Kolmogorov goodness-of-fit test for $\vec{S}$ -symmetric distributions in climate and weather modeling

National Research Tomsk State University

Authors:
Z.N. Zenkova
L.A. Lanshakova

Reporter: Dr. Zhanna Zenkova
PhD, MBA, TSU Associate Professor

Agenda

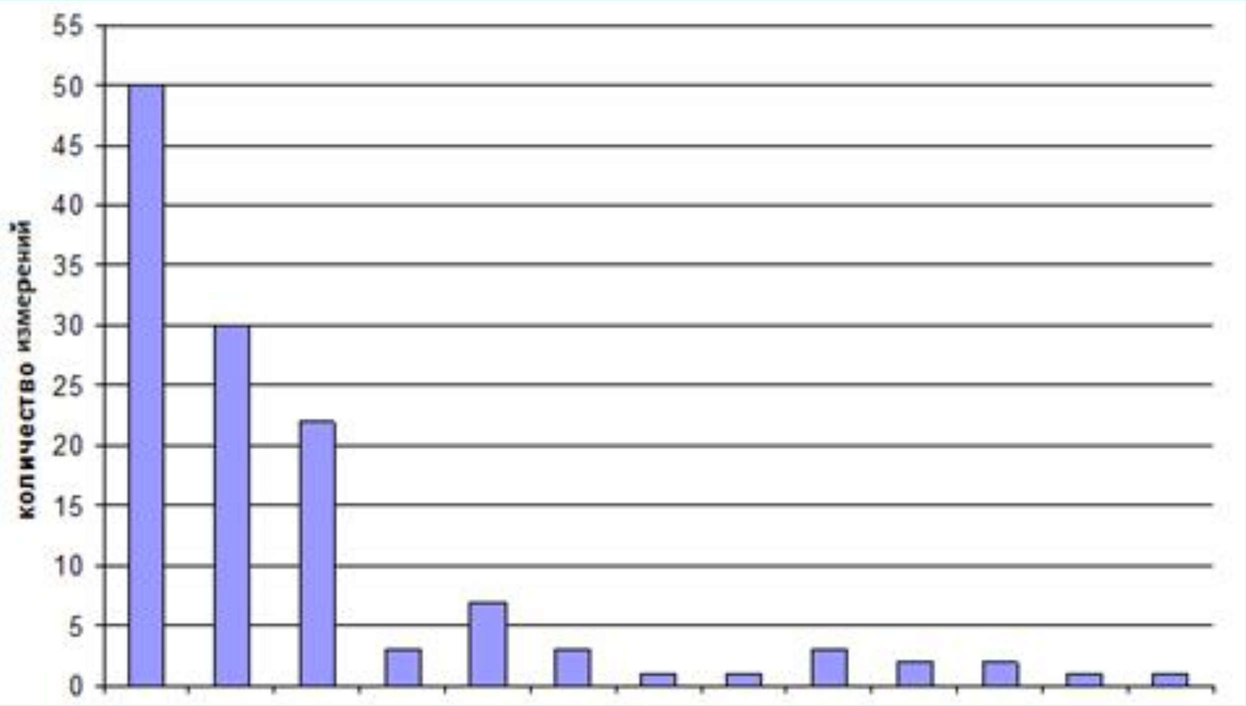
Title: Kolmogorov goodness-of-fit test for $\vec{S}$-symmetric distributions in climate and weather modeling

1. Mathematical statistics in climate and weather modeling;
2. Kolmogorov goodness-of-fit test;
3. $\vec{S}$-symmetric distributions;
4. Example.

Mathematical statistics in climate and weather modeling

Water lifting

Mathematical statistics in climate and weather modeling



Уровень подъема воды

Kolmogorov goodness-of-fit test

$$X = \left(X_1, X_2, \ldots, X_N\right)$$

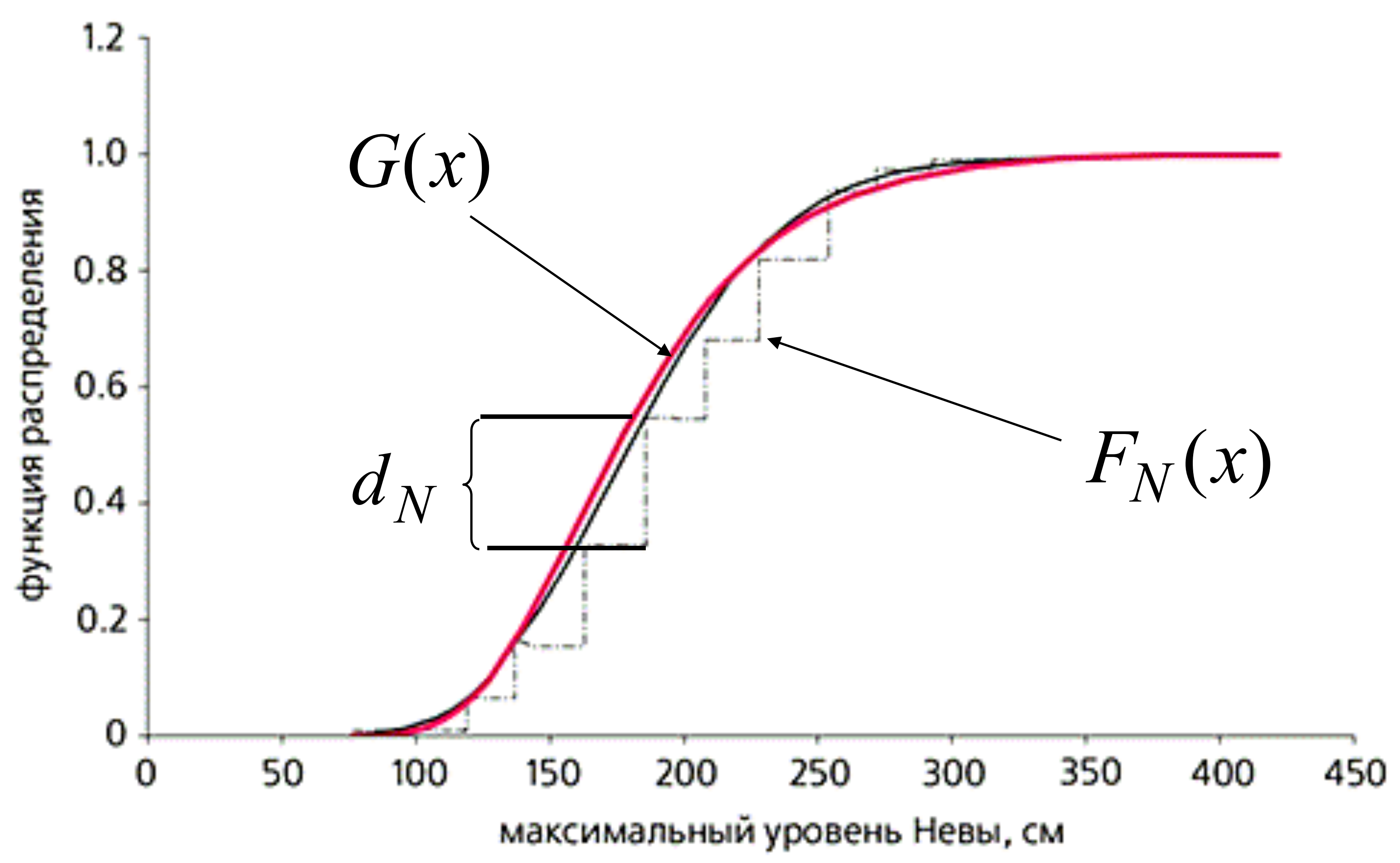
$$H_0 : \; F(x) = G(x)$$

$$d_N = \sup_{x \quad \mathbf{R}} \left| F_N(x) \quad G(x) \right| \qquad (1)$$

$$F_N(x) = \frac{1}{N} \sum_{i=1}^{N} I_{(\quad ;x)}\left(X_i\right) \qquad (2)$$

$$\lim_{N} \mathrm{P}\left(\sqrt{N} d_N < z\right) = K\left(z\right) = \sum_{k=}^{+} (\quad 1)^k e^{\quad 2k^2 z^2} \qquad (3)$$

Kolmogorov goodness-of-fit test



$\vec{S}$-symmetric distribution

Definition. If for $c_1 < c_2 < \ldots < c_k, i = \overline{1,k}, k > 1, \; p_i = F(c_i) = F(c_i + 0)$
$p_0 = F(c_0) = 0, \; p_{k+1} = F(c_{k+1}) = 1,$ for $j = \overline{1,k}$ and $x \quad c_{j+1}$
c.d.f. $F(x)$ satisfies the conditions:

$$F\left(\max\left\{x, S_j(x) + 0\right\}\right) = p_{j+1} \quad \frac{p_{j+1} \quad p_j}{, \; p_j} F\left(\min\left\{x, S_j(x) + 0\right\}\right), \qquad (4)$$

where $S_j(x)$ are continuous monotonically decreasing functions,

$$\left(S_j\right)^{\;1}(x) = S_j(x)$$

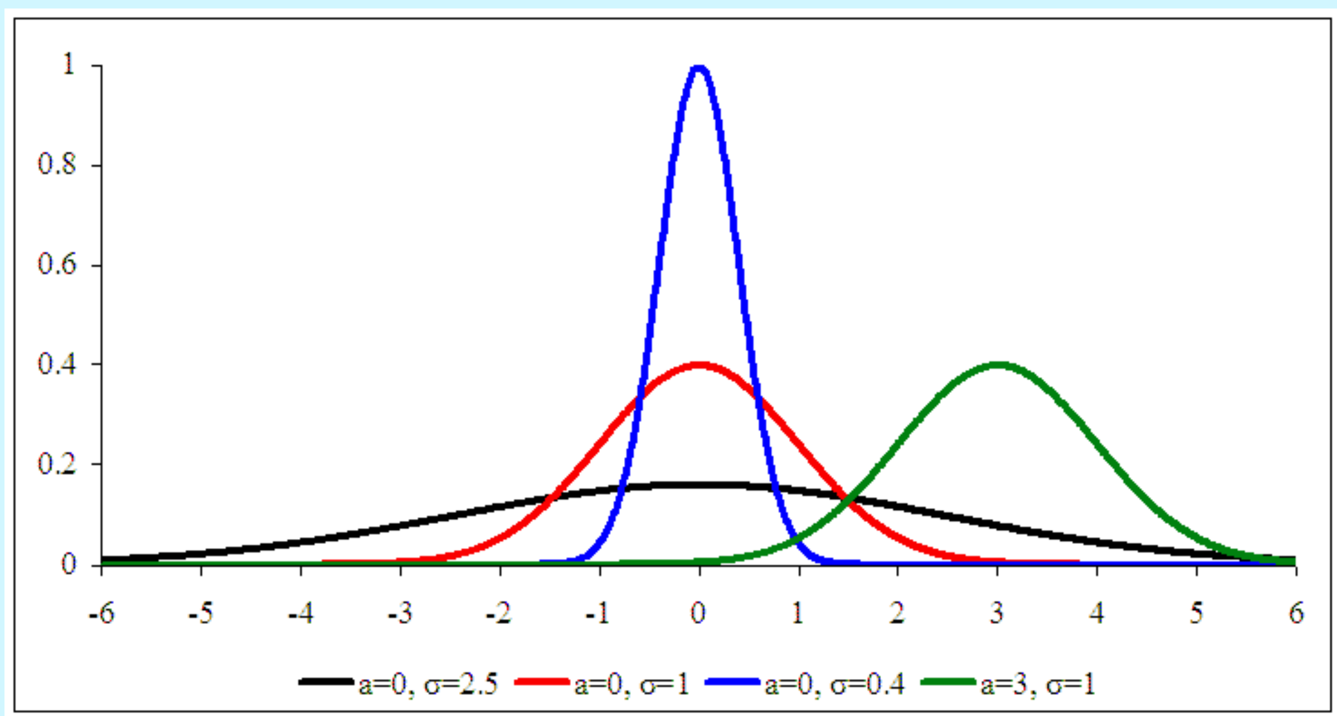
$x \quad c_{j+1}$, $S_j\left(c_i\right) = c_i, \quad S_j\left(c_{j+1}\right) = c_0,$
then c.d.f. $F(x)$ is **-symmetric c.d.f.**

$\vec{S}$-symmetric distribution

For $k = 1$, $p_1 = F(c_1) = 0.5$, $S_1(x) = 2c_1 \quad x$

we can obtain classical symmetry of c.d.f. around c_1 :

$$F(x) = 1 \quad F\left(2c_1 \quad x\right) \qquad (5)$$



Test modification

$$H_0^S : F(x) = G(x), \; F, G \qquad {}^S, \qquad x \quad \mathbf{R}$$

against

$$H_1^S : F(x) = V\big(G(x)\big), \; V, F, G \qquad {}^S, \qquad x \quad \mathbf{R}$$

$$d_N^S = p_1 \sup_{x \quad \mathbf{R}} \left| F_N^S(x) \quad G^*(x) \right| \tag{6}$$

$$G^*(x) = {}^{G(x)}\!\big/_{p_1} \tag{7}$$

Test modification

$F_N^S(x)$ is e.d.f. based on k times symmetrized sample

$$X^* = \left(X_1^*, \ldots, X_N^*\right)$$

$$X_i^{(j)} = \min\left\{X_i^{(j\ \ 1)}, S_{k\ \ j+1}\left(X_i^{j\ \ 1}\right)\right\} \quad j = \overline{1, k}, \qquad (8)$$

$$X_i^* = X_i^{(k)}$$

$$X_i^{(0)} = X_i, \quad i = \overline{1, N}$$

Test modification

For $F(x) = V\big(G(x)\big), \;\; y > 0,$

$$P\big(d_N^S < y\big) = N!\det \frac{0 \quad 1 \quad \dfrac{V\big(p_1 b_i\big)}{p_1} \qquad 1 \quad \dfrac{V\big(p_1 a_j\big)}{p_1} \quad {}^{j \quad i+1}}{(j \quad i+1)!} \qquad (9)$$

$$_{i,j=\overline{1,N},}$$

$$b_i = 0 \quad 1 \quad \frac{i \quad 1}{N} + \frac{z_1}{p_1} \;\; , \qquad a_i = 1 \quad 0 \quad \frac{i}{N} \quad \frac{y_1}{p_1} \;\; , \qquad i = \overline{1,N}$$

$$z_j = \min \;\; z_{j+1}, \frac{p_j}{p_{j+1} \quad p_j} y_{j+1} \;\; , \qquad y_j = \max \;\; y_{j+1}, \frac{p_j}{p_{j+1} \quad p_j} z_{j+1} \;\; , \qquad j = \overline{1,k}$$

$$y_{k+1} = z_{k+1} = y$$

Test modification

For $z > 0$

$$\lim_{N} \mathrm{P}\left(\sqrt{N} d_N^S < z\right) = K\left(\frac{z}{p_1}\right), \qquad (10)$$

Example

The uniform c.d.f. $U(x)$ is satisfied by (4), if for arbitrary

$0 < p_1 < 1,\ \ c_1 = p_1$

$$S_1(x) = \begin{array}{l} 1 \quad \dfrac{1 \quad p_1}{p_1} \; x,\ 0 \quad x \quad c_1, \\[2ex] \dfrac{1 \quad x}{1 \quad p_1} \; p_1,\ \ c_1 < x \quad 1. \end{array} \qquad (11)$$

Example

$$H_0^S: \quad F(x) = U(x) = x, \quad x \quad [0,1],$$

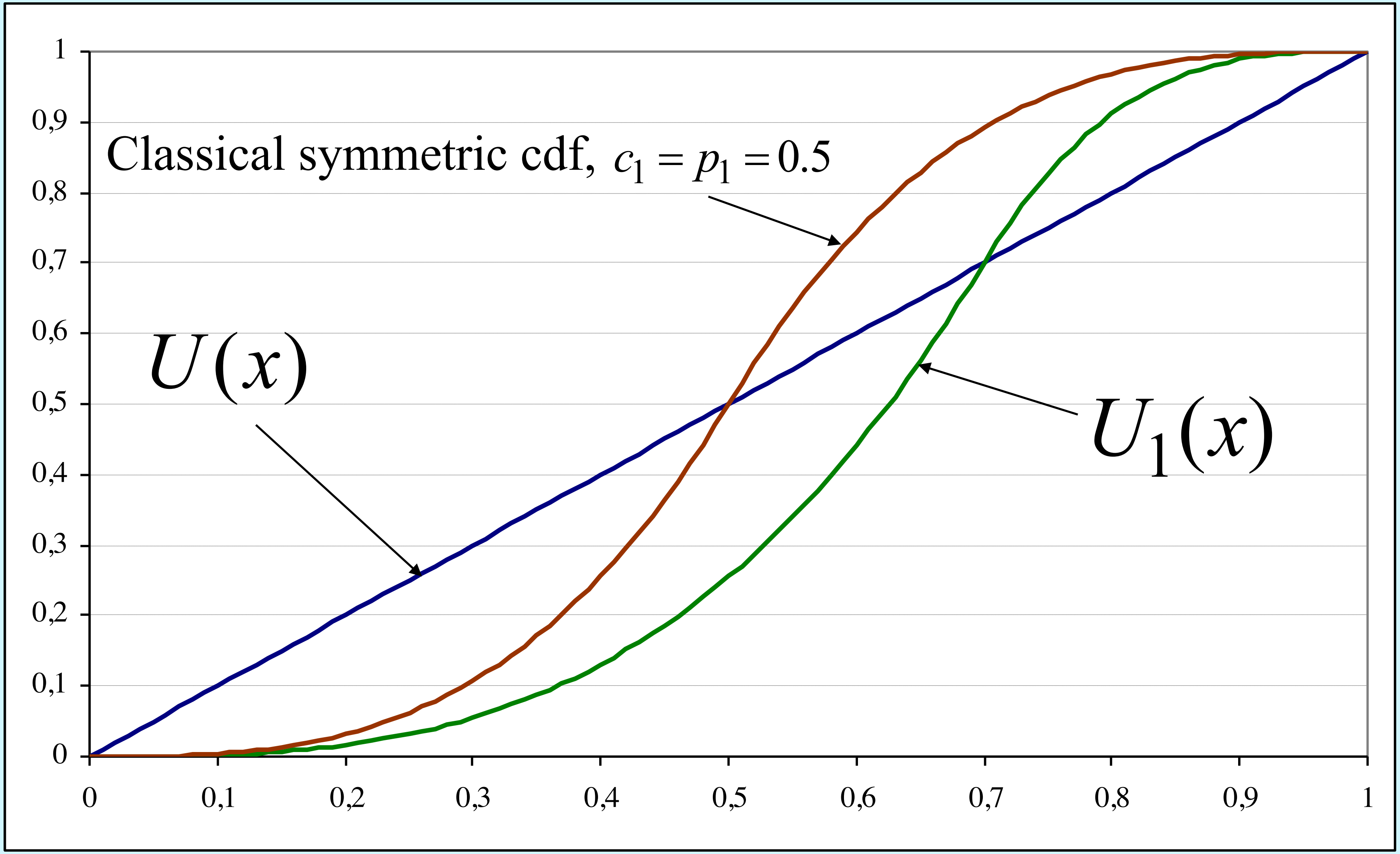
against

$$H_1^S \quad : \quad F(x) = U_1(x), \quad x \quad [0,1],$$

$$U_1(x) = \begin{cases} p_1 \quad \dfrac{x}{p_1} \quad ^{m}, & x \quad [0, p_1], \\[2ex] 1 \quad (1 \quad p_1) \quad \dfrac{1 \quad x}{1 \quad p_1} \quad ^{m}, & x \quad (p_1, 1]. \end{cases} \qquad (12)$$

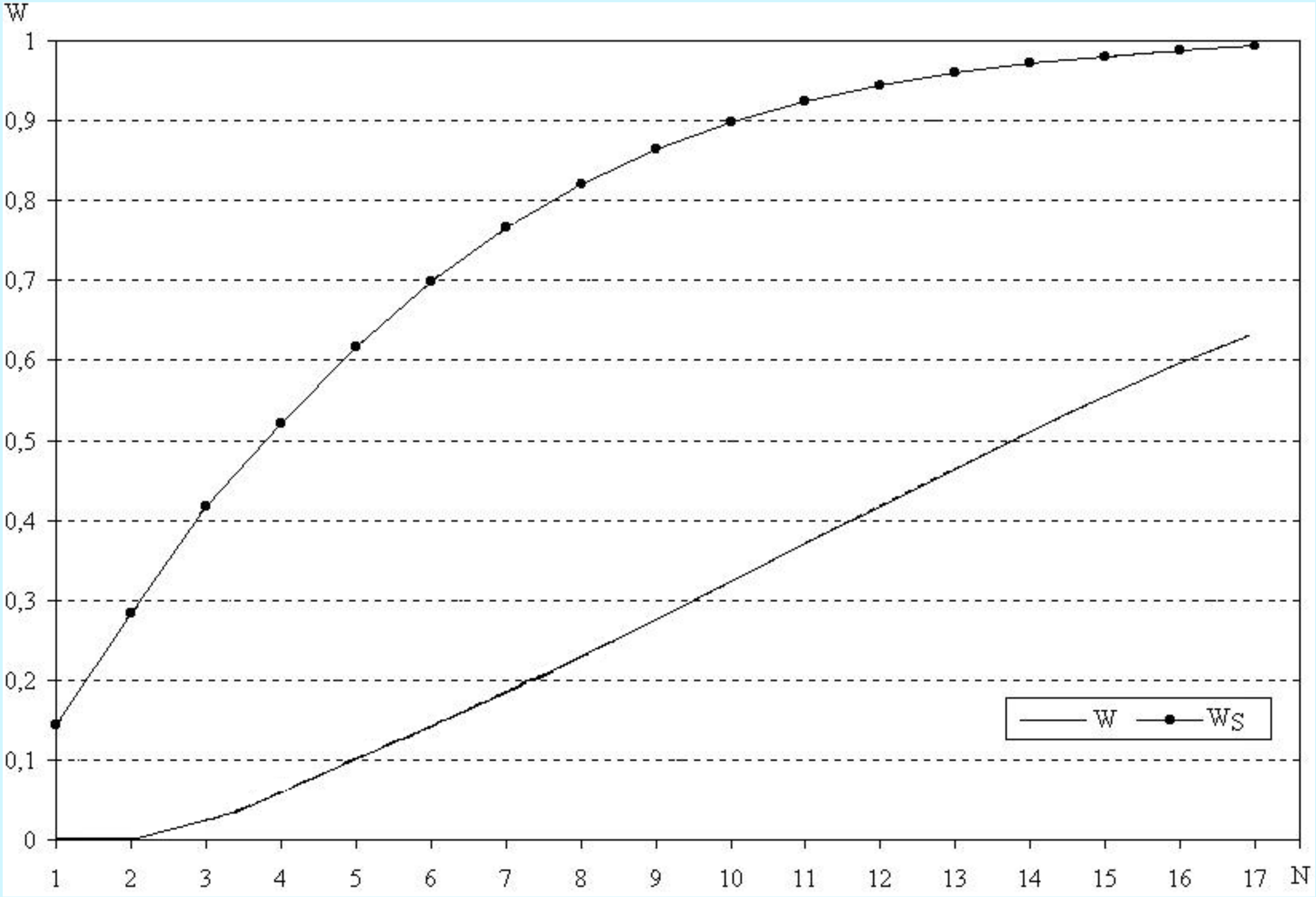
Example

$m = 3, \;\; c_1 = p_1 = 0.8$



Example

$m = 3,\ p_1 = 0{,}7$



Thank you
for your attention!