Estimation of the probability of extreme intra-monthly anomalies of atmospheric values

Loginov S.V.[1], Eliseev A.V.[2,3,4], Loginov A.S.[5]

[1]IMCES SB RAS, 10/3 Academichesky ave., Tomsk, 634055, (logsv13@imces.ru)
[2]M.V. Lomonosov MSU, Moscow, [3]KFU, Kazan, [4]A.M.Obukhov IAP RAS, 3, Pyzhevsky, Moscow, 119017 (eliseev@ifaran.ru)
[5]NR TSU; 36, Lenin Avenue, Tomsk, 634050; (as_loginov@phys.tsu.ru)

Motivation and goals

Deviations from the normal (Gaussian) distribution of atmospheric variables may reflect nonlinear aspects of the atmospheric dynamics (Monahan, 2006; Petoukhov et al., 2008; Sura, Hannachi, 2015; Loginov et al., 2017). Moreover, such deviations affect 'tails' of the sampled probability density functions (PDFs) of weather-related anomalies of atmospheric variables. The latter may either enhance or diminish frequency of extreme weather events.

The goals of the present work are

- to study non-Gaussian features in weather variability based on the state-of-the-art reanalysis data,
- to construct a statistical model for non-Gaussian PDFs for weather variability in the atmosphere.

The asymmetry of PDF of weather anomalies - characteristic of nonlinear processes

$$X' = X - \langle X \rangle$$

X' and $-X'$ *is* equally probable

Linear Process → PDF is symmetric about E(X)

Nonlinear process → PDF is nonsymmetric about E(X)

skewness $S = \overline{x'^3}$ $x' = X'/\sigma$

kurtosis K$= \overline{x'^4 - 3}$

The unimodal probability density W (x) can be approximated by the Edgeworth series:

$$W(x) = W_G(x) + \sum_{k=3}^{\infty}(-1)^k \frac{\beta_k}{k!} W_G^{(k)}(x) \approx W_G(x) + \sum_{k=3}^{N}(-1)^k \frac{\beta_k}{k!} W_G^{(k)}(x)$$

$$W_G^{(k)}(x) = H_k(x) W_G(x)$$

$W_G(x)$ – Gaussian PDF,
$H_k(x)$ – Chebyshev-Hermite polynomials,
$\varkappa_k$ – Seminvariants, k – order,
β_k – quasi-moments of distribution, calculated from the recurrence relation

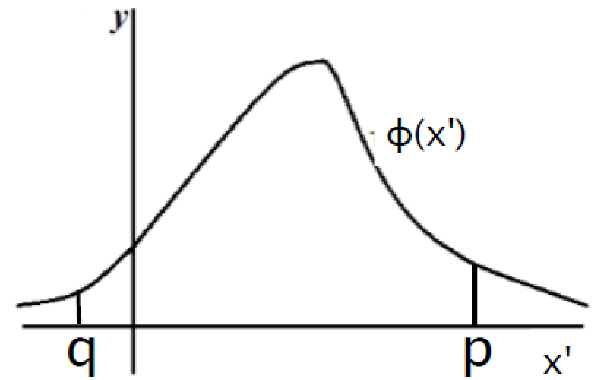
$$\beta_k = \sum_{i=0}^{k-1} \beta_{k-1-i} \varkappa_{i+1} \binom{k-1}{i}$$

Statistical model for probability of weather extremes

The Edgeworth expansion for PDF:

$$\phi = \phi_N(x') - \frac{S}{6}\phi_N^{(3)}(x') + \frac{K}{24}\phi_N^{(4)}(x')$$

(ϕ_N is a Gaussian PDF)

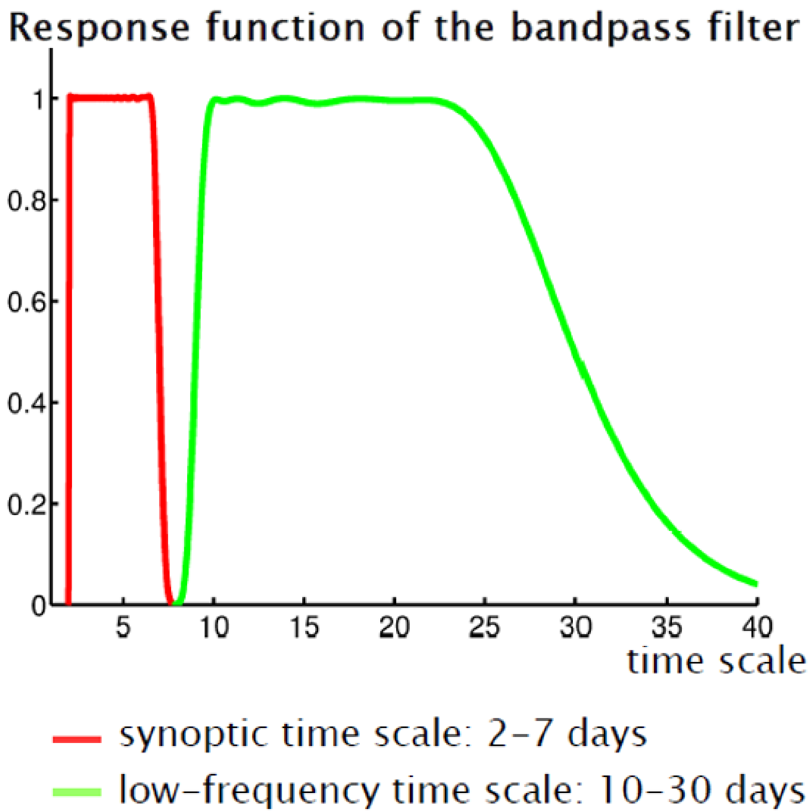


$$\int_p^{+\infty}\phi\,dx' = \frac{1}{2}\left[1-\mathrm{erf}\left(\frac{p}{\sqrt{2}}\right)\right] + \frac{S}{6}\frac{p^2-1}{\sqrt{2\pi}}e^{-p^2/2} + \frac{K}{24}\frac{p\left(3-p^2\right)}{\sqrt{2\pi}}e^{-p^2/2}$$

$$\int_{-\infty}^{q}\phi\,dx' = \underbrace{\underbrace{\underbrace{\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{q}{\sqrt{2}}\right)\right]}_{\text{Gaussian}} - \frac{S}{6}\frac{q^2-1}{\sqrt{2\pi}}e^{-q^2/2}}_{3^{\text{rd}}\text{ order}} - \frac{K}{24}\frac{q\left(3-q^2\right)}{\sqrt{2\pi}}e^{-q^2/2}}_{4^{\text{th}}\text{ order}}$$

p = −q $\Rightarrow$ $\mathbf{P}(\,|X'| > p\,\boldsymbol{\sigma}\,) = \int_{-\infty}^{q}\phi\,dx' + \int_p^{+\infty}\phi\,dx'$

Filter characteristics



- Nonrecursive finite impulse response filter(FIR)
- the Hamming weight window

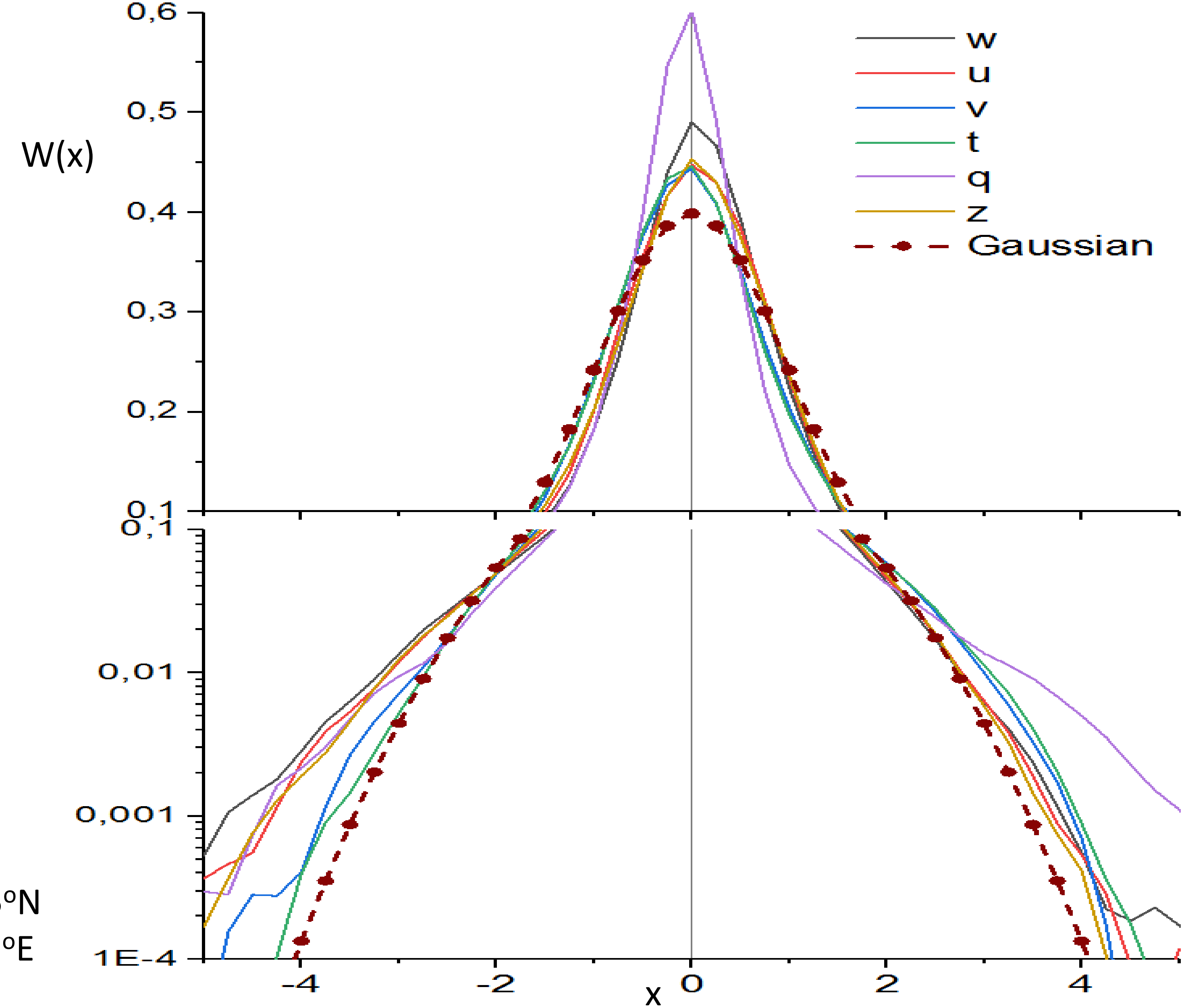
- Quantity decay
  - ~25 dB in low-frequency absorption band
  - ~40 dB in high-frequency absorption band

- Transition zones (0.5 of W function)

- Phase shift was removed by passing of filtered data through the filter in the forward and backward directions

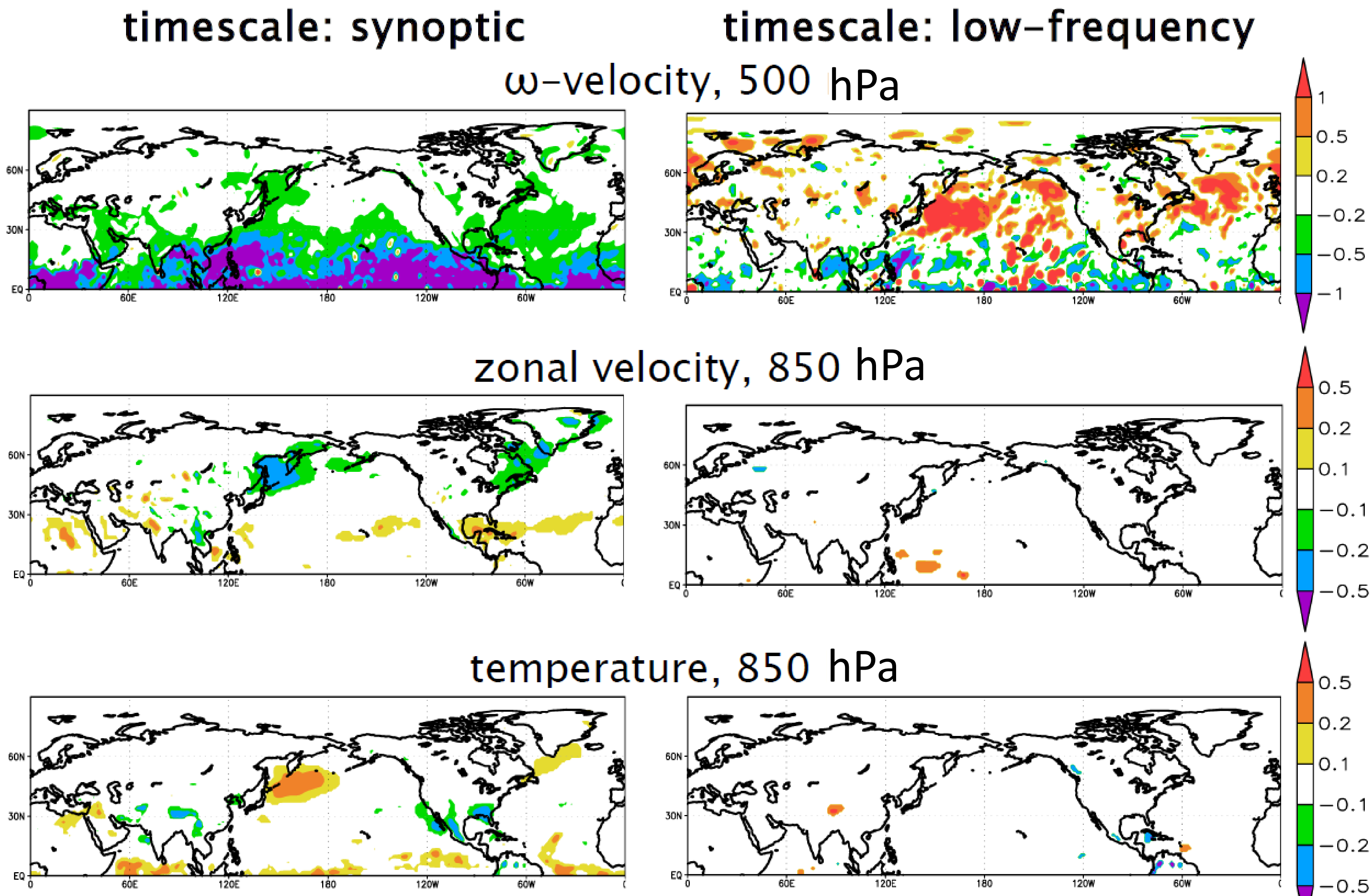
Example of PDF

in October–March for the interval of time scales of 2–7 days at the level of 850 hPa



47.5°N
170 °E

Skewness of weather anomalies

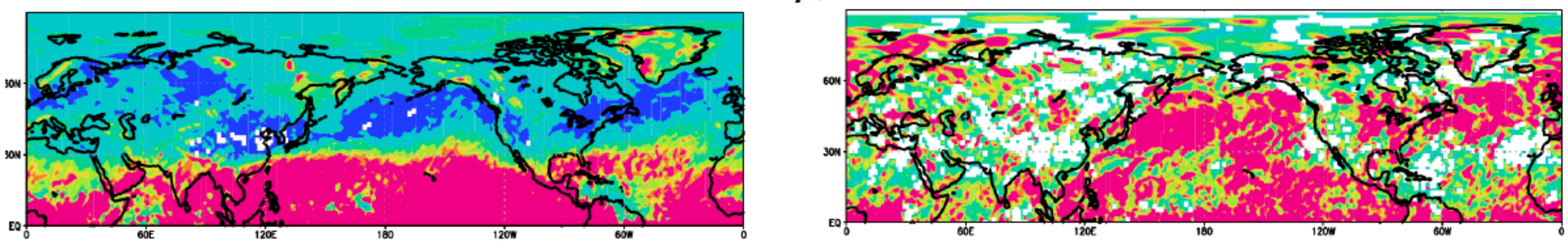


only values, which are statistically significantly differ from zero, are shown

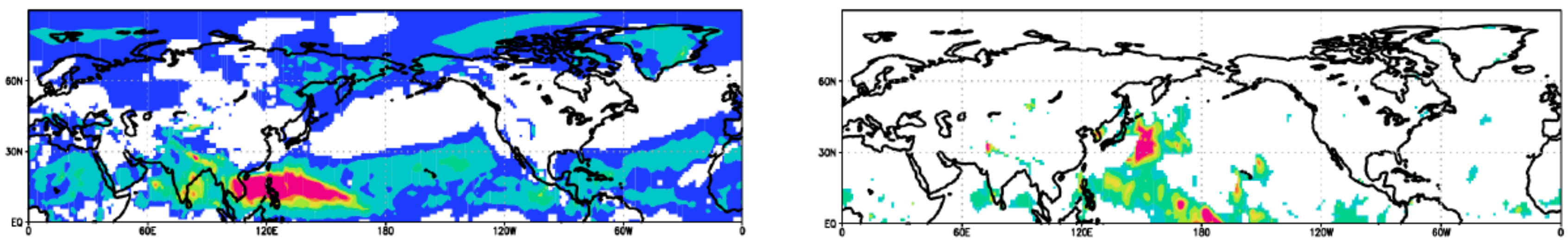
Kurtosis of weather anomalies

timescale: synoptic — timescale: low–frequency

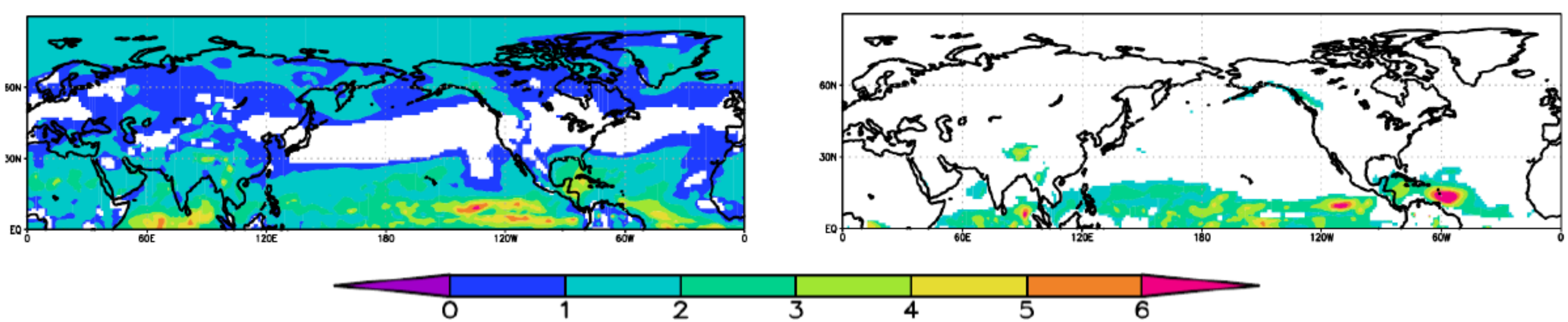
ω–velocity, 500 hPa



zonal velocity, 850 hPa

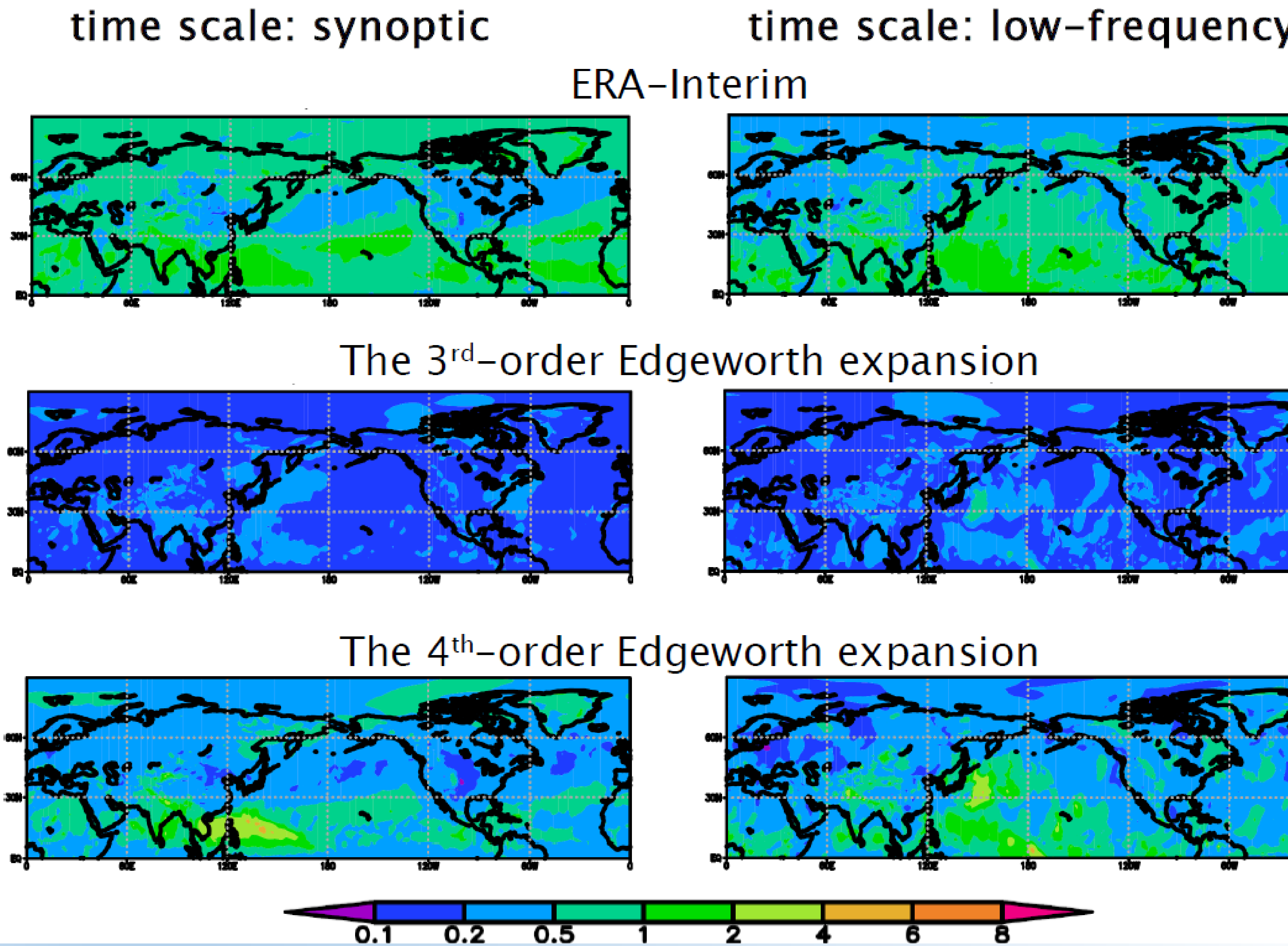


temperature, 850 hPa



only values, which are statistically significant differ from zero, are shown

Probability of development of zonal wind anomalies with $|u'| > 3\sigma_u$, 850 hPa



The difference (δ) of the empirical and model probability of weather anomalies in the Northern Hemisphere

		Synoptic variability				Low-frequency variability			
Level	**Value**	**N=3**	**N =4**	**N=5**	**N=6**	**N=3**	**N =4**	**N=5**	**N=6**
850, hPa									
	ω	0,882	0,419	0,415	0,433	0,708	0,388	0,49	0,558
	u	0,455	0,234	0,233	0,236	0,303	0,159	0,173	0,176
	v	0,505	0,256	0,255	0,258	0,357	0,199	0,203	0,205
	T	0,726	0,348	0,324	0,306	0,394	0,222	0,226	0,23
	q	1,028	0,429	0,480	0,513	0,729	0,308	0,329	0,342
	H	0,466	0,203	0,171	0,169	0,320	0,209	0,198	0,200
300, hPa									
	ω	0,927	0,467	0,508	0,567	0,717	0,466	0,614	0,711
	u	0,486	0,245	0,234	0,232	0,288	0,182	0,199	0,203
	v	0,563	0,282	0,267	0,263	0,395	0,246	0,254	0,26
	T	0,562	0,244	0,242	0,240	0,368	0,201	0,221	0,224
	q	1,383	0,674	0,785	0,883	1,119	0,675	0,681	0,686
	H	0,558	0,270	0,232	0,224	0,332	0,226	0,223	0,222

$$\delta = med(|P - \tilde{P}|_{j\in NH})$$

Yellow - cells with the least deviation δ
N – order in Edgeworth expansion

For normally distributed time series of finite length, the standard deviation of *S* estimate for normal process is (V.Petoukhov et al., 2008; M.Holzer, 1996) :

$$\sigma_{S,G} = \left(6/N_{eff}\right)^{1/2}$$

$$N_{eff} = N(1 - r_1)$$

where N_{eff} is the number of degrees of freedom of the filtered series calculates by

N is a length of the series and r_1 is its correlation coefficient with a unit lag

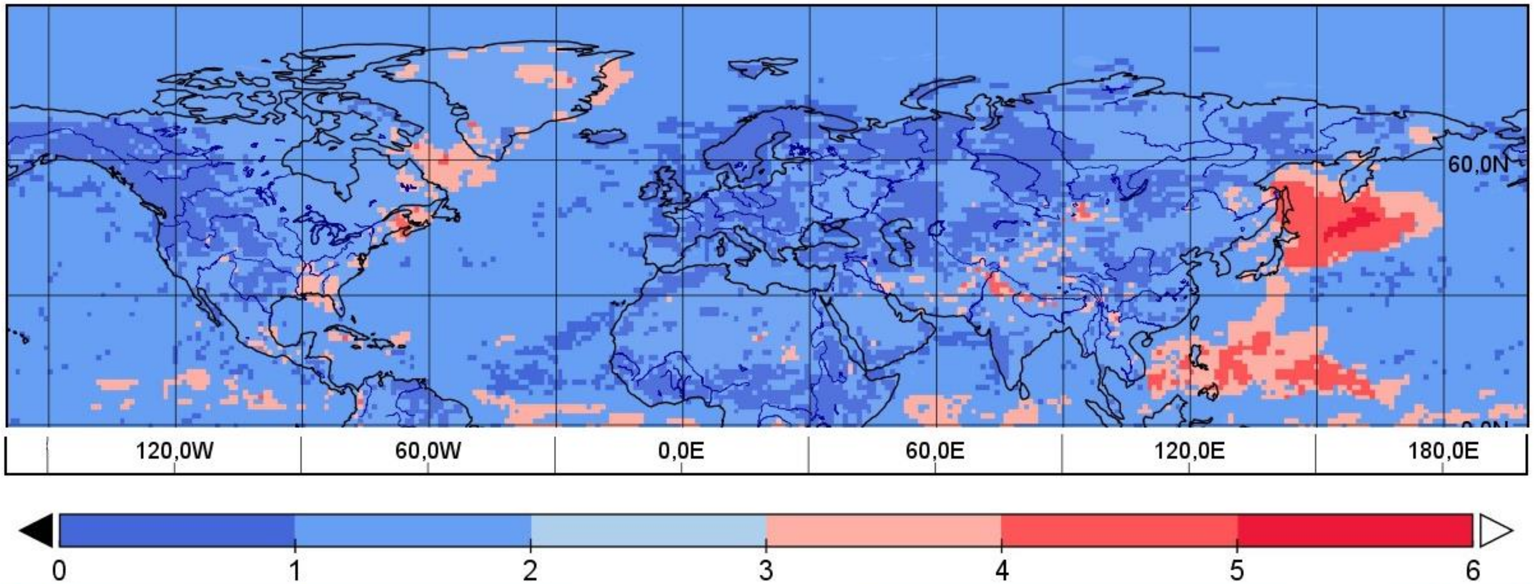
A characteristic of the deviation from the normal PDF

$$s = \left(S_X/\sigma_{S,G}\right)^{1/2}$$

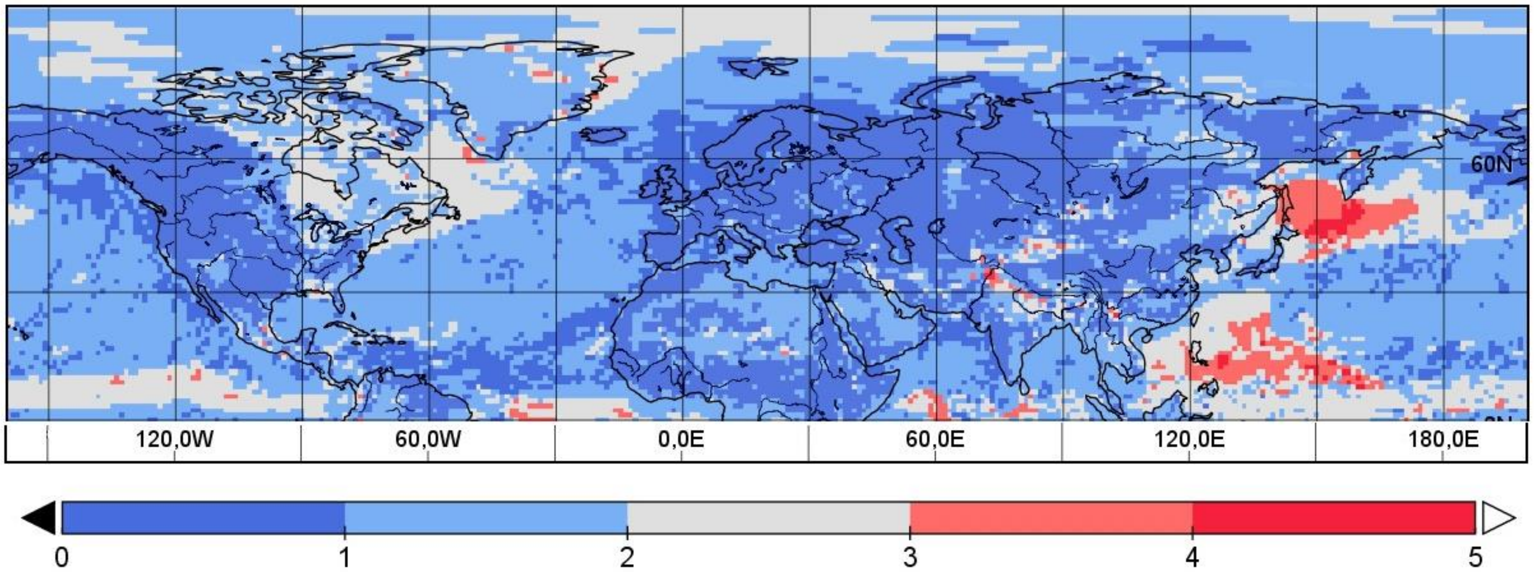
$$\theta = \sum_{1}^{6} \delta_j \qquad \delta_j = \begin{cases} 1, & s \geq M \\ 0, & s < M \end{cases}$$

Example of θ in October–March for the interval of time scales of 2–7 days at the level of 850 hPa

M=3



M=4



Results

- Marked deviations from Gaussianity are found, especially in the regions of the preferred formation of baroclinic eddies.
- The deviations of the atmospheric variability from the Gaussian distribution impact probabilities of extreme events.
- An approximation to probability distribution function, which is based on the Edgeworth expansion and accounts only for the two leading terms of this expansion, reasonably reproduces such probabilities compared to the reanalysis data.