all *u(0)*.

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T W(u(t))dt = \langle W \rangle = \int_A W d\mu$$

SYSTEM STATISTICAL CHARACTERISTICS

ARE

AVERAGES OVER INVARIANT MEASURE ON THE

SYSTEM ATTRACTOR

(ergodic hypothesis is implied here)

How about annual cycle?

$$\frac{du}{dt} = F(u,t) + f$$

$$F(u,t+T) = F(u,t)$$

global solvabilty, attractor existence for 2D NS (Chepyzhov, Vishik, 1993,4)

(not sure about SRB measure)

Let us change forcing of the system (apply external perturbation δf)

$$\frac{du^1}{dt} = F(u^1) + f + \delta f .$$

New system statistics will be

$$\left\langle W(u^1) \right\rangle = \int_{A_1} W d\mu^1 = \lim_{T\to\infty} \frac{1}{T}\int_0^T W(u^1(t))dt .$$

Response of the system depends on δf somehow

$$\delta \left\langle W \right\rangle = < W(u^1) > - < W(u) > = M(\delta f)$$

$$\delta\langle W\rangle = M\delta f\,.$$

Knowing *M* we can solve a number of important inverse and sensitivity problems

(i.e. "what forcing produces prescribed response of the system?" or "what forcing gives the largest response?")

Can we find *M*?

For small δf we may expect linearity of the response (formal Taylor expansion)

$$\delta\langle W\rangle(\delta f) = <W(u^1(\delta f))> - <W(u)>$$

$$<W(u^1)>(\delta f) = <W(u)> + \left[\frac{\partial(\int W d\mu^1)(\delta f)}{\partial \delta f}\right]_{\delta f=0}$$

$$\delta\langle W\rangle = M\delta f, \quad M = \left[\frac{\partial(\int W d\mu^1)(\delta f)}{\partial \delta f}\right]_{\delta f=0}$$

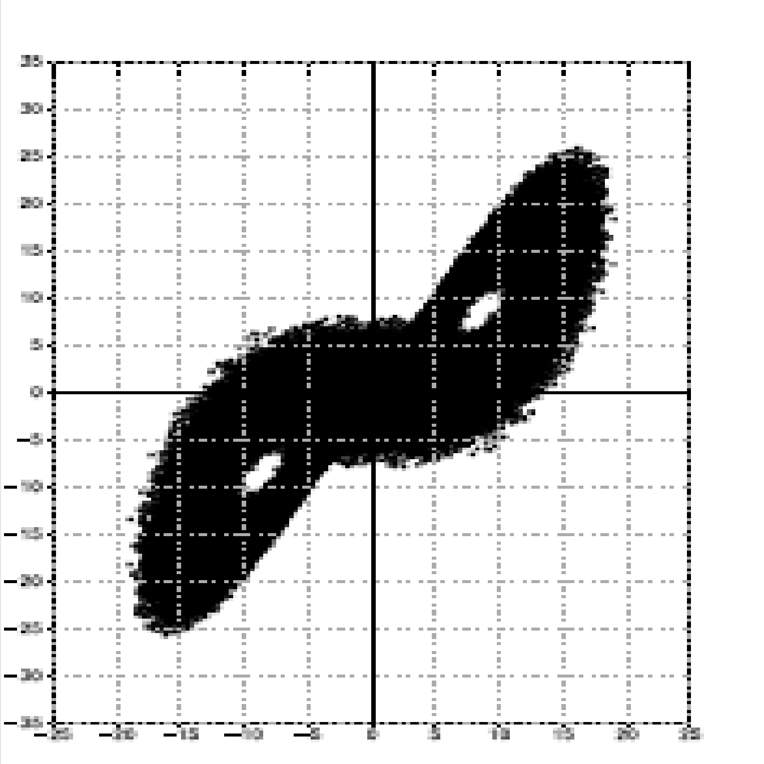
PROBLEM!

SYSTEM ATTRACTOR IS A FRACTAL SET

MEASURE IS SINGULAR

$$\frac{du}{dt} = F(u) + f$$

$$\delta f$$



Distribution of states on the attractor (PDF) depends on system parameters, response operator relates changes in PDF moments to changes in external forcing.

$$\delta\langle W \rangle = M\delta f$$

Change system by adding regularization (noise) into RHS.

Add small noise into the system right hand side (*Zeeman, 1988*) to avoid PDF singularities

$$\frac{du}{dt} = F(u) + f + \varepsilon\eta(t),$$

$\eta(t)$ **is normally distributed white noise.**

Physically reasonable!

stochastic parameterizations in GCMs, rounding errors in algorithms etc…

PDF of the system can be obtained from the Fokker-Planck equation (*dμ=ρdV*).

$$\frac{\partial \rho}{\partial t} + div((F + f)\rho) = \varepsilon \Delta \rho, \ \rho(0) = \rho_0$$

For a wide class of systems (including 2D Navier-Stokes) it has unique stationary solution ρ_{st} **(see *Kuksin, Shirikyan, Mattingli*)**

and $\rho(t) \xrightarrow[t \to \infty]{} \rho_{st}$ **for any** ρ_0

PDF of the perturbed system also obey Fokker-Planck equation

response could be obtained from FP equation theory in the form of general fluctuation-dissipation theorem (FDT)

FDT: to the first order in *δf* *(Dekker, Haake (1974), Risken (1984))*

$$\delta < W > (t) = \int W(u)\rho^{1}(u,t)du^{1} - \int W(u)\rho(u)du =$$

$$\int < W(u(t' + \tau))B(u(t')) > d\tau$$

Where *B(u)* is defined as

$$B(u) = (1/\rho)(-div(\rho\delta f)).$$

For normal pdf $\rho = c\exp(-C^{-1}(0)u,u)/2,$

$$M = \int_{0}^{t} < W(u(t' + \tau))u(t')^{T} > C^{-1}(0)d\tau$$

For quasi-gaussian PDF of the system we arrive at

$$M \approx \int_0^t < W(u(t' + \tau))u(t')^T > C^{-1}(0)d\tau$$

If *M=u* (response of the first moments) the classical *(Kraichnan (1959))* formula takes place.

$$M = \int_0^t C(\tau)C^{-1}(0)d\tau$$

How about annual cycle?

$$M(t) = \int_0^t < W(u(t))u(t-\tau)^T C_{t-\tau}^{-1} >_{t-\tau} d\tau$$

general and quasi-gaussian cases (Majda, Wang, 2008)

IN ALL CASES WE NEED DATA ONLY! COULD WORK FOR REAL CLIMATE SYSTEM!

$$\boxed{\delta < W >= M\delta f \quad M = \int_0^t < W(u(t'+\tau))u(t')^T > C^{-1}(0)d\tau}$$

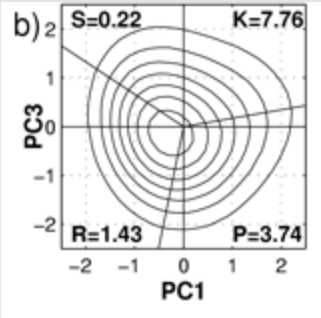
Not so good:

1. **Inversion of covariance matrix may produce large numerical errors**

$$C_0 \approx \hat{C}_0 = \frac{1}{L}\sum_{l=1}^{L} h'(l)(h'(l))^T, C_0 - \hat{C}_0 \approx \frac{B}{L^{1/2}}$$

- **need lot of data!**
- **calculation of *M* may not be possible for full phase space dimension**

2. **May not work for system with nongaussian pdfs**



However NONparametric PDF approximation (*Cooper, Hynes, 2011*) could work

PDF of NCAR CCM0 (EOF1, EOF3 plane), (*Berner, Branstator,2007*)

3. **Time dependent right hand side could make problems**

time-periodic case will require even more data! Aperiodic time dependence?

$$\longrightarrow \quad M(t) = \int_0^t < W(u(t))u(t-\tau)^T C_{t-\tau}^{-1} >_{t-\tau} d\tau$$

$$\delta < W >= M\delta f$$

$$M = \int_0^t < W(u(t+\tau))u(t)^T > C^{-1}(0)d\tau$$

Numerical procedure:

1. **Run the model as long as you can. (1 million days should be enough).**
2. **Reduce operator dimension to ensure accurate inversion of C(0).**
3. **Calculate covariances and produce operator.**

Idea - *(Leith, 1975)***;** **barotropic T5 system -** *(Bell, 1980)***;** **barotropic T12 system, 2x QG model -** *(Gritsun,Dymnikov, 1999, 2001)***;** **NCAR GCM model CCM0, INM RAS GCM model 5x4x21 -** *(Gritsun, Dymnikov,Branstator(2002)),(Dymnikov,Gritsun,2005), (Gritsun,Branstator, 2007), (Gritsun, Branstator, Majda, 2008),* **NCEP/ NCAR reanalysis data** *(Dymnikov,Gritsun,2005),* **time-periodic INM RAS GCM** *(Gritsun,2010).*

1. Fluctuation-dissipation theorem (in quasi-gaussian time-periodic or time-constant forms) seems to give robust method for calculating response operators for atmospheric systems.

2. Construction of approximate response operator provides method for analysis some important problems (estimation of model sensitivity to external forcing, finding most dangerous forcing, solving inverse problems)

SENSITIVITY OF THE AIR SURFACE TEMPERATURE

to EXTERNAL HEATING ANOMALIES on

SEASONAL INTERVALS

REAL SYSTEM (DATA) / ATMOSPHERIC GCMS

Is it possible to get reliable estimate for the response operator having 63 winters only?

$$M = \int_0^T C_{t^0-\tau}(\tau)(C_{t^0-\tau}(0))^{-1} d\tau$$

1. **Have to calculate inverse covariance matrix for every date.**

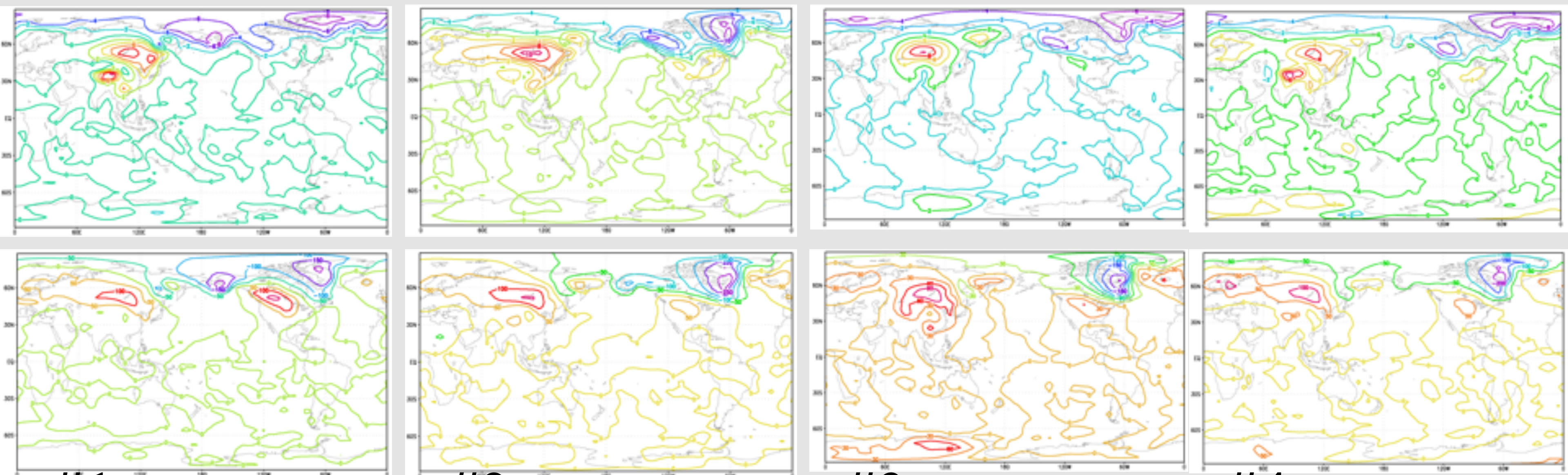
2. **63yrs of NCEP/NCAR could not be enough (annual cycle, El-Nino variability, PDO).**

3. **Aperiodic forcing acting on real system could be a problem.**

Sensitivity of the response operator for INM A5421 model on data

4 operators calculated for different 5400day datasets from INM A5421 output, 50EOFs were used to invert C(0)

Leading forcing /surface temperature response patterns



#1 **#2** **#3** **#4**

To get stable results the response operator must be truncated to the space of 30-50 leading EOFs of C(0) (i.e. the forcing should be in this space).

$$W(u) = T_{surf}$$

$$\delta < W >= M\delta f$$

$$\boxed{M = \int_0^{\Theta} C_{t^0 - \tau}(\tau)(C_{t^0 - \tau}(0))^{-1} d\tau}$$

calculate response for Jan and Feb; forcing is acting 5 months

$$M = P\Lambda Q$$

next pictures: leading response patterns (surface temperature);

leading perturbation (heating at 200,400,600,700,800,900mb).

(80 EOFs are used for inversion of C(0))

NCAR CAM3

Atmospheric general circulation model CAM3 ([Collins, 2006])

- prognostic variables – (u,v) wind components, temperature, relative humidity, surface pressure
- hybrid vertical coordinate, 26 levels, T42, dim=200000
- DATA: 12000 yrs, (fixed) annual cycle

INM A5421

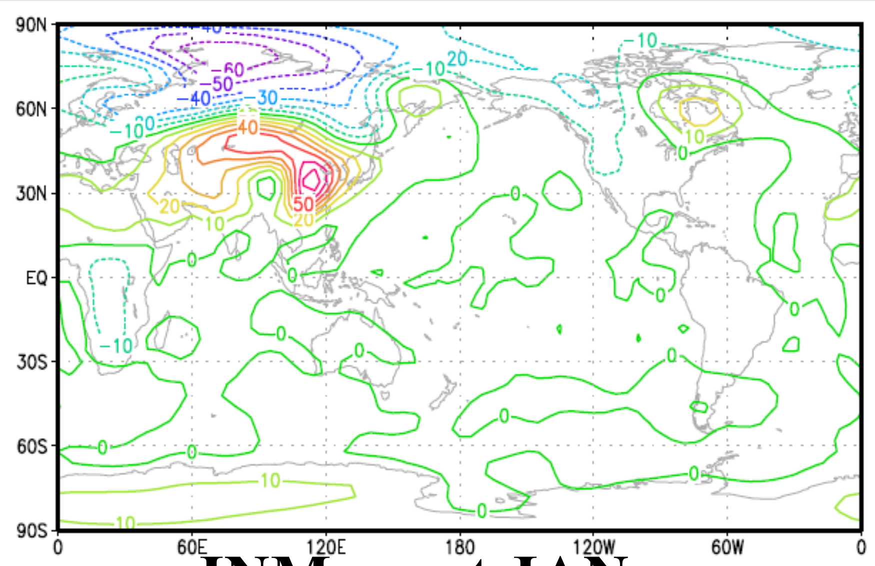
Atmospheric general circulation model A5421 ([Alexeev et.al, 1998])

- prognostic variables – (u,v) wind components, temperature, relative humidity, surface pressure
- sigma vertical coordinate, 21 levels, 5x4 horizontal, dim=200000
- DATA: 200x60 yrs, (fixed) annual cycle AND 200x60 yrs constant January

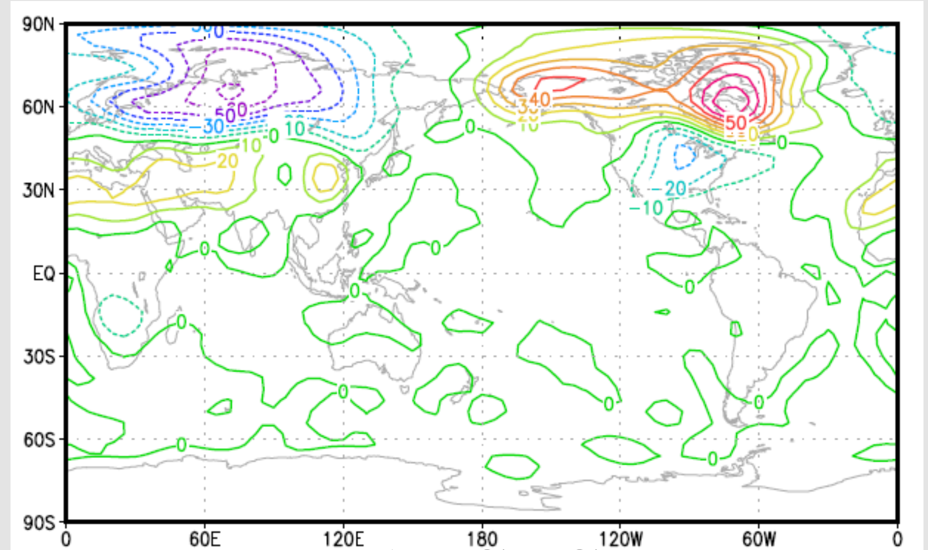
NCEP/NCAR reanalysis data

- (u,v) wind components, temperature
- pressure vertical coordinate, 21 levels, interpolated to 5x4 horizontal
- DATA: 64 yrs, (variable) annual cycle

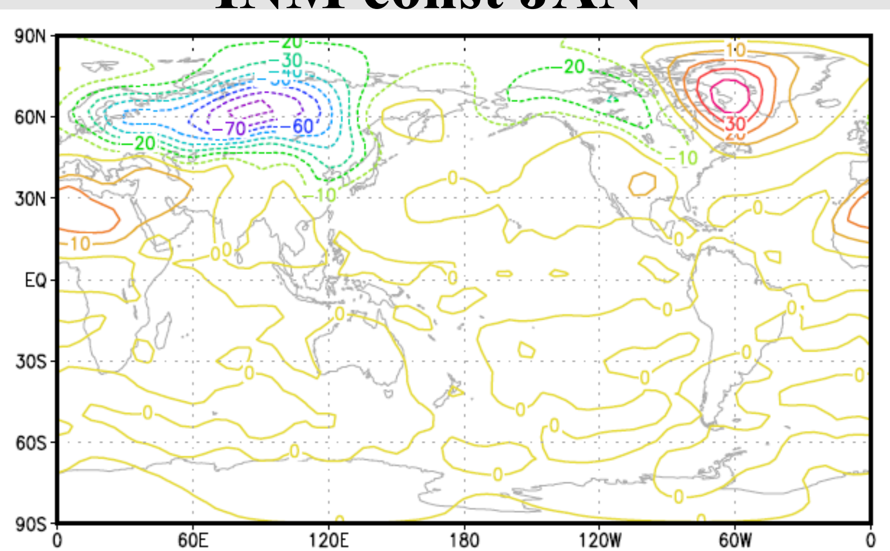
Surface temperature responses



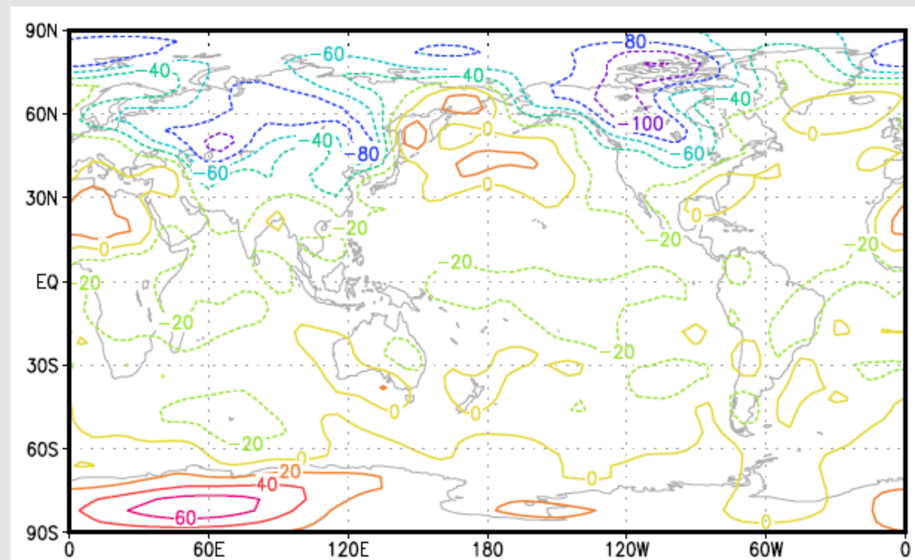
INM const JAN



INM ANCYC

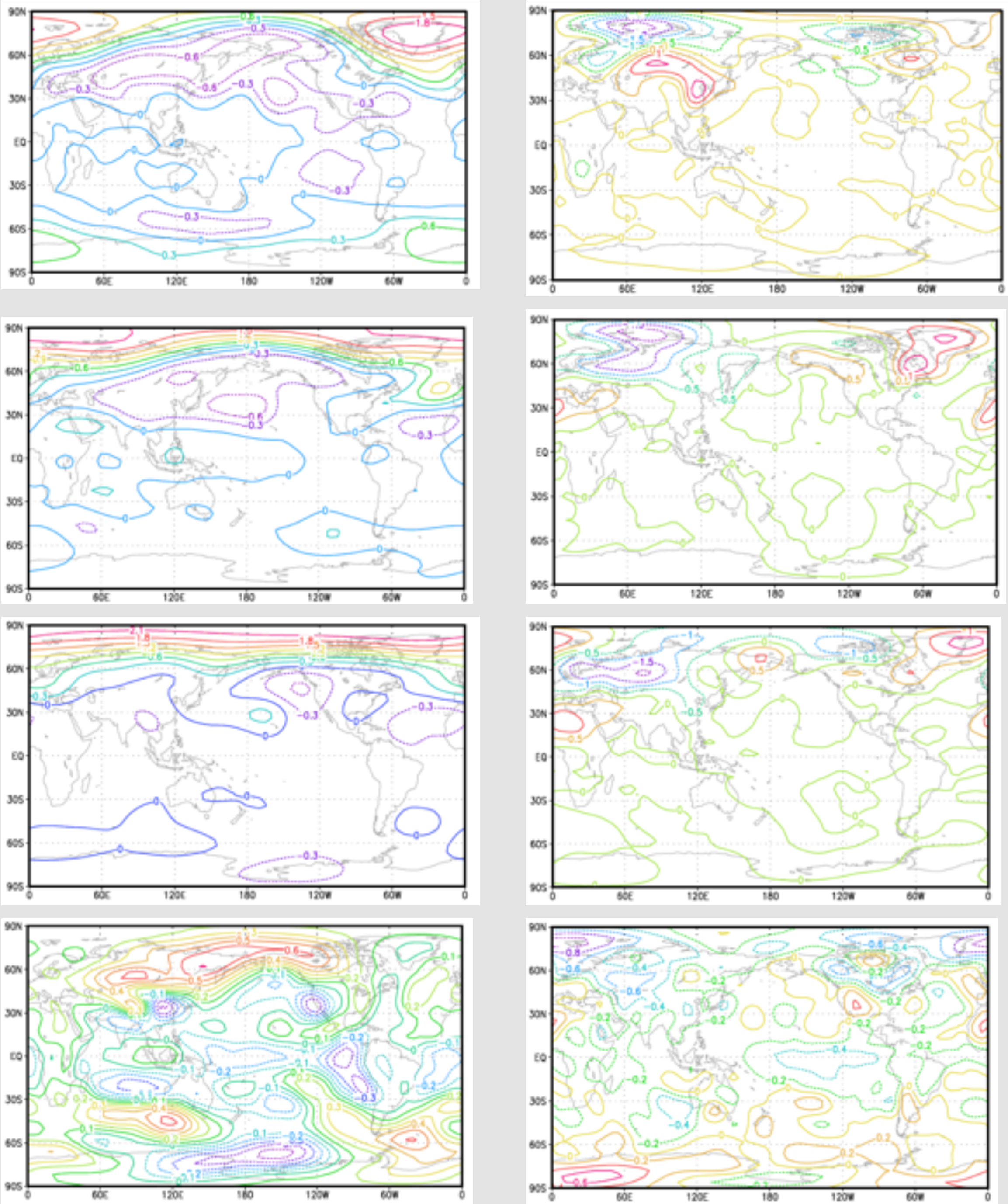


CAM3 ANCYC



NCEP/NCAR data

Operator from data is 2.5x more sensitive than INMconst, 3.2x more sensitive than INMcyc and 3.0x more sensitive than CAM3cyc



200/800mb optimal forcing

INM constJAN

INM ANCYC

CAM3 ANCYC

NCEP/NCAR data

RESULTS 1:

1. **Sensitivity of atmospheric systems of comparable complexity (i.e. CAM3 and INM A5421) seems to be close.**

2. **Sensitivity analysis for response operator of real climate system derived from NCEP/NCAR data should be done more carefully.**

SENSITIVITY OF THE AIR SURFACE TEMPERATURE

to SST ANOMALIES on

ANNUAL INTERVALS

COUPLED CLIMATE MODELS

NCAR CCSM3

Coupled climate model CCSM3 ([Bliesner et.al., 2006])

CAM3 model:
- prognostic variables – (u,v) wind components, temperature, relative humidity, surface pressure
- hybrid vertical coordinate, 26 levels, T31

POP ocean model:
- 25 vertical levels
- 3x1.5 horizontal resolution

•DATA: 5000yrs

- **maximize average Oct-Nov-Dec response**
- **to forcing that is acting 3months**
- **forcing is starting T months before the response is calculating**
- **T=4,6,8,10,12,18,24…..240.**

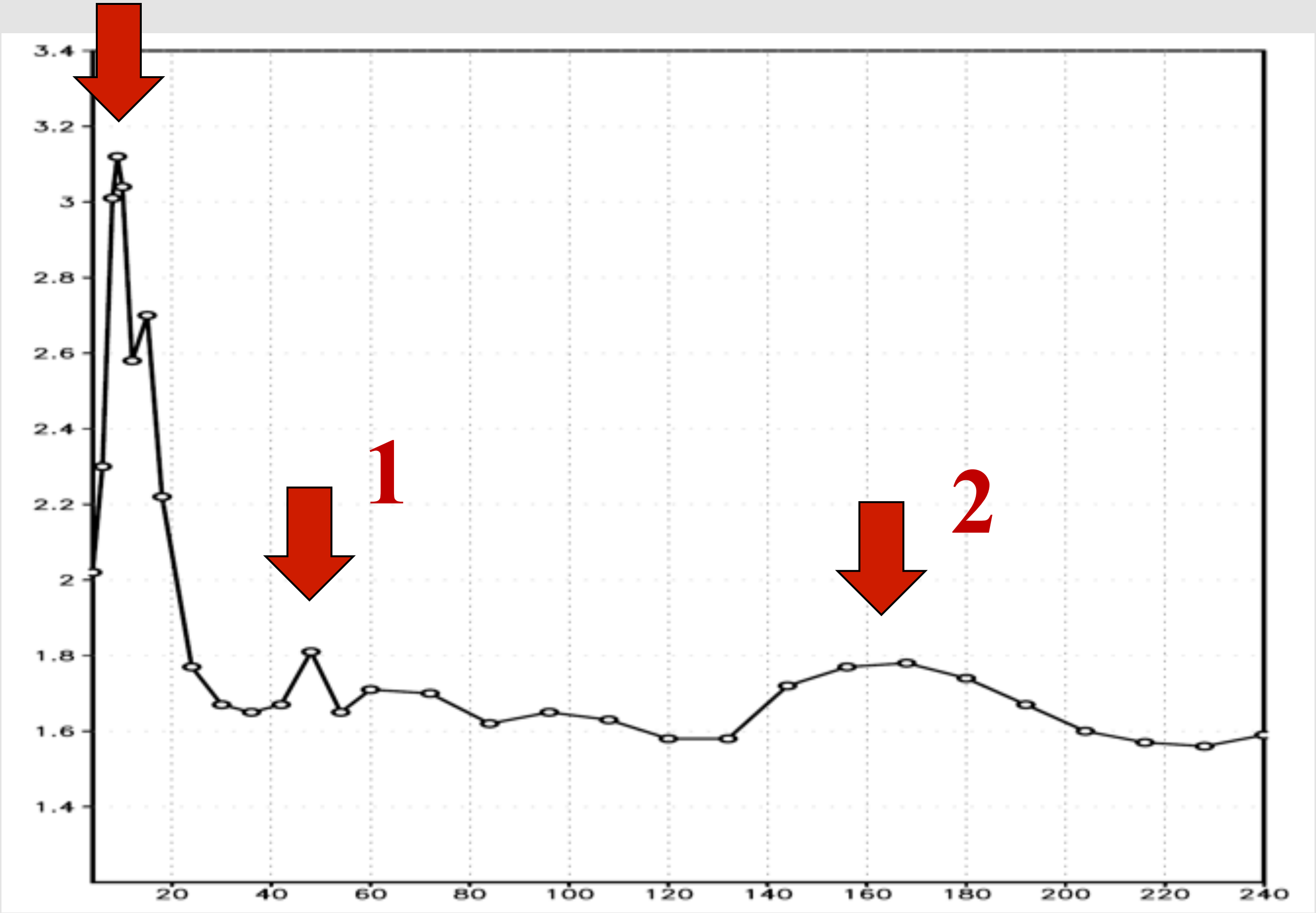
$$\delta < W > = M\delta f$$

$$W(u) = T_{surf}$$

$$\boxed{M(t^0) = \int_0^T C_{t^0-\tau}(\tau)(C_{t^0-\tau}(0))^{-1}\Theta(\tau+\Delta-T)d\tau}$$

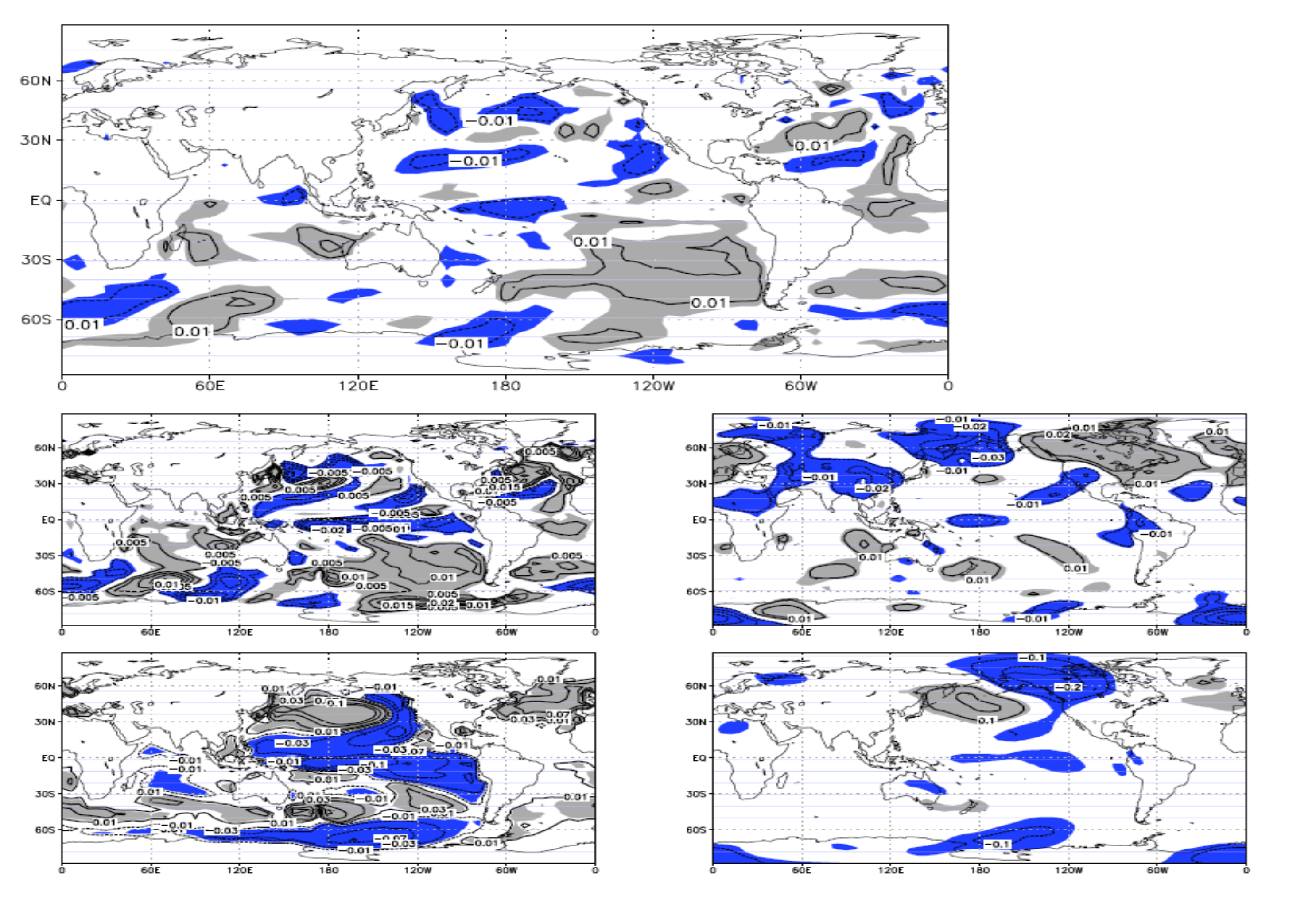
$$M = P\Lambda Q$$

Leading singular value of the response operator



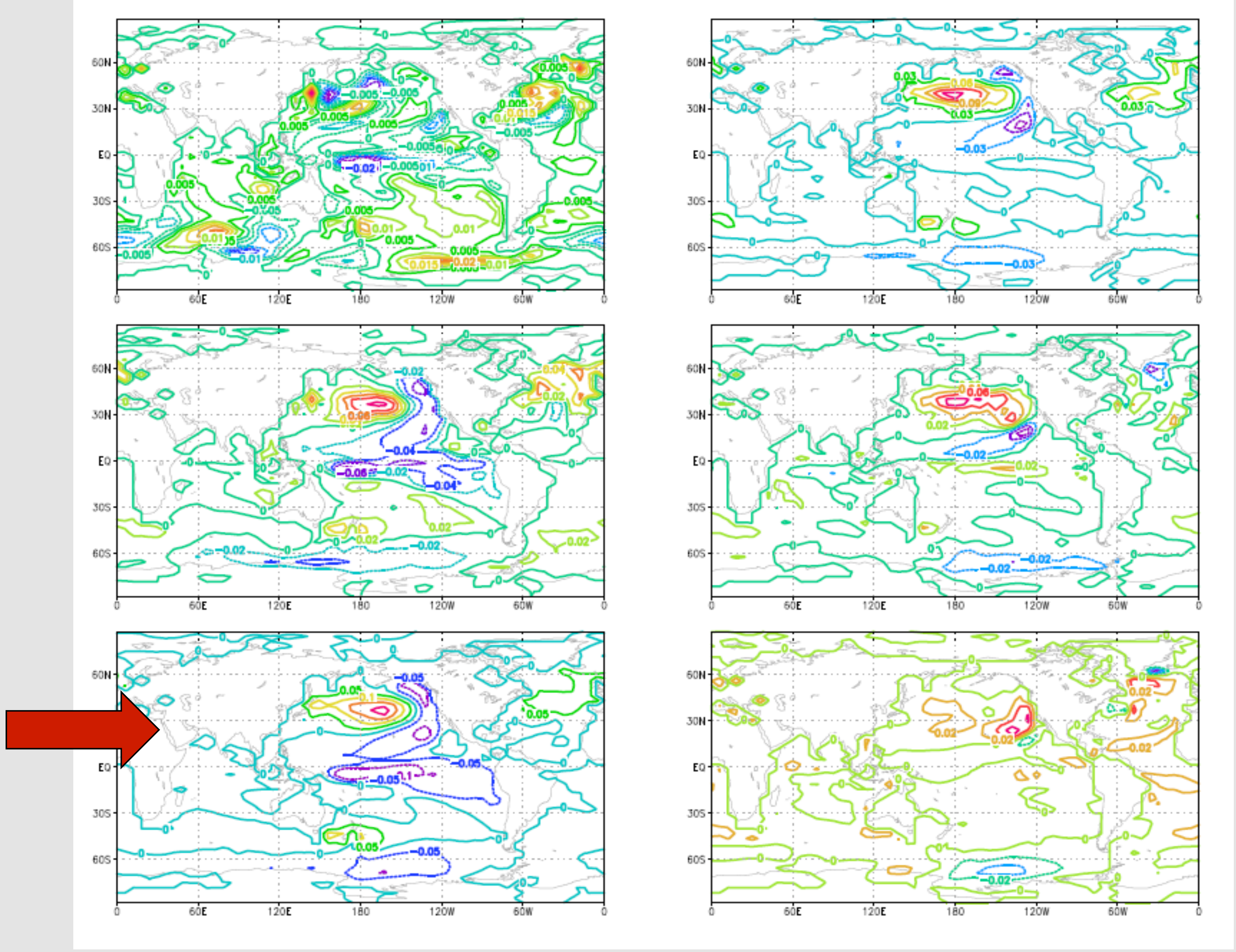
X axis - lag between response and forcing (months)
Y axis - response norm

#1

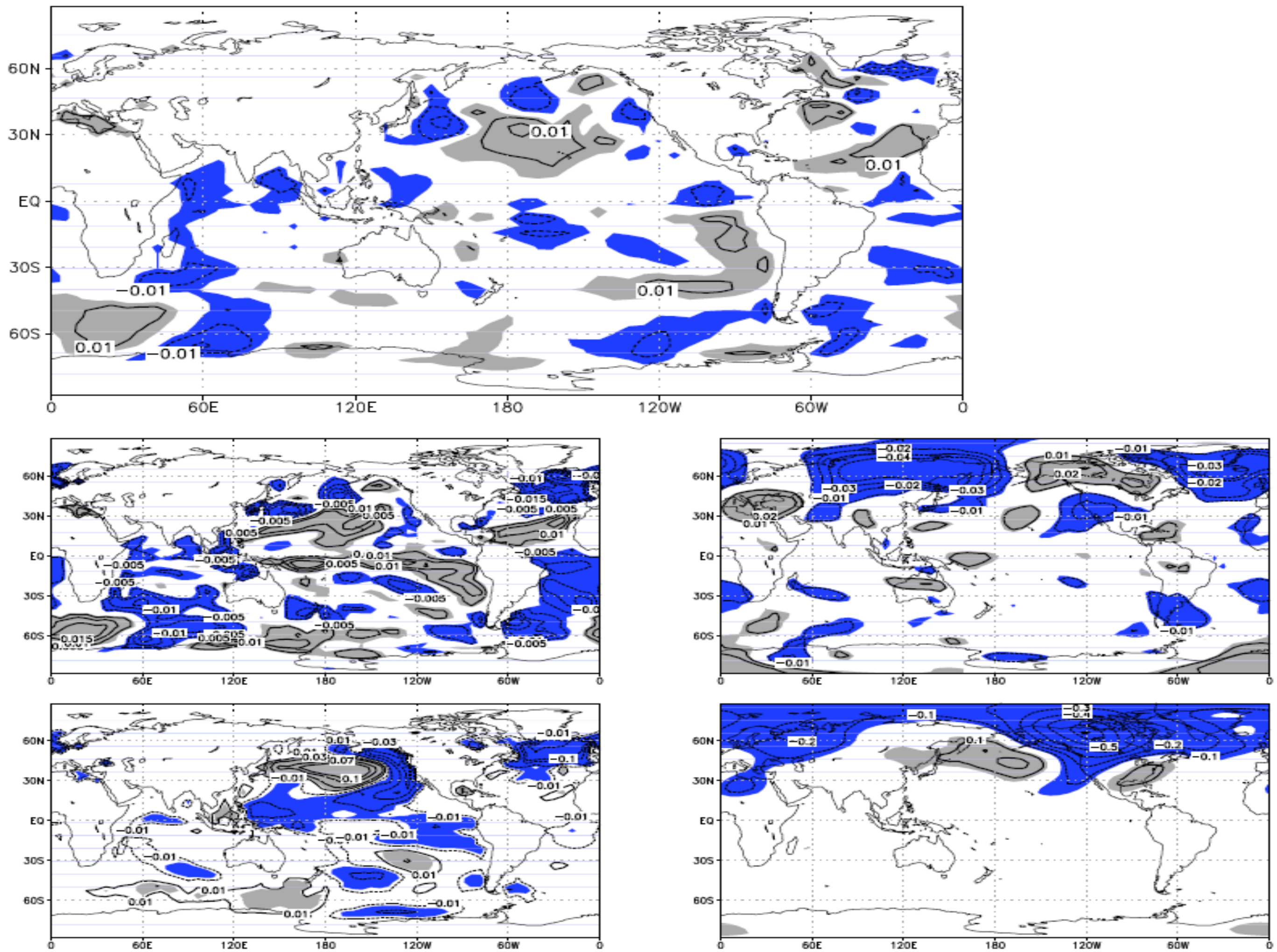


Upper panel – optimal forcing (K/mon);
Left: response air surface temperature after 3 months (top) and 48 months (bottom);
Right: correspondent SST responses.

#1

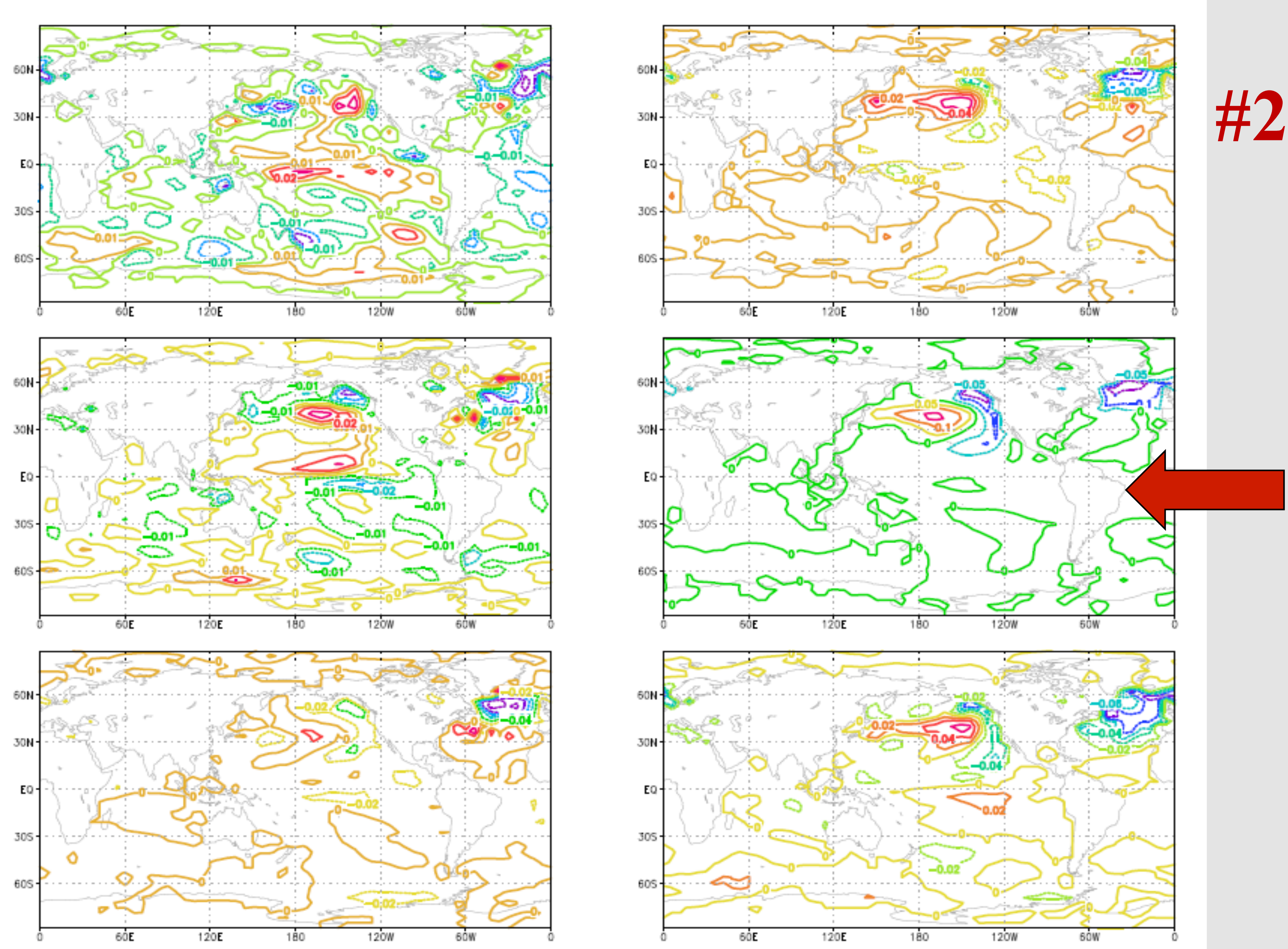


SST responses after 0/24/48/72/96/120 months



#2

Upper panel – optimal forcing (K/mon);
Left: response air surface temperature after 3 months (top) and 156 months (bottom);
Right: correspondent SST responses.



SST responses after 12/48/84/120/156/192 months

RESULTS 2:

1. **Sensitivity of coupled model from quasi-gaussian FDT operator seems to be reasonable.**

2. **More testing (direct sensitivity experiments should be done with coupled models) should be done.**