Propagation of a gravity current in the atmosphere over a steep obstacle: applications of the artificial compressibility method

M.S. Yudin
Institute of Computational Mathematics
& Mathematical Geophysics
Prospekt Akad. Lavrentyeva 6
Novosibirsk 630090
Russia

Artificial compressibility method :

Yanenko N.N.: The method of fractional steps, Springer,N.Y.,1971

Splitting method:

Marchuk G.I.: Numerical methods in weather prediction, Academic Press, N.Y.,1974

$$\frac{dU}{dt}+\frac{\partial P}{\partial x}=f_1(V-V_g)-f_2W+R_u,$$

$$\frac{dV}{dt}+\frac{\partial P}{\partial y}=-f_1(U-U_g)+R_\upsilon,$$

$$\frac{dW}{dt}+\frac{\partial P}{\partial z}+\frac{gP}{C_s^2}=f_2U+g\,\frac{G^{1/2}\,\overline{\rho}\theta^{/}}{\theta}+R_\varpi$$

$$\frac{d\theta}{dt}=R_\theta,$$

$$\frac{ds}{dt}=R_s,$$

$$\frac{1}{C_s^2}\frac{\partial P}{\partial t}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=\frac{\partial}{\partial t}(\frac{\overline{\rho}\theta^{/}}{\theta})$$

$$U=\overline{\rho}u,V=\overline{\rho}\upsilon,P=\overline{\rho}p^{/},$$

$$\bar{f}^{\tau\beta} = \frac{1+\beta}{2} f^{\tau+\Delta\tau} + \frac{1-\beta}{2} f^{\tau}$$

$$\bar{f}^{\tau\gamma} = \frac{1+\gamma}{2} f^{\tau+\Delta\tau} + \frac{1-\gamma}{2} f^{\tau}$$

$$\delta\tau f = \frac{f^{\tau+\Delta\tau} - f^{\tau}}{\Delta\tau}$$

$$\delta\tau U + \frac{\partial}{\partial x}P + \frac{\partial}{\partial \xi}(G^{13}\ P) = -ADVU$$

$$\delta\tau V + \frac{\partial}{\partial y}P + \frac{\partial}{\partial \xi}(G^{13}\ P) = -ADVV$$

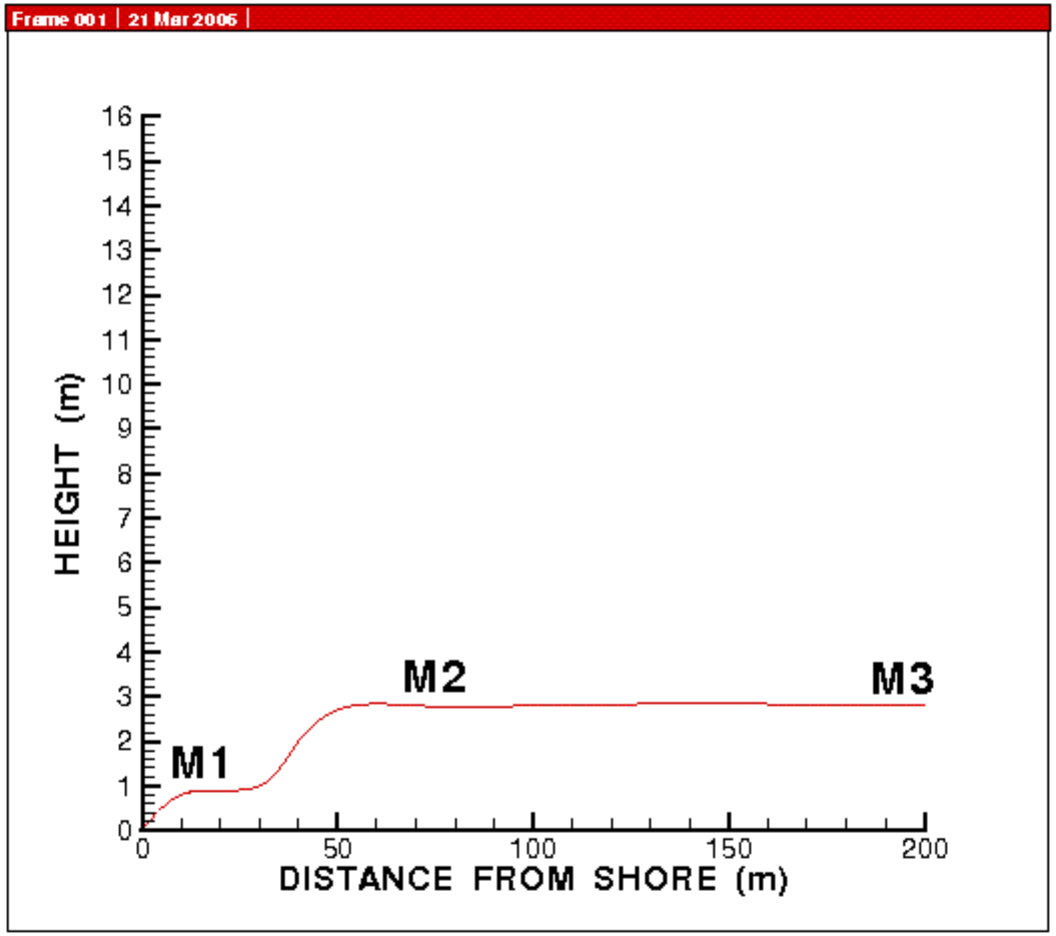
$$\delta\tau W + \frac{1}{G^{1/2}}\frac{\partial \bar{P}^{\tau\beta}}{\partial \xi} + \frac{g\bar{P}^{\tau\beta}}{Cs^2} = BUOY - ADVW$$

$$\frac{1}{Cs^2}\ \delta\tau P + \frac{\partial}{\partial x}\bar{U}^{\tau\gamma} + \frac{\partial}{\partial y}\bar{V}^{\tau\gamma} + \frac{\partial}{\partial \xi}(G^{13}\ \bar{U}^{\tau\gamma}) + \frac{\partial}{\partial \xi}(G^{13}\ \bar{V}^{\tau\gamma}) + \frac{1}{G^{1/2}}\frac{\partial \bar{W}^{\tau\beta}}{\partial \xi} = PFT$$

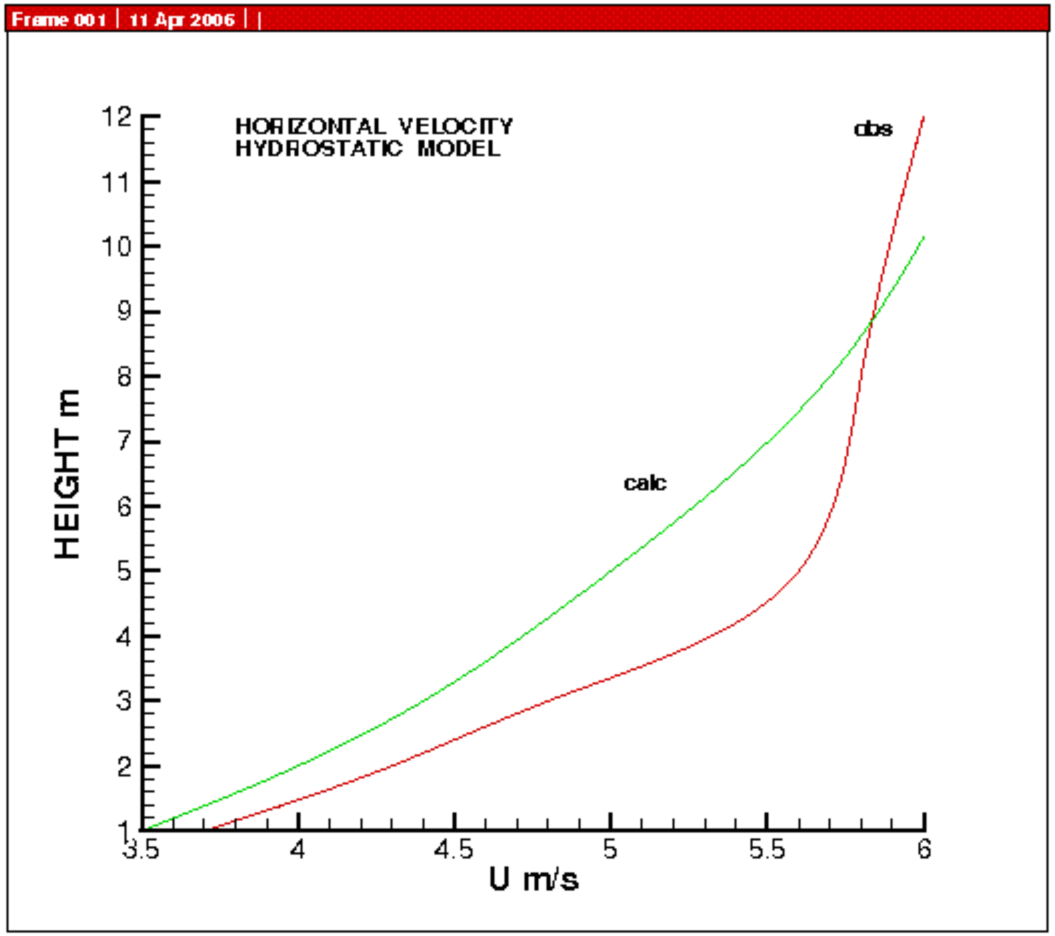
Flow over a low hill at a coastal site

2D cold front propagation over a valley. Formation of a hydraulic jump

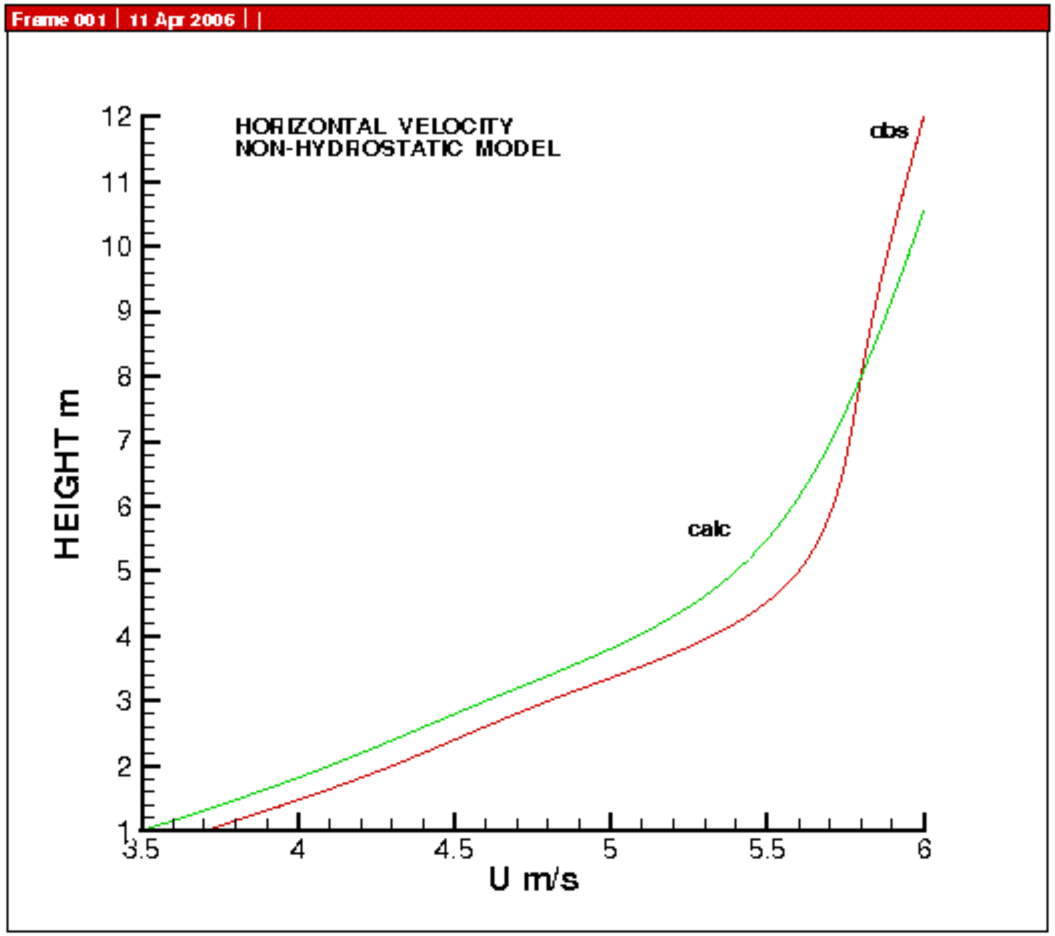
3D mountain effects on a cold front in neutrally and stably stratified atmosphere



Frame 001 | 21 Mar 2006 |
HEIGHT (m)
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
M1
M2
M3
0
50
100
150
200
DISTANCE FROM SHORE (m)

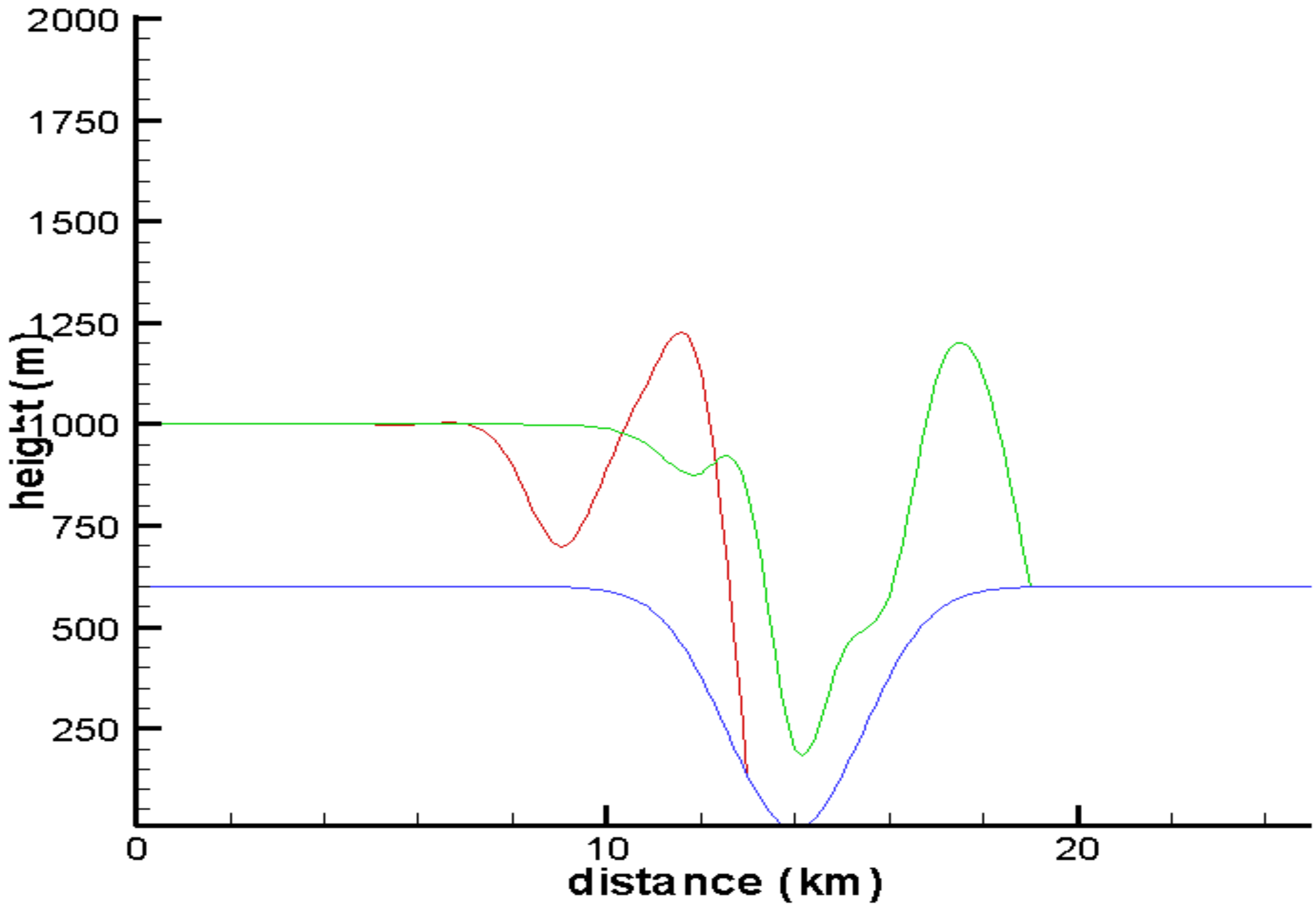


Frame 001 | 11 Apr 2006 | |
HORIZONTAL VELOCITY
HYDROSTATIC MODEL
obs
calc
HEIGHT m
U m/s
12
11
10
9
8
7
6
5
4
3
2
1
3.5
4
4.5
5
5.5
6



Frame 001 | 11 Apr 2006 | |
HORIZONTAL VELOCITY
NON-HYDROSTATIC MODEL
obs
calc
HEIGHT m
U m/s
1
2
3
4
5
6
7
8
9
10
11
12
3.5
4
4.5
5
5.5
6

2D cold front propagation over a 600 m valley , Formation of a hydraulic jump
Time interval=15 min, Delta T=2K



Semi - Lagrangian Advection

$$x_D = x - \int u\,dt$$

$$f(x, t+\Delta t) = f(x_D, t)$$

$$f(t+\Delta t) = f_i(1-\lambda/2-\lambda^2+\lambda^3/2)$$

$$+ f_{i+1}(\lambda+\lambda^2/2-\lambda^3/2)$$

$$+ f_{i+2}(-\lambda/6-\lambda^2+\lambda^3/6)$$

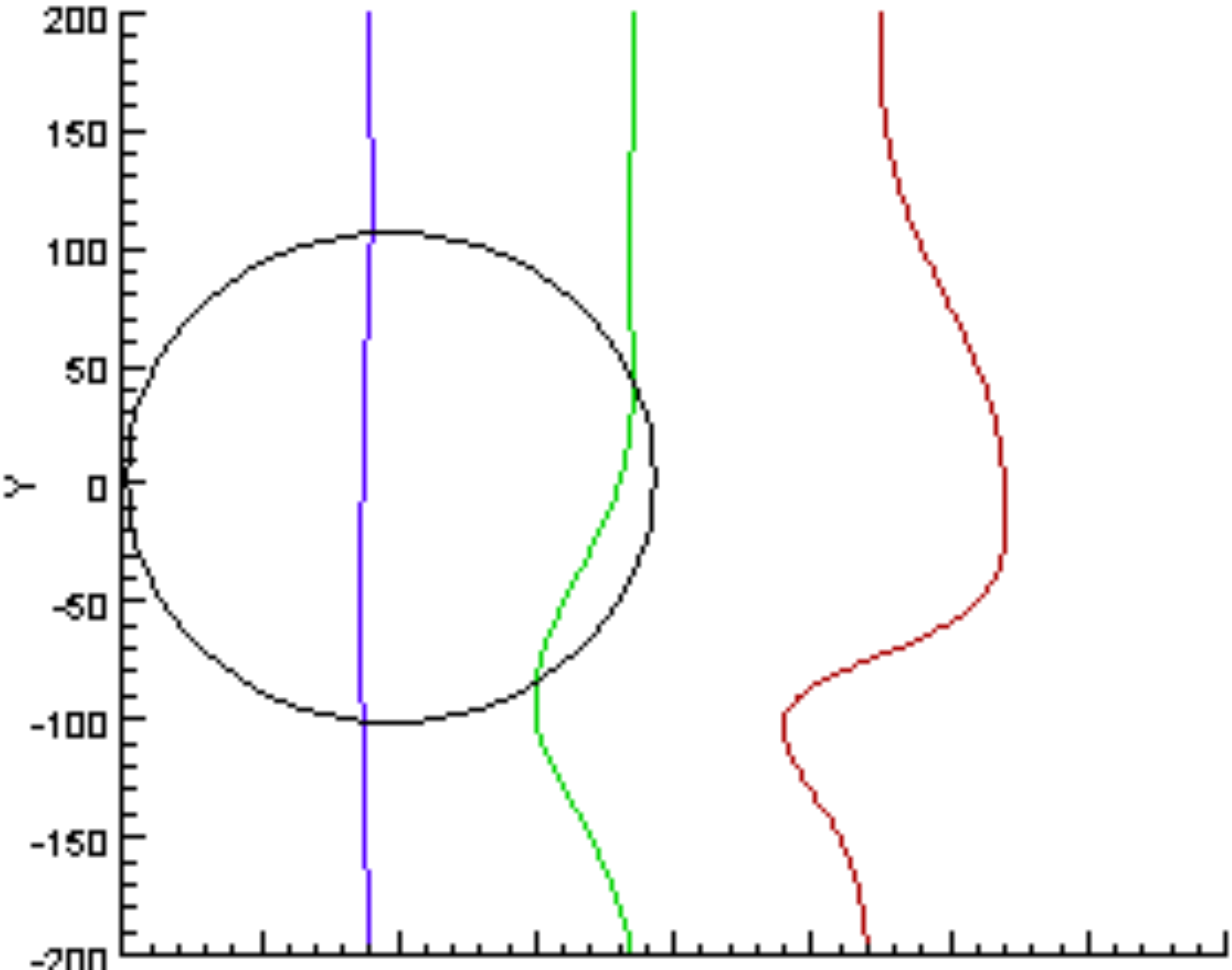
$$+ f_{i-1}(-\lambda/3+\lambda^2/2-\lambda^3/6)$$

The first-order schemes have
large numerical diffusion.

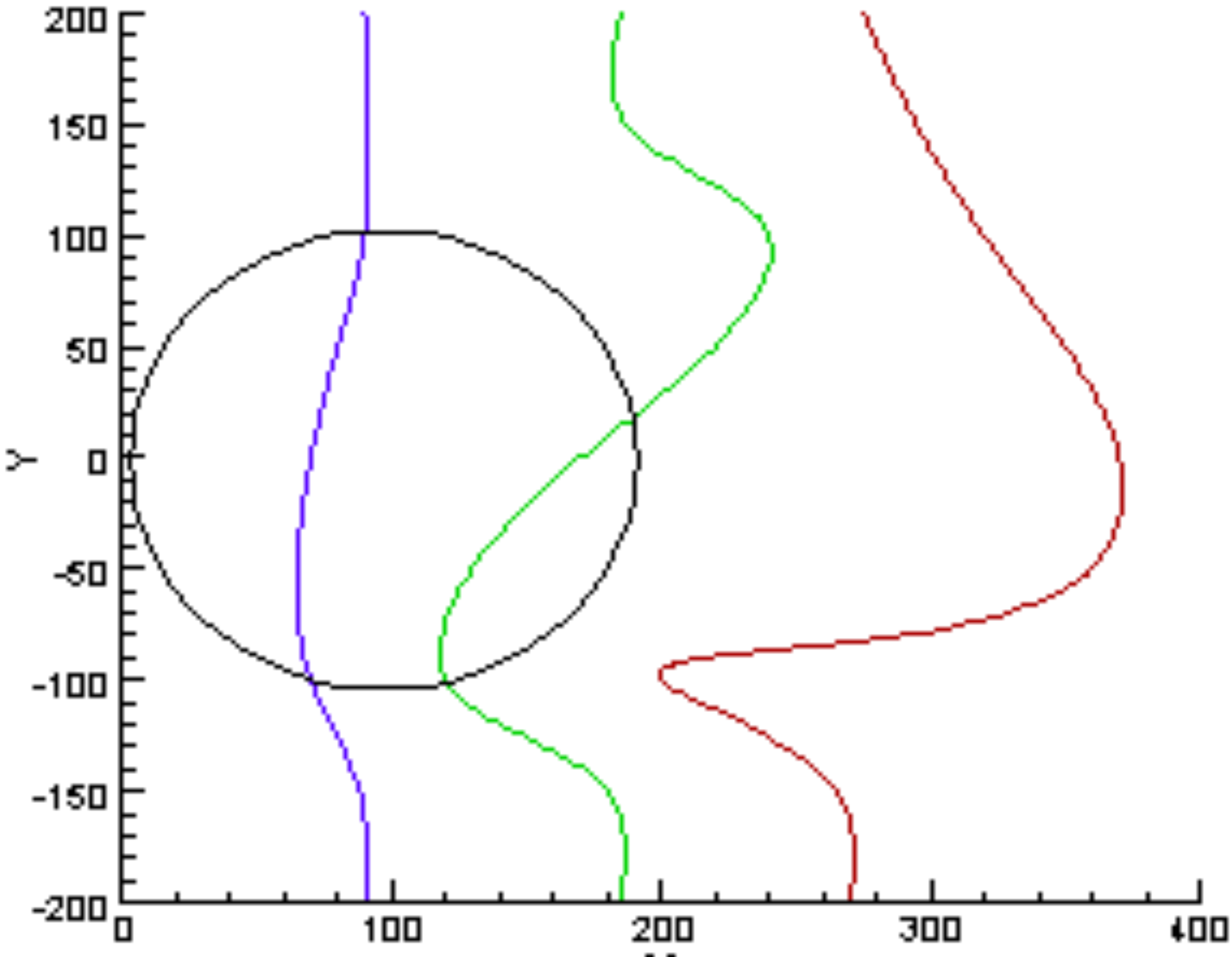
The second-order schemes are nonmonotonic
and have a small-scale wavelike structure.

In the third-order schemes,
the above two effects are essentially reduced.

The schemes of order higher than 3
have a significant increase in cost
but only a small increase in the solution quality.



200
150
100
50
Y
0
-50
-100
-150
-200



200
150
100
50
0
-50
-100
-150
-200
Y
0
100
200
300
400

PHYSICAL PARAMETERS

• Hm	2.5	2.5	km
• L	200.0	200.0	km
• Hf	4.5	9.0	km
• DeltaT	6.0	7.0	K
• U	12.0	10.0	m/sec
• V	40.0	15.0	m/sec
• C	30.0	45.8	m/sec
• R	300.0	458.0	km

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