**Institute of Computational Technologies
of the Siberian Branch
of the Russian Academy of Sciences
6 Acad. Lavrentjev avenue
630090 Novosibirsk**

Estimation of concentration and fluxes of passive gases on the basis of data assimilation system for Siberian region

N. Kilanova,

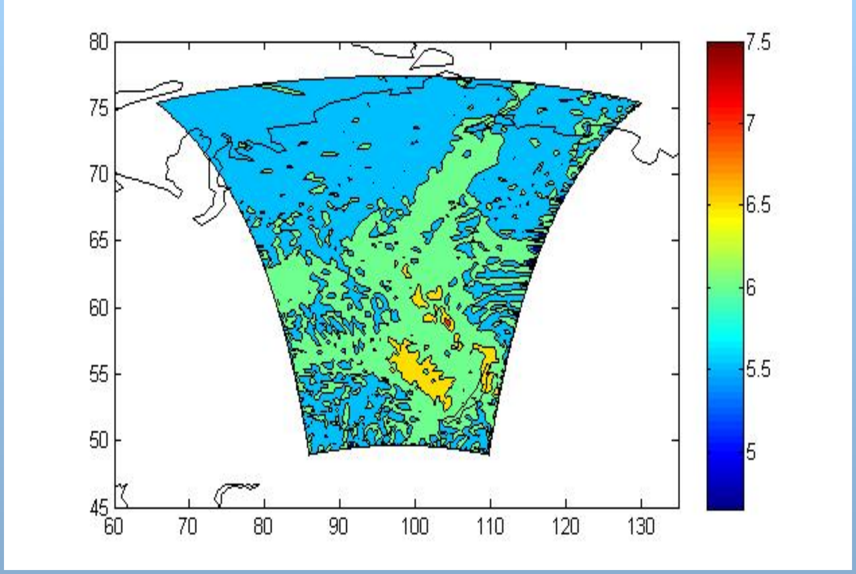
E. Klimova, A. Zudin

Passive pollution transport and diffusion model

$$\frac{\partial s}{\partial t}+u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}+(w-w_g)\frac{\partial s}{\partial z}= \frac{\partial}{\partial x}\left(k_1\frac{\partial s}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_2\frac{\partial s}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_3\frac{\partial s}{\partial z}\right)+\eta$$

The problem is considered in $D_t=G\times[0,T]$,
$G=S\times[h,H]$; $S=\{0\le x\le X; 0\le y\le Y\}$.

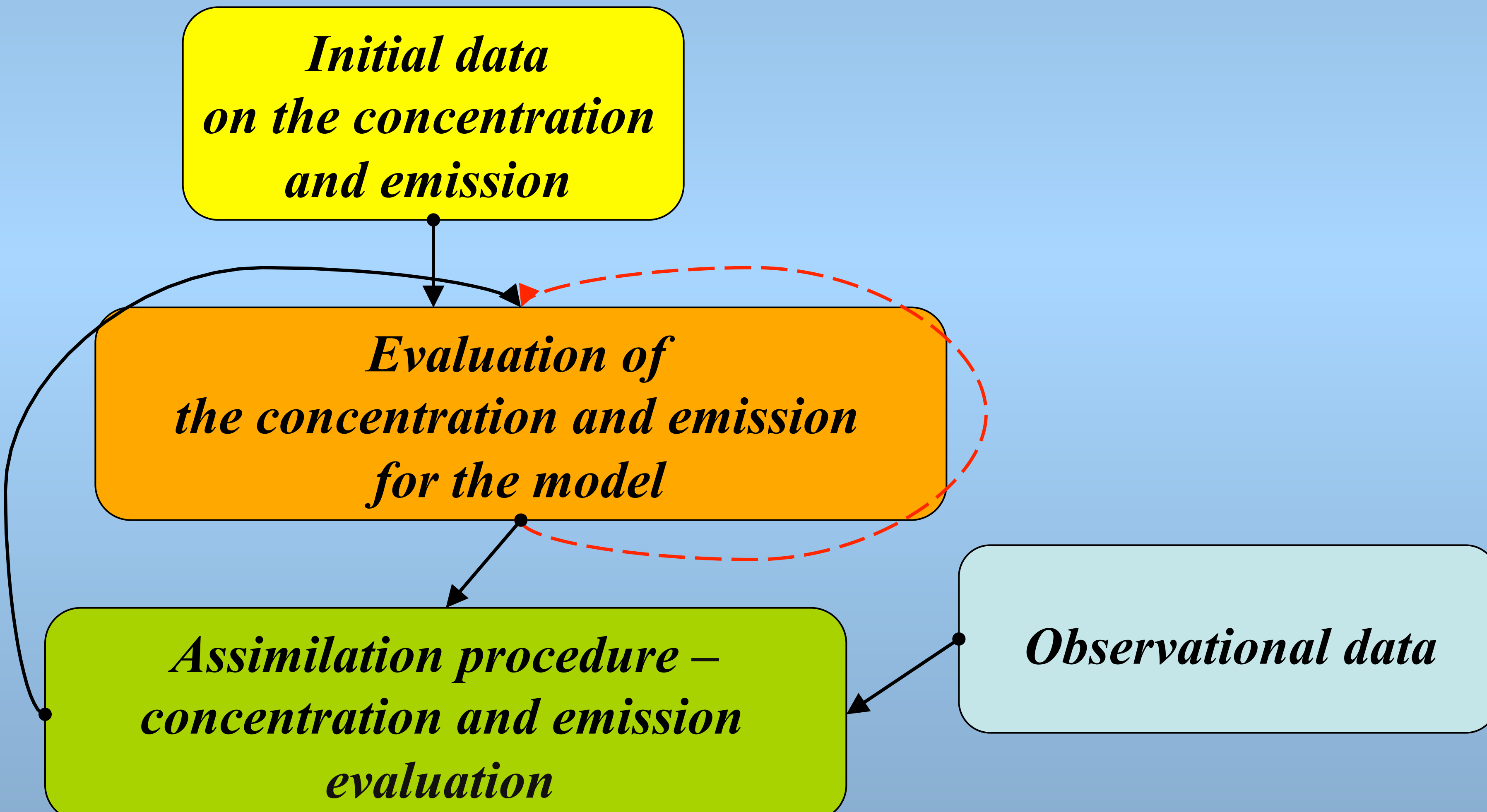
Boundary conditions: $s=s_g$



$$\nu\frac{\partial s}{\partial z}=\alpha\left(s-s_0\right) \qquad z=h$$

$$\nu\frac{\partial s}{\partial z}=0 \qquad z=H$$

Scheme of the experiment for evaluation of passive pollution concentration and emission



Kalman filter algorithm

$$x_k^f = A_{k-1} x_{k-1}^a;$$

$$P_k^f = A_{k-1} P_{k-1}^a A_{k-1}^T + Q_{k-1;}$$

$$K_k = P_k^f M_k^T (M_k P_k^f M_k^T + R_k)^{-1};$$

$$P_k^a = (I - K_k M_k) P_k^f;$$

$$x_k^a = x_k^f + K_k (y_k^0 - M_k x_k^f);$$

$$k = 0, \ldots, K.$$

$$P_k^f = E(x_k^f - x_k^t)(x_k^f - x_k^t)^T; P_k^a = E(x_k^a - x_k^t)(x_k^a - x_k^t)^T.$$

The suboptimal Kalman filter algorithm

Passive pollution transport and diffusion model :

$$s_{k+1} = As_k + \eta_k$$

(s – passive pollution concentration, η- emission)

$$\eta_0 = \tilde{\eta} \qquad \eta_{k+1} = \eta_k$$

Forecast errors covariance matrix:

$$P_k^f = \overline{\Delta s_k (\Delta s_k)^T} \cong \frac{1}{N-1} \sum_{i=1}^{N} \Delta s_i (\Delta s_i)^T$$

$$\Delta s_k = s_k^f - s_k^t$$

The concentration and emission in the data assimilation procedure

Experiment 1 - the absolute value of emissions restores.

The initial approximation of the field emission is zero.

Forecast of concentration and emission:

$$\eta_0^f = 0$$

$$\eta_k^f = \eta_{k-1}^f \qquad s_k^f = As_{k-1}^f + \eta_{k-1}^f$$

The concentration and emission in the data assimilation procedure

Experiment 2 - value of the correction factor restores

Forecast of concentration and emission:

$$\eta_0^f = \tilde{\eta}(1 + \delta\eta_0^f)$$

$$\eta_k^f = \tilde{\eta}(1 + \delta\eta_k^f)$$

$$\delta\eta_k^f = \alpha\delta\eta_{k-1}^f + \sqrt{1-\alpha^2} * \chi_{k-1}^f$$

$$s_k^f = As_{k-1}^f + \eta_{k-1}^f$$

A.W.Heemink, A.J.Segers "Modeling and prediction of environmental data in space and time using Kalman filtering", Stochastic Environmental Research and Risk Assessment 16 (2002), 225-240 ,Springer-Verlag 2002.

The concentration and emission in the data assimilation procedure

"True" field of concentration and emission:

$$s_k^t = As_{k-1}^t + \eta_{k-1}^t + \varepsilon_{k-1}$$

$$\eta_0^t = \overline{\eta} \qquad \eta_0^t = \widetilde{\eta}(1 + \delta\eta_0^t)$$

$$\eta_k^t = \eta_{k-1}^t \qquad \delta\eta_k^t = \alpha\delta\eta_{k-1}^t + \sqrt{1-\alpha^2} * \chi_{k-1}^t$$

Observational data :

$$y_k^o = M_k s_k^t + \xi_0$$

Evaluation of the concentrations and emissions:

$$s_k^a = s_k^f + P_k^f M_k^T (M_k P_k^f M_k^T + R_k)^{-1} (y_k^o - M_k s_k^f)$$

$$P_k^f = \overline{\Delta s_k (\Delta s_k)^T} \cong \frac{1}{N-1} \sum_{i=1}^{N} \Delta s_i (\Delta s_i)^T$$

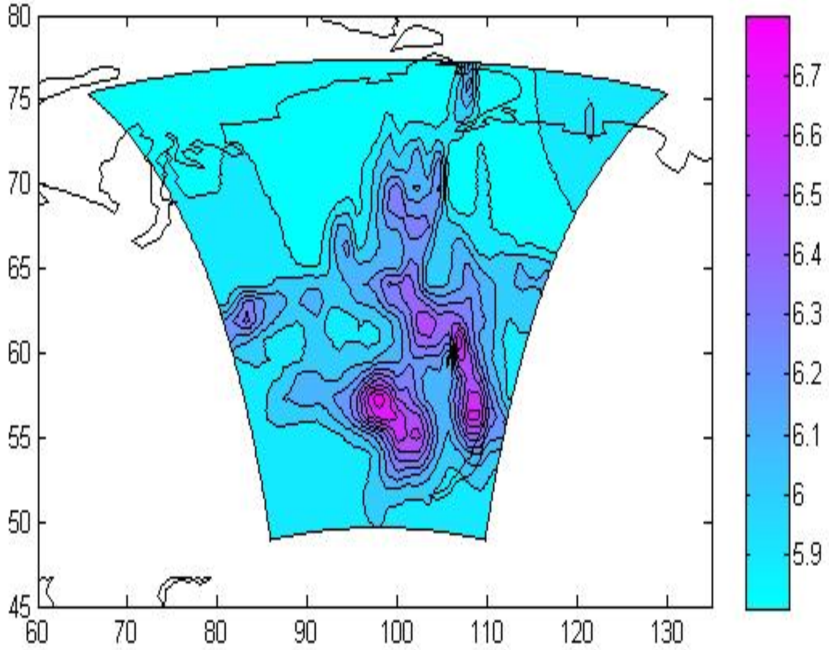
$$\eta_k^a = \eta_k^f + \overline{\Delta s (\Delta \eta)^T} (M_k P_k^f M_k^T + R_k)^{-1} (y_k^o - M_k s_k^f)$$

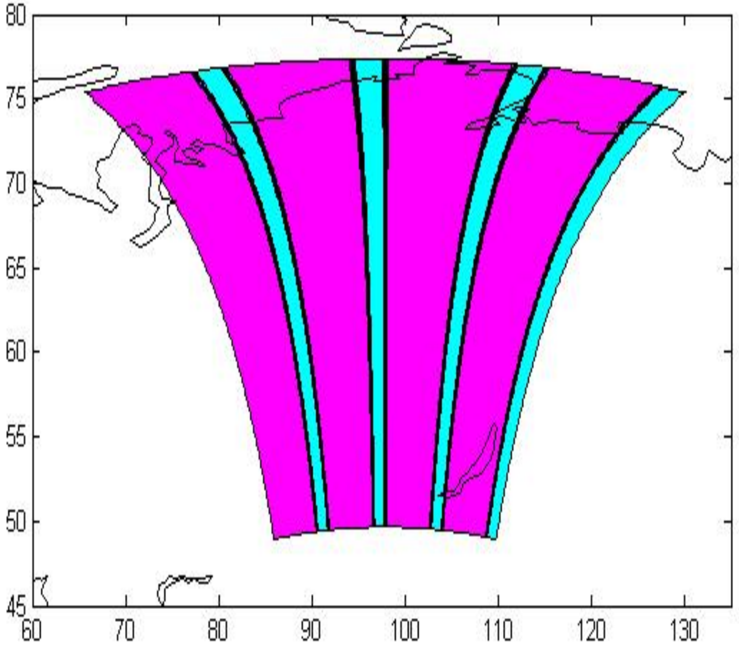
$$\Delta \eta_k = \eta_k^f - \eta_k^t$$

$$\delta\eta_k^a = \delta\eta_k^f + \overline{\Delta s (\Delta \delta\eta)^T} (M_k P_k^f M_k^T + R_k)^{-1} (y_k^o - M_k s_k^f)$$

$$\Delta \delta\eta_k = \delta\eta_k^f - \delta\eta_k^t$$

The initial field of CO_2





Concentration field,
((kg / kg)*10^{-4},
height of 1200 m)

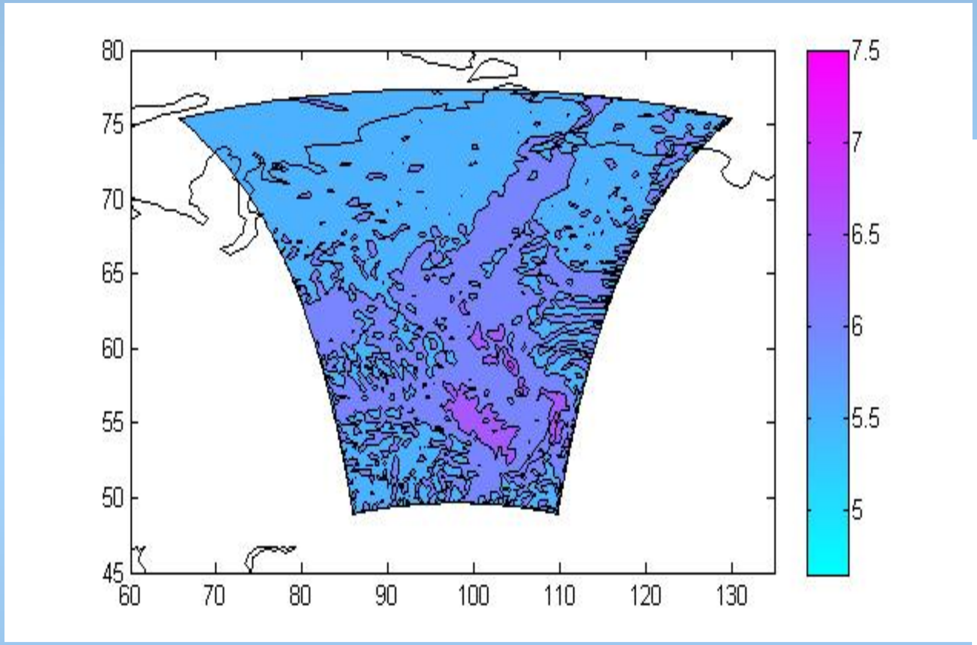
The region (48.9^o N, 85.9^o E; 76.9^o N, 109^o E)

Modeling data

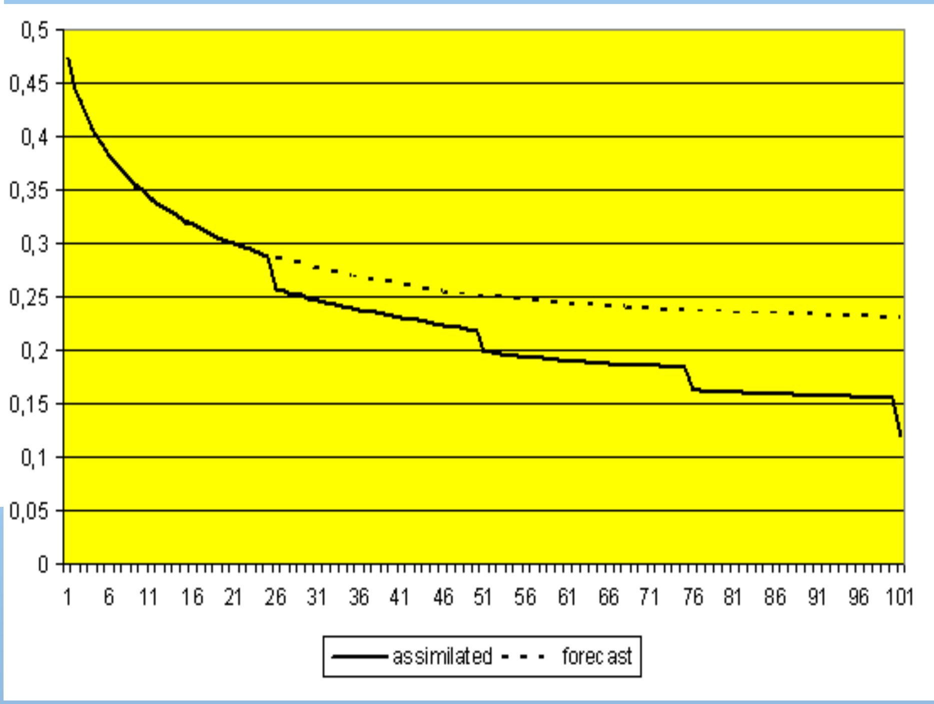
each pink band is the observational data every 6 hours

http://www.gmes-atmosphere.eu/data

Estimate of the concentration CO_2 for 24 hours



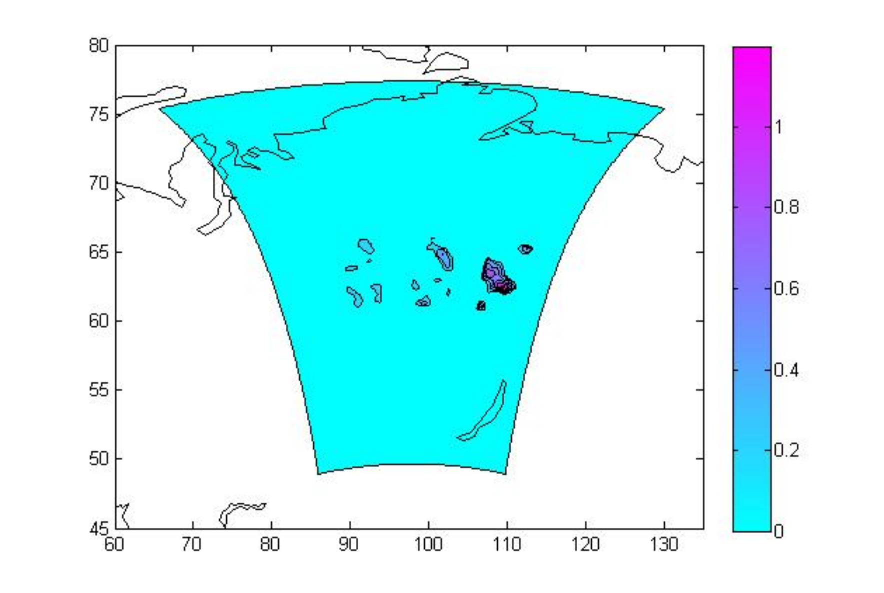
Concentration field, $((kg / kg)*10^{-4}$, height of 1200 m)



Root mean square error for concentration, $((kg / kg)*10^{-4})$

Estimate of CO_2 emission for 24 hours

The initial emission field is zero



$$\eta_0^f = 0 \qquad \eta_k^f = \eta_{k-1}^f$$

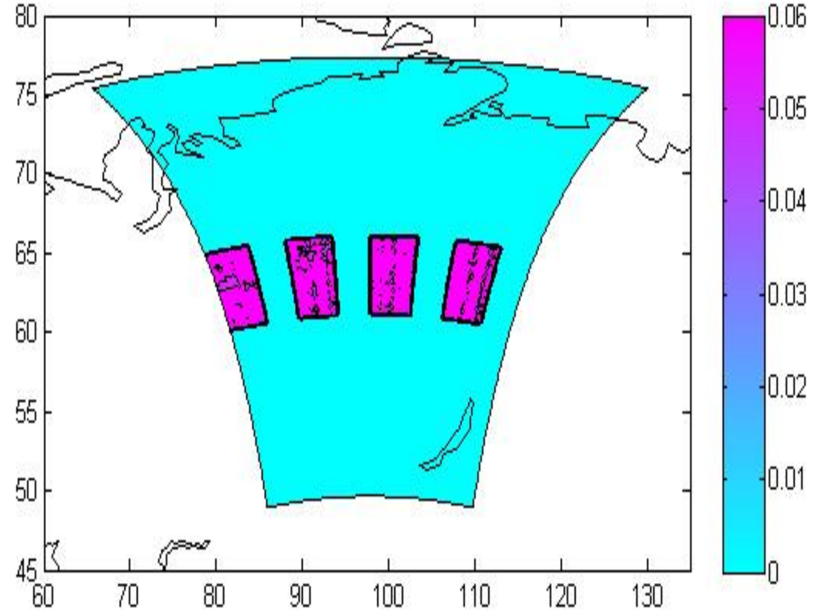
$$\eta_k^a = \eta_k^f + \overline{\Delta x(\Delta\eta)^T} *$$
$$*(M_k P_k^f M_k^T + R_k)^{-1}(y_k^o - M_k x_k^f)$$

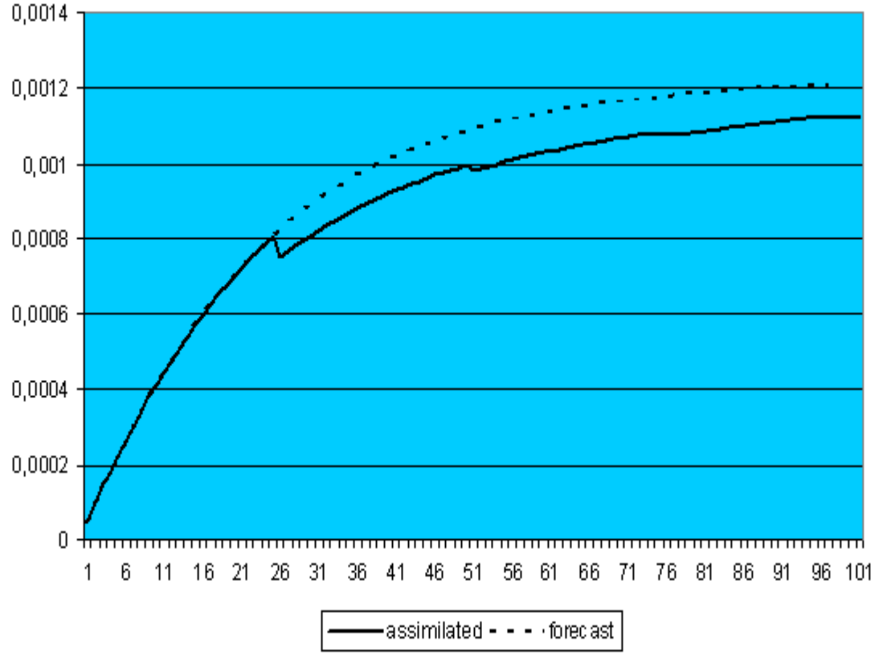
$$\Delta\eta_k = \eta_k^f - \eta_k^t$$

CO_2 Emission field,
$((kg / kg)*10^{-4})$

Estimate of CO_2 emission for 24 hours

The initial emission field is $\eta_0 = \tilde{\eta}$

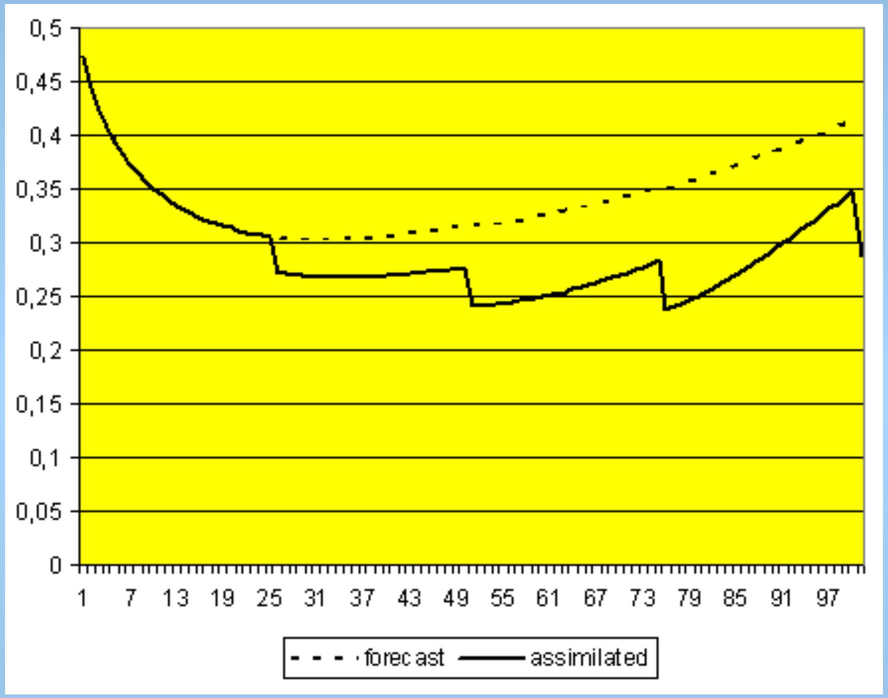


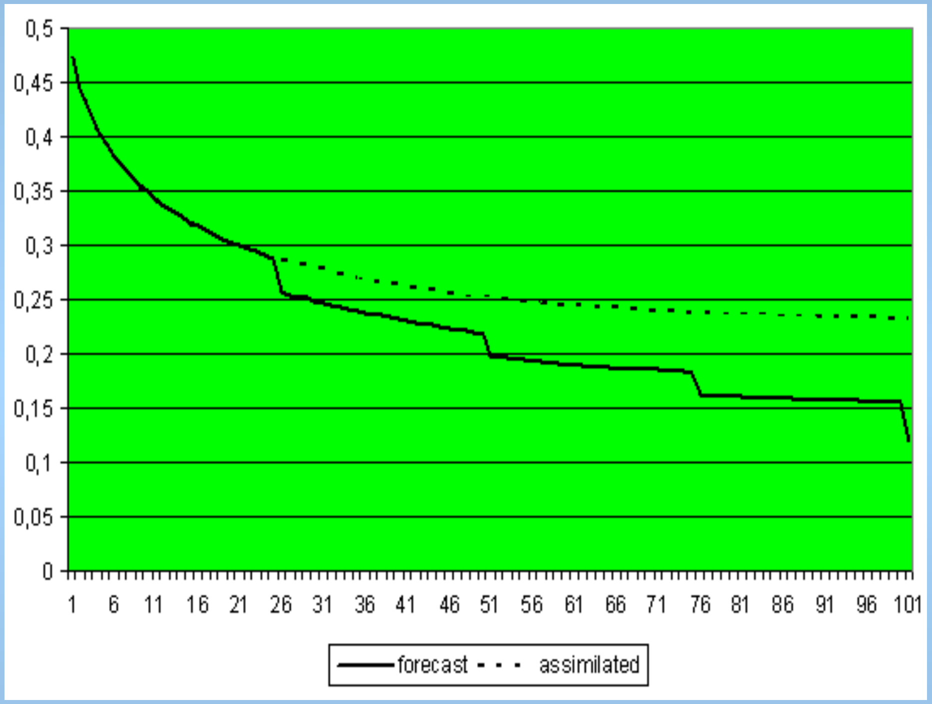


CO_2 Emission field, $((kg / kg)*10^{-4})$

Root mean square error for emission, $((kg / kg)*10^{-4})$

Estimate of CO_2 emission for 24 hours





Root mean square error for concentration, ((kg / kg)*10^{-4}),

$$\eta_0 = 0$$

Root mean square error for concentration, ((kg / kg)*10^{-4})

$$\eta_0 = \tilde{\eta}$$

Conclusions

- *The algorithm for the joint estimation of the passive polution concentration and emission is suggested.*
- *Numerical experiments have shown that this algorithm reduces the root mean square error estimation of the polution concentration.*
- *This algorithm allows to select the emission fields according to the observational data .*

Thank you for your attention!