

**RESEARCH OF HYDROTHERMAL REGIME  
IN THE KRASNOYARSK HYDROELECTRIC  
POWER STATION DOWNSTREAM  
BY MATHEMATICAL MODELING AND REMOTE SENSING**

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# Introduction

The Yenisei River is the largest in Russia in terms of runoff (599 km<sup>3</sup>/year) and seventh largest in world (1.5% of global runoff).

The stream flows in the meridian direction through various climatic zones. The river basin of total area  $2.6 \times 10^6$  km<sup>2</sup> houses the largest region in Russia – Krasnoyarsk Krai.

There are 6 hydroelectric power plants (HPP) built in the basin of the river. One of the them – Krasnoyarsk HPP is among the top ten world's most powerful hydroelectric power plants (6000 MW) and is the key anthropogenic factor influencing the Yenisei river.

# Krasnoyarsk hydroelectric power plant



The river dam is 124 m in height and 1065 m in width.

# Abstract

In this report we consider the hydrothermal regime in a 35-km river reach downstream of the Krasnoyarsk HPP on July 3, 2016.

The physical heat exchange processes include absorption of direct and scattered solar radiation by water, absorption of downwelling thermal infrared radiation (TIR) from the atmosphere by the water surface, TIR back from the water surface, convection of heat and heat loss due to evaporation of water.

To carry out mathematical simulation, we use the Fourier equation which allows downstream water temperature at various times to be estimated.

The water temperatures found by modeling are compared against water surface temperatures obtained by remote sensing satellite data.

# Mathematical modeling of the hydrothermal regime

The hydrothermal river regime in this situation can be described by the Fourier equation

$$\frac{\partial T_w(x,t)}{\partial t} = V(x) \frac{\partial T_w(x,t)}{\partial x} + D \frac{\partial^2 T_w(x,t)}{\partial x^2} + \frac{W(t) B(x)}{\rho c S(x)}. \quad V = \frac{Q}{S}.$$

The first term on the right-hand side refers to the rate change in temperature caused by advection, the second one is associated with the rate change in temperature due to dispersion, and the third term describes heat exchange between water and the surrounding environment.

# Mathematical modeling...

The  $W$  ( $\text{Wm}^{-2}$ ) is the heat transfer power between water and the surroundings which equals

$$W(t) = W_s + W_{ss} + W_a - W_w + W_c - W_e,$$

where

$W_s$ ,  $W_{ss}$  are, respectively, the direct and scattered downwelling solar radiation absorbed by water;

$W_a$  is atmospheric TIR absorbed by water;

$W_w$  is TIR from water surface to the atmosphere;

$W_c$  is convective heat transfer from water to the atmosphere, and

$W_e$  is the loss of heat due to evaporation.

# Mathematical modeling...

The typical flow velocity in the downstream reach of  $2 \text{ m sec}^{-1}$ , and the thermal conductivity can be neglected. In the system of coordinates moving at a velocity  $V(x)$  equation for temperature is rewritten as

$$\frac{\partial T_w(x,t)}{\partial t} = \frac{W(t) B(x)}{\rho c S(x)},$$

solution of which is found from the expression

$$T_w(x,t) = \frac{1}{\rho c} \int_{t_0}^t \frac{B(x(t))}{S(x(t))} W(t) dt + T_w(0,t_0), \quad x(t) = Q \int_{t_0}^t \frac{dt}{S(x(t))}.$$

Here  $T_w(0,t_0)=7,2^\circ\text{C}$  is the outflow temperature of water leaving the dam at time  $t_0$ .

$$Q=2,9 \times 10^3 \text{ m}^3/\text{сек}$$

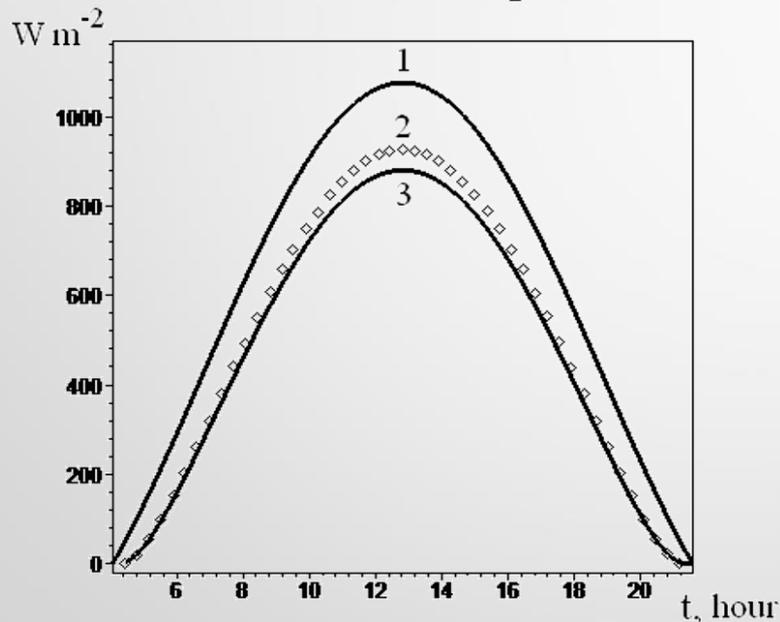
# Physical modeling of a hydrothermal regime

## Solar radiation

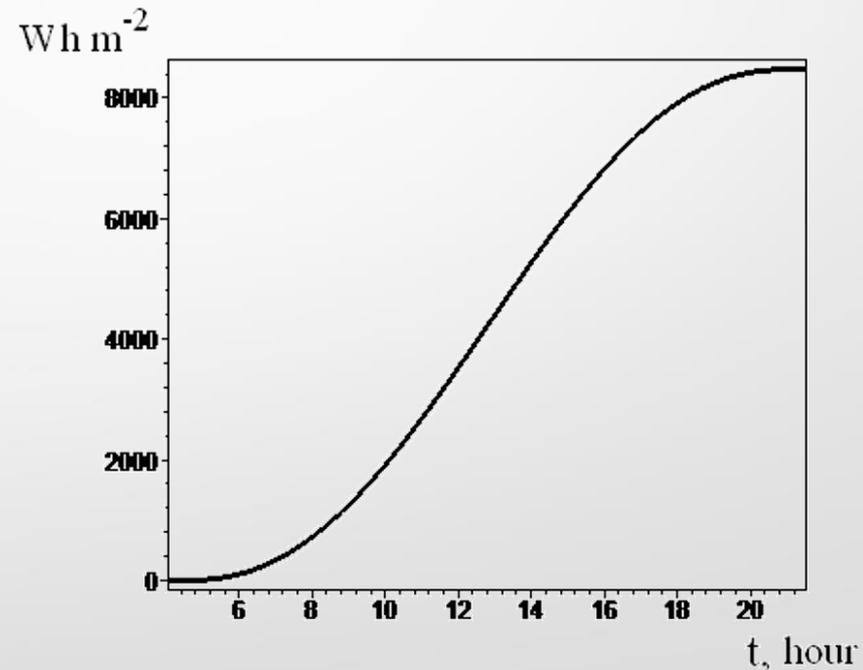
$$F = F_0 E \cos \theta, \quad \cos \theta = \sin \varphi \cdot \sin \delta + \cos \varphi \cdot \cos \delta \cdot \cos \omega t', \quad \tau_0 = 0,125$$

The temporal dependence of the solar radiation power and solar energy versus time of the day presented on Figures:

a) the solar radiation power



b) solar energy absorbed by water



- 1 – incident on the Earth's atmosphere,
- 2 – transferred through the atmosphere,
- 3 – absorbed by water

# Thermal Infrared Radiation (TIR)

## Emission from water

Water surface emits TIR defined by the Stefan-Boltzmann law as

$$W_w = \varepsilon_w \sigma (273 + T_w)^4,$$

where  $\varepsilon_w$  is emissivity of the water surface, which is  $\varepsilon_w = 0.995$  according to (Handcock et al., 2012).

The Stefan-Boltzmann constant is  $\sigma = 5.67 \cdot 10^{-8} (\text{W m}^{-2} \text{K}^{-4})$ .

For  $T_w = 7.2^\circ \text{C}$  we have  $W_w = 290 \text{ W m}^{-2}$

While emitting this energy the water gets colder.

# Thermal Infrared Radiation (TIR)

## Atmospheric emission

Our analysis has shown that the best suited formula for our situation is the one proposed by (Iziomon et al., 2003) because the optical thickness of the atmosphere in the wavelength range  $9.8 \mu\text{m}$  is close to unity (Rees, 2001).

We now can carry out calculations for July 3, 2016.

At noon  $T_a = 26^{\circ}\text{C}$  and the humidity was  $H=45\%$ ,  
at midnight  $T_a = 14^{\circ}\text{C}$ ,  $H = 85\%$ .

Then we have  $\varepsilon_a = 0.99$ ,  $W_a = 460 \text{ W m}^{-2}$  at noon  
and  $\varepsilon_a = 0.99$ ,  $W_a = 390 \text{ W m}^{-2}$  at midnight.

Atmospheric thermal infrared radiation is absorbed by water surface and increases the water temperature.

# Heat exchange between water surface and atmosphere

## Evaporation

The energy spent on water evaporation  $W_e$  is estimated as (Shulyakovskii, 1969; Ryan and Harleman, 1973; Gulliver and Stefan, 1986)

$$W_e = \rho L f(w)(e_s - e_a),$$

where  $L=2.26 \cdot 10^6 \text{ J kg}^{-1}$  is the latent heat of evaporation,  
 $e_a$  (mb) is the atmospheric water vapor pressure,  
 $e_s$  (mb) is the saturation vapor pressure.

When the wind velocity is  $w=0$ , we have  $f \approx 3 \cdot 10^{-9} (\text{mb}^{-1} \text{m s}^{-1})$  according to the data from (Oregon, 2010).

For the quoted data we have  $W_e = 31 \text{ W m}^{-2}$  at noon and  $W_e = 4.6 \text{ W m}^{-2}$  at midnight, which results in the drop of the water temperature.

# Heat exchange between water surface and atmosphere

## Convective heat flux

Convective heat flux is estimated as (Bowen, 1926):

$$W_c = 0.61\rho L f (T_w - T_a).$$

Thus, convective heating is  $25 \text{ W m}^{-2}$  at noon and  $1.3 \text{ W m}^{-2}$  at midnight.

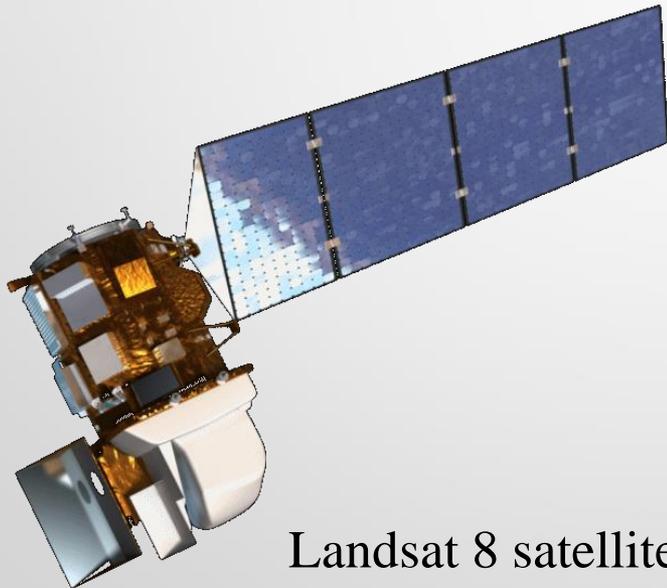
The difference between evaporation and convection is  $6 \text{ W m}^{-2}$  during the daytime and  $3.3 \text{ W m}^{-2}$  at night.

So, the heat budget is dominated by evaporation and the water gets colder.

# Water temperature from remote sensing data

Landsat 8 satellite Thermal InfraRed Sensor (TIRS) data were used to estimate the water surface temperature.

The input data of the TIR channels are converted into brightness temperature, which, in turn, into the water surface temperature.



Landsat 8 satellite

## **Landsat 8 TIRS data:**

Spatial resolution = 100 m/pixel

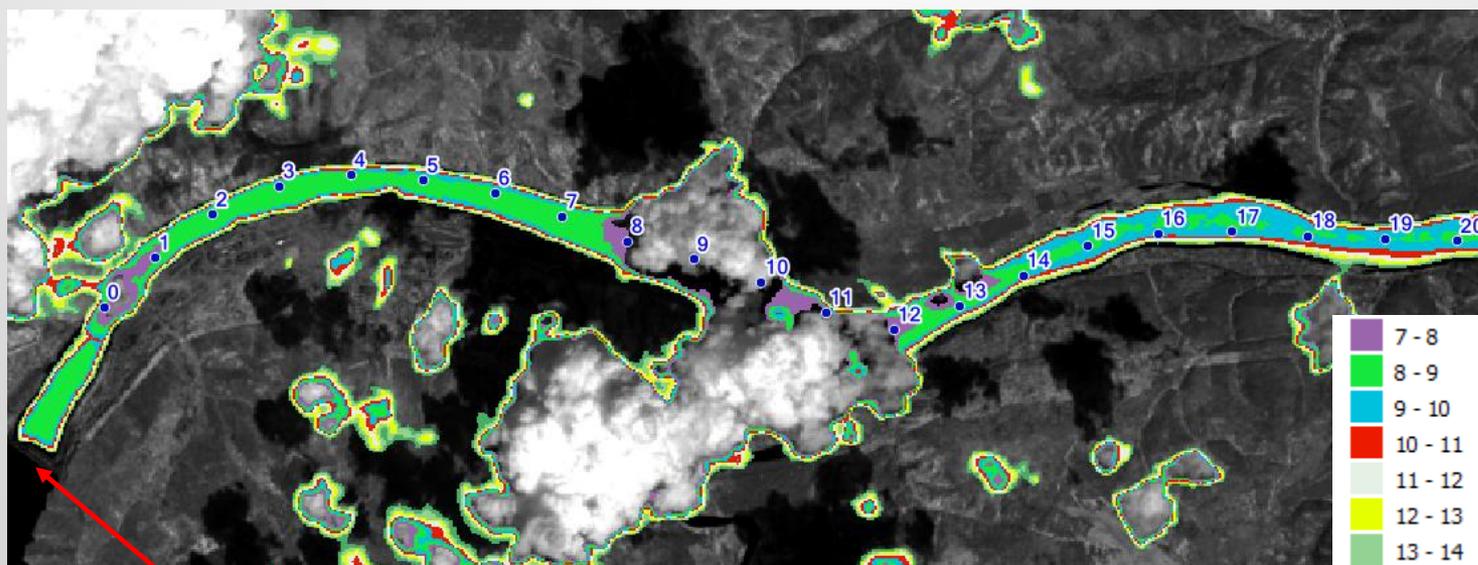
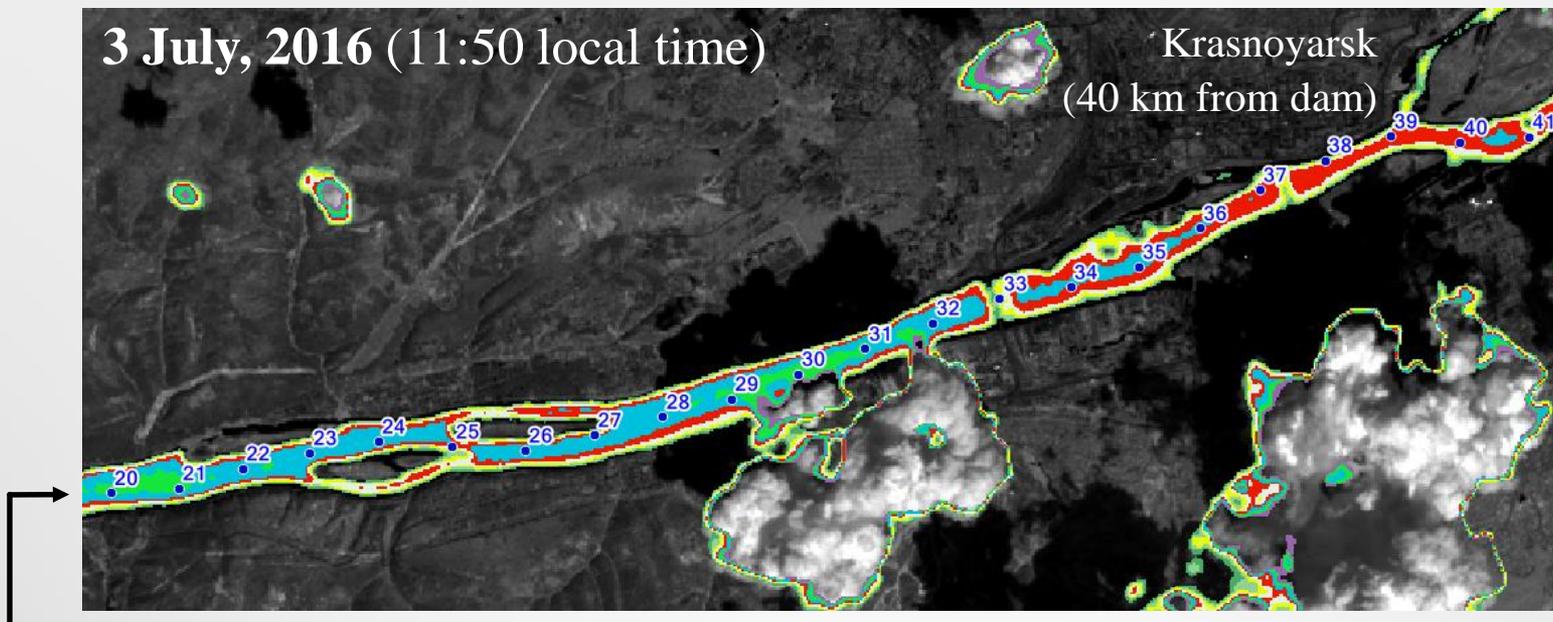
TIR1 channel: 10,3 – 11,3  $\mu\text{m}$

TIR2 channel: 11,5 – 12,5  $\mu\text{m}$

# Remote sensing water surface temperature

3 July, 2016 (11:50 local time)

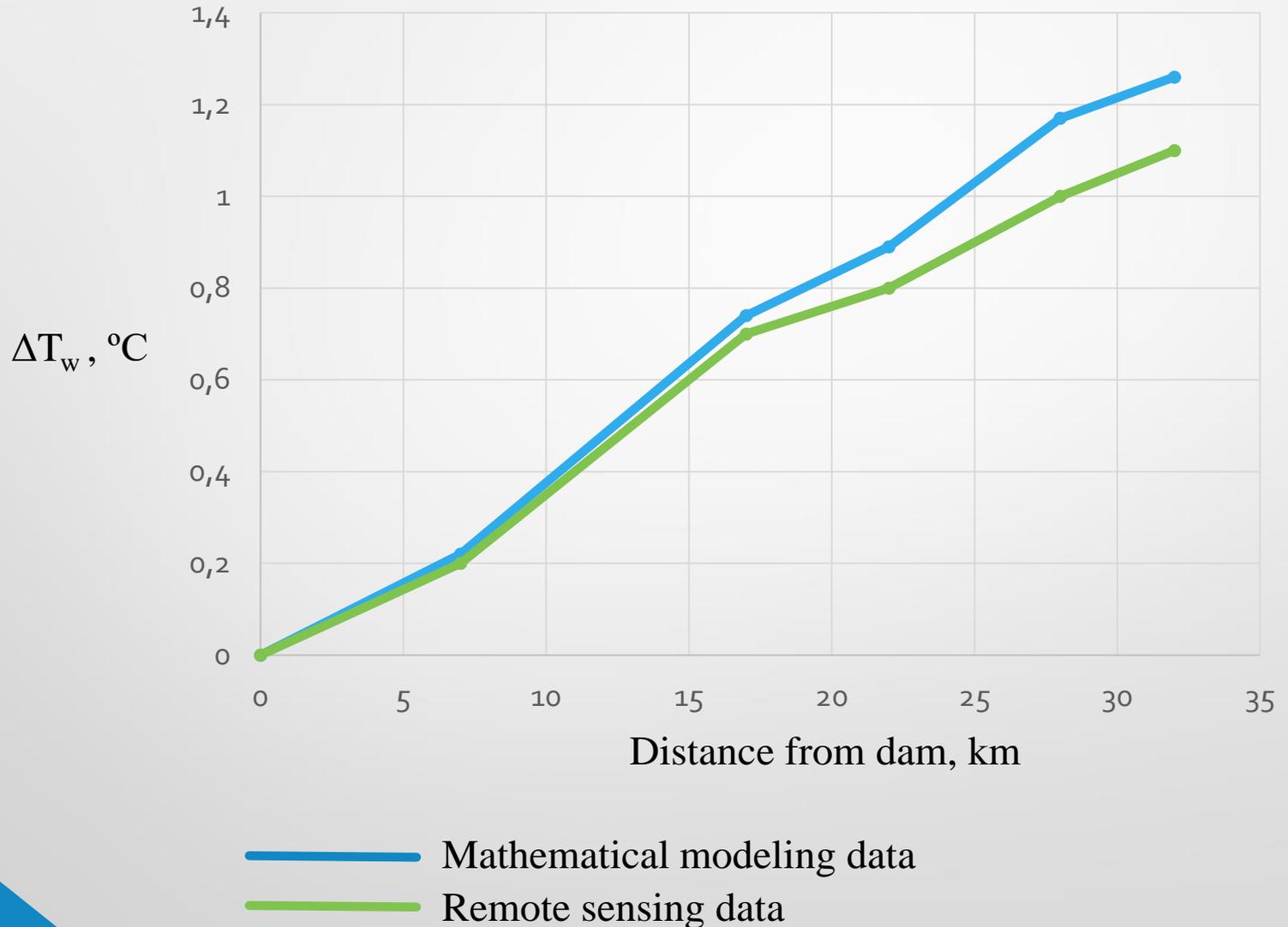
Krasnoyarsk  
(40 km from dam)



Krasnoyarsk hydroelectric power plant dam

Temperature, °C

# Water temperature depending on the distance: Mathematical modeling & Remote sensing



# Conclusions

We have proposed a simple model for simulating summertime hydrothermal regime of a river based on calculation of water temperature in a coordinate system moving with water.

The physically based estimation of water heat budget takes into account absorption of solar radiation by water surface, emission and absorption of atmospheric TIR by water, convective heating of water as well as heat loss due to evaporative processes.

The temporal fluctuation pattern of direct and scattered solar radiation depends on the zenith angle and atmospheric absorption.

The dominant water heating factor is solar radiation during the daytime and atmospheric TIR at night.

Water temperatures 35 km downstream of the Krasnoyarsk HPP on the Yenisei River computed using the proposed model with consideration of morphometric characteristics are close to the temperatures obtained from remote sensing data, which proves that the deployed physical-mathematical model provides an adequate description of the actual hydrothermal processes.