Cold front speed calculation with a hyperbolic model of the atmosphere

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In the present study, a numerical meteorological model is applied to the calculation of the speed of propagation of a cold front in the atmosphere over an artificial obstacle in the form of a hill, as well as along flat terrain.

The model is constructed on some basic principles developed by S. K. Godunov and E. I. Romenski :

Hyperbolicity

Fully divergent form of the governing equations

Consistency with the laws of thermodynamics

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Changes in surface ozone concentration after atmospheric front propagation (372 fronts,1989-1993, Tomsk)

	Front type	Decrease %	Increase %	No change %
	Cold	70	24	6
	Warm	43	53	4
	Occlusion	35	48	17
	Surface cold	47	37	16
	Upper warm	12	56	32
	All types	49	40	11

(Belan B.D., Ozone in the troposphere., IAO SB RAS,Tomsk,2010.-488 pp.)

The front speed in the propagation of an atmospheric gravity current (cold front) over flat terrain and over a steep hill is estimated in the present study with a non-hydrostatic finite-difference model of atmospheric dynamics. Artificial compressibility is introduced into the model in order to make its equations hyperbolic. For comparison with available simulation data, the physical processes under study are assumed to be non-viscous and adiabatic. The influence of orography is also estimated. The results of simulations of front speed under stable stratification are presented and compared with an empirical formula.

A general form of the basic equations .
With specially-chosen variables the system can be transformed to symmetric form .

$$\frac{\partial L_{q_i}}{\partial t} + \frac{\partial (u_k L)_{q_i}}{\partial x_k} = 0,$$

$$\frac{\partial L_{u_i}}{\partial t} + \frac{\partial [(u_k L)_{u_i} - r_{i\alpha} L_{r_{k\alpha}} - b_i L_{b_k} - d_i L_{d_k} + j_k L_{j_i} - \delta_{ik} j_\alpha L_{j_\alpha}]}{\partial x_k} = 0,$$

$$\frac{\partial L_{r_{il}}}{\partial t} + \frac{\partial [u_k L_{r_{il}} - u_i L_{r_{kl}}]}{\partial x_k} = 0,$$

$$\frac{\partial L_{d_i}}{\partial t} + \frac{\partial [u_k L_{d_i} - u_i L_{d_k} - e_{ikl} b_l]}{\partial x_k} = 0,$$

$$\frac{\partial L_{b_i}}{\partial t} + \frac{\partial [u_k L_{b_i} - u_i L_{b_k} + e_{ikl} d_l]}{\partial x_k} = 0,$$

$$\frac{\partial L_n}{\partial t} + \frac{\partial [u_k L_n + j_k]}{\partial x_k} = 0,$$

$$\frac{\partial L_{j_k}}{\partial t} + \frac{\partial [u_\alpha L_{j_\alpha} + n]}{\partial x_k} = 0$$

$$\frac{\partial}{\partial t}(q_i L_{q_i} + u_i L_{u_i} + r_{il} L_{r_{il}} + d_i L_{d_i} + b_i L_{b_i} + n L_n + j_k L_{j_k} - L) +$$

$$\frac{\partial}{\partial x_k}(u_k (q_i L_{q_i} + u_i L_{u_i} + r_{il} L_{r_{il}} + d_i L_{d_i} + b_i L_{b_i} + n L_n)) = 0$$

$$div\, L_r = 0,\; div\, L_b = 0,\; div\, L_d = 0, rot\, L_j = 0$$

Relation between Potential Temperature and Entropy

Air follows the ideal gas laws quite closely, and these are sufficiently accurate for most purposes.

For an ideal gas *cp is independent of pressure and temperature, so*

$$\eta = c_p \ln\theta + const.$$

(Adrian E. Gill Atmosphere-Ocean Dynamics *1982 Academic Press)*

$$\frac{dU}{dt}+\frac{\partial P}{\partial x}=f_1(V-V_g)-f_2W+R_u,$$

$$\frac{dV}{dt}+\frac{\partial P}{\partial y}=-f_1(U-U_g)+R_\upsilon,$$

$$\frac{dW}{dt}+\frac{\partial P}{\partial z}+\frac{gP}{C_s^2}=f_2U+g\frac{G^{1/2}\bar{\rho}\theta'}{\theta}+R_\varpi$$

$$\frac{d\theta}{dt}=R_\theta,$$

$$\frac{ds}{dt}=R_s,$$

$$\frac{1}{C_s^2}\frac{\partial P}{\partial t}+\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=\frac{\partial}{\partial t}\left(\frac{\bar{\rho}\theta'}{\theta}\right)$$

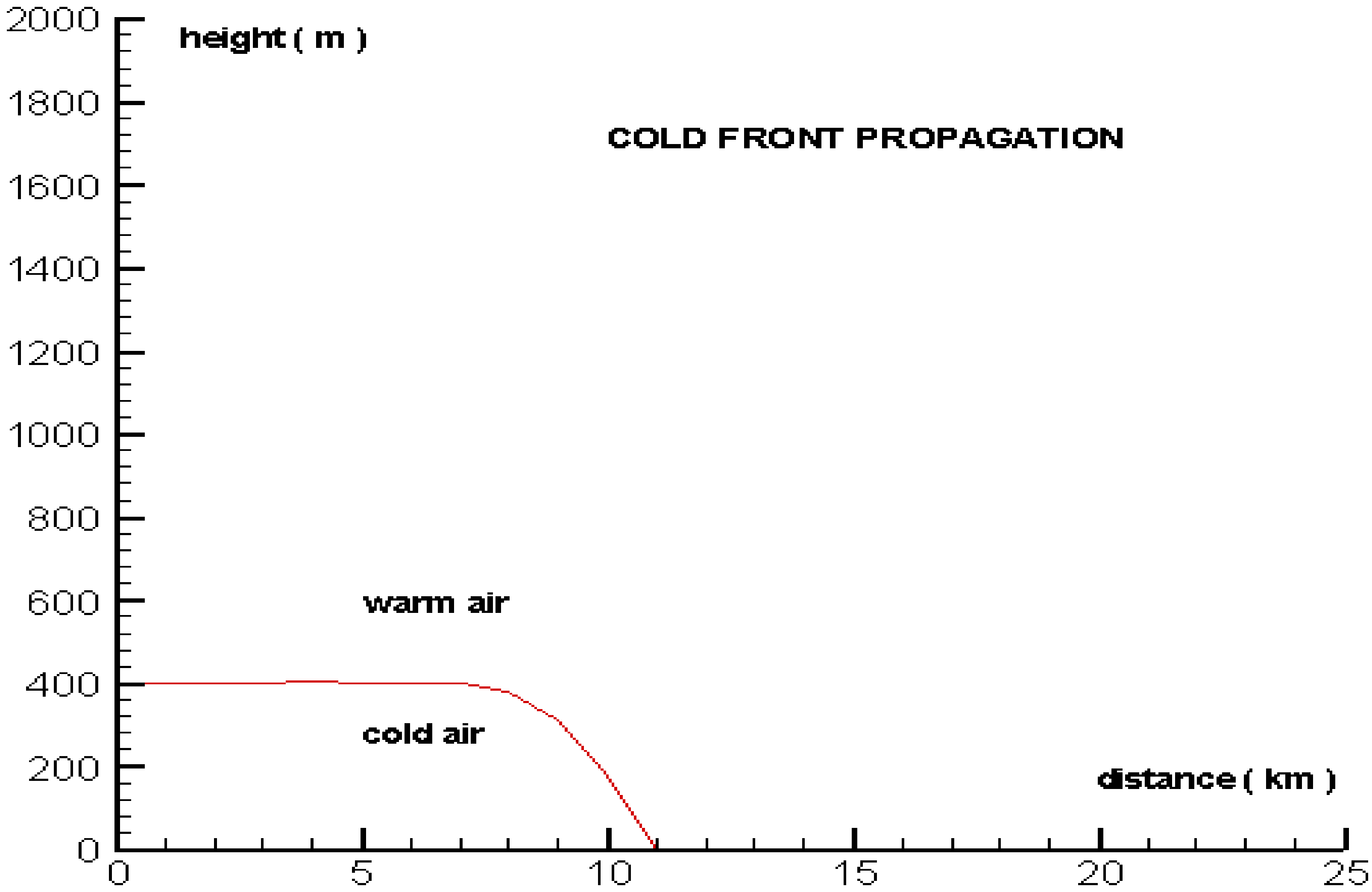
$$U=\bar{\rho}u, V=\bar{\rho}\upsilon, P=\bar{\rho}p', \; W=\bar{\rho}\omega$$

$$\delta\tau U + \frac{\partial}{\partial x}P + \frac{\partial}{\partial \xi}(G^{13}\ P) = -ADVU$$

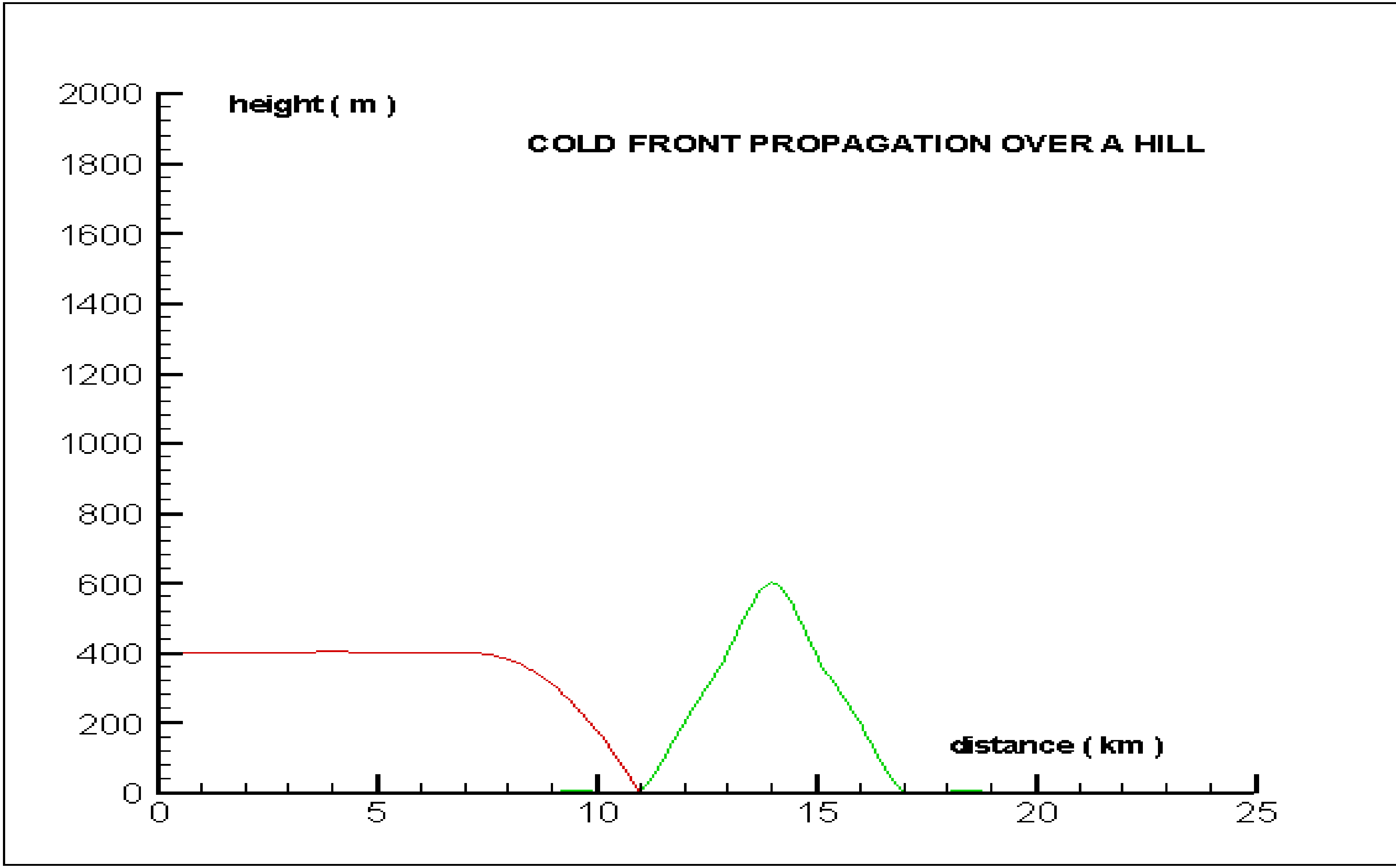
$$\delta\tau V + \frac{\partial}{\partial y}P + \frac{\partial}{\partial \xi}(G^{13}\ P) = -ADVV$$

$$\delta\tau W + \frac{1}{G^{1/2}}\frac{\partial \overline{P}^{\tau\beta}}{\partial \xi} + \frac{g\overline{P}^{\tau\beta}}{Cs^2} = BUOY - ADVW$$

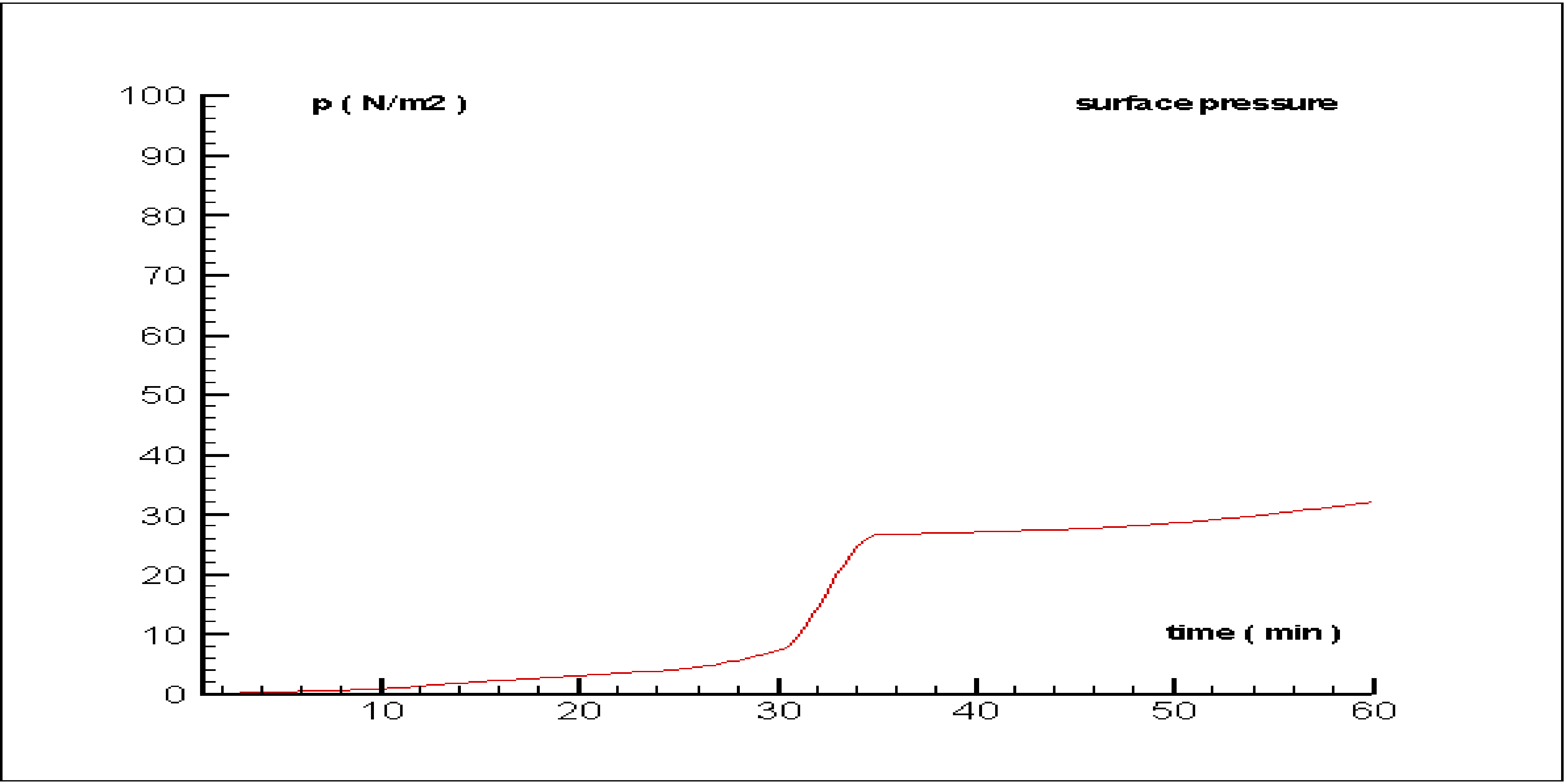
$$\frac{1}{Cs^2}\ \delta\tau P + \frac{\partial}{\partial x}\overline{U}^{\tau\gamma} + \frac{\partial}{\partial y}\overline{V}^{\tau\gamma} + \frac{\partial}{\partial \xi}(G^{13}\ \overline{U}^{\tau\gamma}) + \frac{\partial}{\partial \xi}(G^{13}\ \overline{V}^{\tau\gamma}) + \frac{1}{G^{1/2}}\frac{\partial \overline{W}^{\tau\beta}}{\partial \xi} = PFT$$



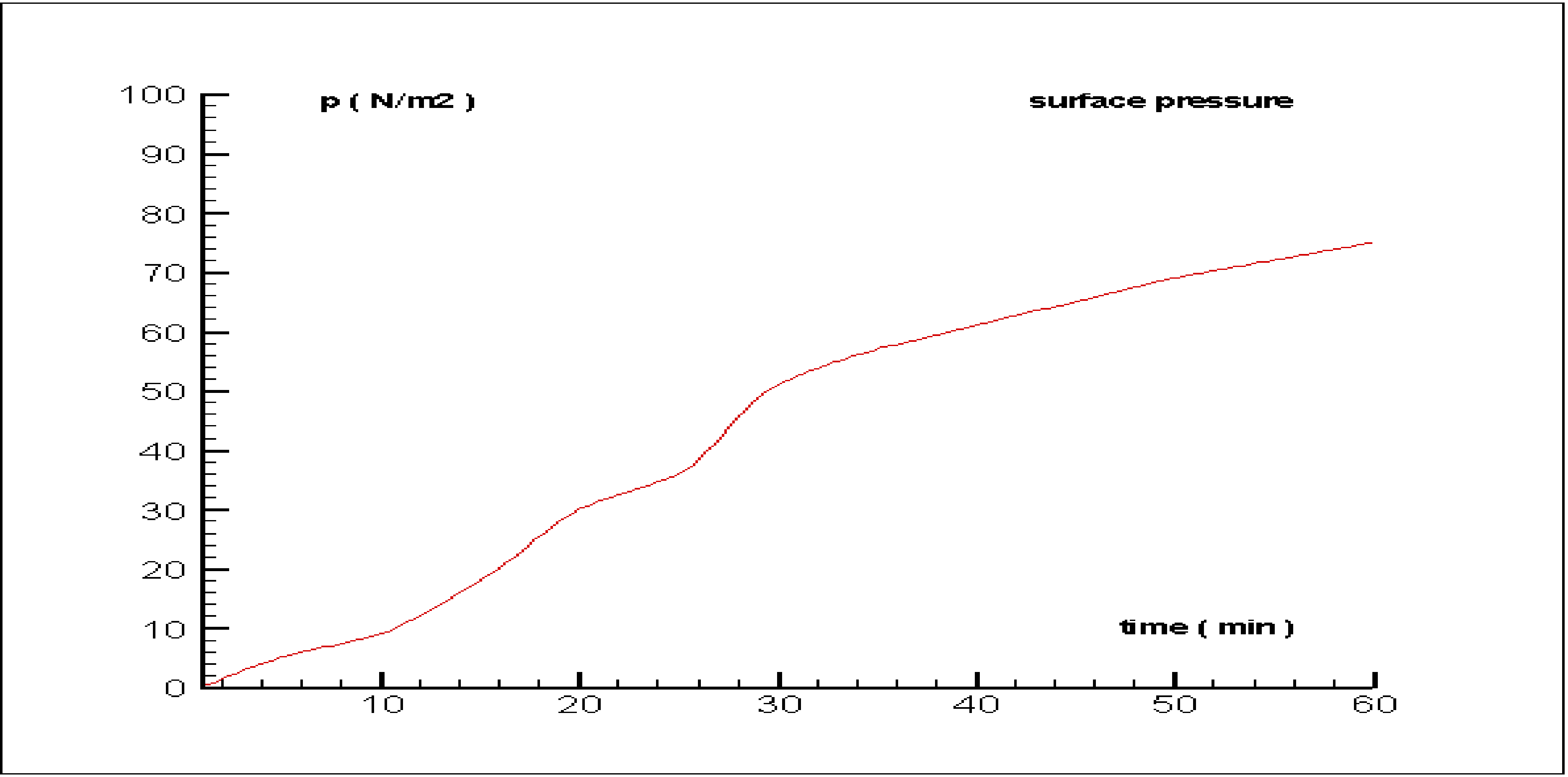
Cold front in the atmosphere over flat orography. Stable stratification



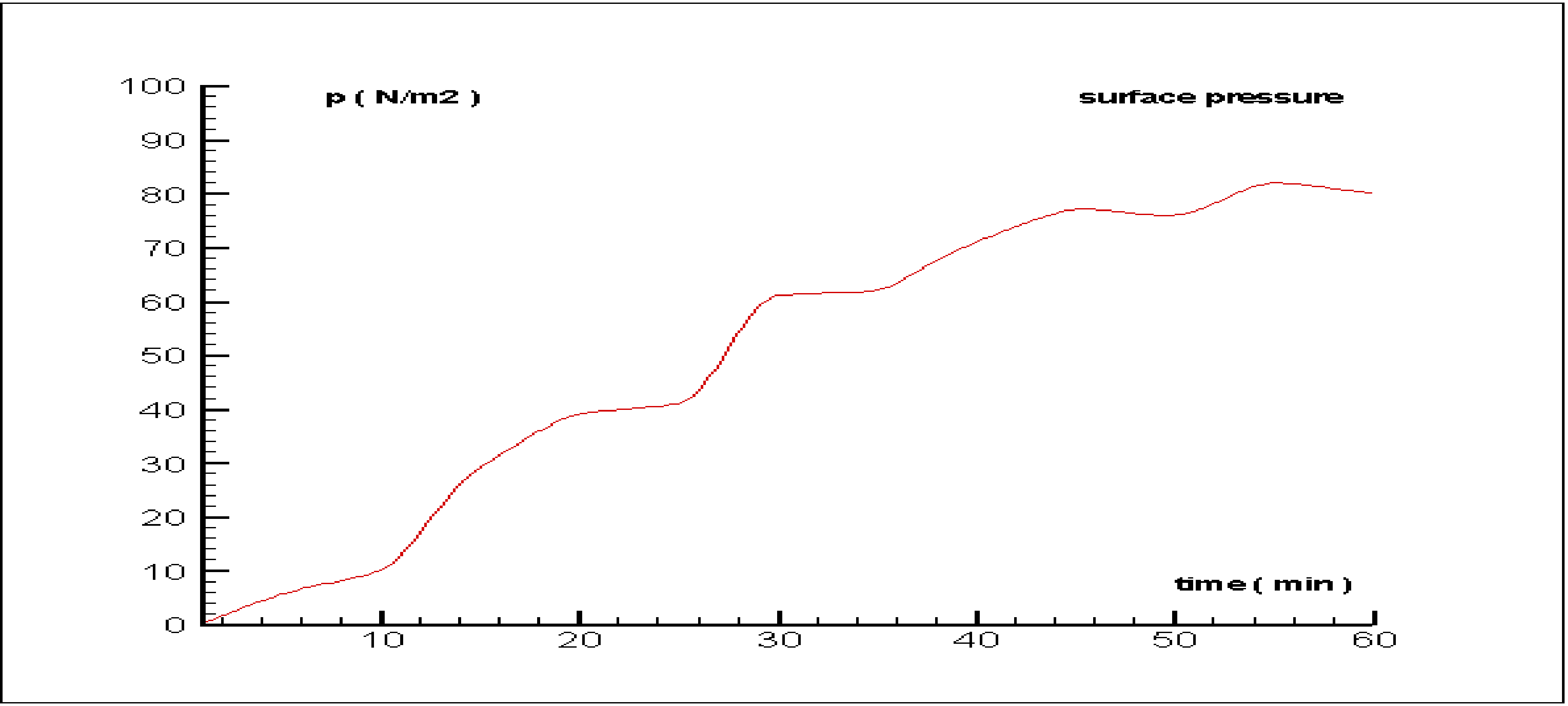
Cold front propagation over a hill. Stable stratification.



Surface pressure at 12 km. Neutral stratification.



Surface pressure at 12 km. Stable stratification.



Surface pressure at 12 km. Stable stratification with inversion.

$$v_F = k(gh_{IFH}\Delta\theta/\theta)^{0.5}$$

$$k = 0.81$$

$$h_{IFH} = 400m$$

$$\theta = 288K$$

$$\Delta\theta = 2K$$

$$v_F = 4.2m/s$$

Cold front propagation over orographic obstacles of various shapes and stratifications

OBSTACLE HEIGHT (m)	INITIAL FRONT HEIGHT (м)	STRATIFICATION (K / 100m)	WINDWARD SPEED (m /sec)	LEEWARD SPEED (m /sec)
0	400	0.0	4.5	4.5
0	400	0.35	5.1	5.1
600	400	0.0	4.4	3.7
600	400	0.35	4.9	2.7
600	100	0.35	3.0	0.0
600	700	0.35	7.5	4.5
- 600	400	0.0	4.5	3.9

Conclusions

The results of the simulations of front speed in the propagation of an atmospheric gravity current (cold front) over flat terrain and over a hill under stable stratification presented above have been compared with an empirical formula. A good agreement between the results of the calculations and the theory has been shown.

The results of the simulations described above have been obtained with two types of models: a 2D non-hydrostatic finite-difference meteorological model and a 2D finite-element model. Both models are dicretized versions of a non-hydrostatic model formulated as the Navier-Stokes equations in the Boussinesq approximation written in a compressible form.

The 2D finite-element model is based on triangular elements. In this study an application of the model was made to simulating cold front propagation over an idealized hill-type obstacle in a stratified atmosphere with an inversion layer over an isolated hill. The study was performed under stable stratification in and beyond the inversion layer. It has been shown that the introduction of the inversion produces a significant decrease in the front speed both for the currents over the obstacle and those over flat orography.

The 2D finite-difference model is based on spatial discretizations that conserve some important quantities of the phenomena under study, atmospheric gravity currents. Also, an efficient procedure is used in both versions of the model to calculate the advection of scalars.

The change in stratification from neutral to stable in the propagation of the cold atmospheric front has shown a time evolution of the surface pressure that is in good agreement with the available observational data. Also, in contrast to the slow evolution of the surface pressure under neutral stratification, there is a considerable pressure jump in a stable atmosphere. This effect is increased by the introduction of an inversion layer. This phenomenon was explained by Charba. The results of calculations of the present study are in good agreement with Charba’s theory.