

# The analysis of the turbulent kinetic energy decay power law in atmospheric boundary layer models

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# Introduction

## In this study

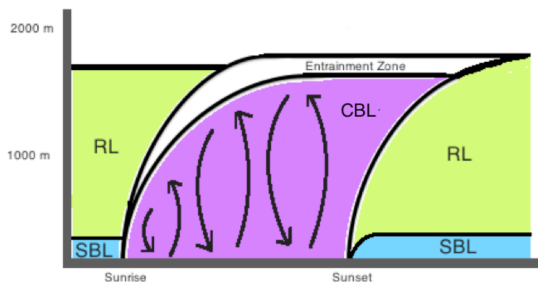
- ▶ The modeling of evening transition is analysed with an emphasis on the pattern of the TKE decay, which follows the power law  $E(t) \propto t^{-\alpha}$ .
- ▶ The effect of different parameters on the outcome of the simulation is explored, along with how the model is influenced by the presence of the geostrophic wind.

## Relevance

Temporal and spatial resolution in weather forecast and climate models has improved greatly throughout the recent decades. Therefore, studying the dynamics that are influenced by the diurnal cycle of the atmospheric boundary layer is more relevant than ever, and existing models would benefit from better understanding of the said dynamics.

## Transitional periods

- ▶ The atmospheric boundary layer (ABL) is the lowest part of the atmosphere, where most of the transport processes take place.
- ▶ One of the key characteristics of the boundary layer is the diurnal variation taking place within it, for example temperature variations, which is insignificant in the free atmosphere. Another is turbulence - one of the most important transport processes.
- ▶ The diurnal cycle consists of the boundary layer changing its state between stably stratified boundary layer (SBL) and convective boundary layer (CBL). Two transitional periods are distinguished: the morning (from SBL to CBL) and the evening (from CBL to SBL) transition.



Sketch of the diurnal cycle in the ABL (Carreras, 2014)

## Turbulence decay rate

The evening transition is characterized by the decay of the convective turbulence. The turbulence decay rate obeys the power law  $E(t) \propto t^{-\alpha}$ , where  $E(t)$  is the normalized TKE and  $t$  is the normalized time. The parameter  $\alpha$  can be found theoretically or empirically.

### Examples

- ▶  $\alpha = 10/7$ , by the hypothesis of the Loitsyansky invariant (Mohamed & Larue 1990; Perot, 2011) and  $\alpha = 2$ , by the assumption of a constant integral turbulence length scale (George & Wang, 2009; Oberlack & Zieleniewicz, 2013) for the decay of the unbounded turbulence
- ▶  $\alpha = 6$  by experimental data (Nadeau et al., 2011; Rizza et al., 2013) for the decay in the ABL

However, values of  $\alpha$  that have been obtained experimentally are not used for ABL turbulence models development at all - these models are mostly based on unbounded turbulence values.

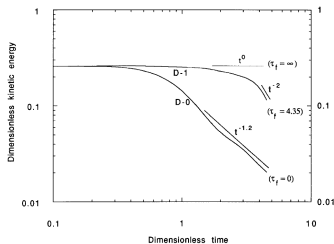
## LES modeling

Large-eddy simulations (LES) are based on resolving "large" - energy containing eddies and modeling the contribution. They are renowned for being a sufficient compromise between direct numerical simulations (DNS), which resolve all scales of motion, however small, and Reynolds-averaged Navier-Stokes (RANS) equations, which rely on ensemble averaging or averaging over time or space.

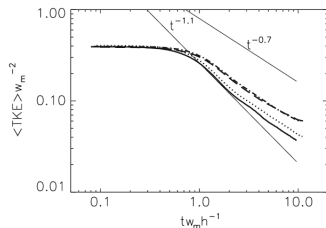
LES models are widely used in studying diurnal cycles in ABL, including transitional periods.

# LES modeling of evening transition

The idea of modeling the evening transition in LES models was pioneered by Nieuwstadt & Brost (1986). Sorbian (1997) expanded on their study, showing that the process of decay is governed by the relation of the external time scale to the convective time scale. Beare et al. (2006) extended the experiment to full transition, while using a significantly finer grid in his study, and presented a problem of a correct representation of the ageostrophic wind in the model. Pino et al. (2006) explored the influence of the wind shear and found the TKE decay to be noticeably slower with the inclusion of geostrophic wind.



TKE decay. Sorbian, 1997



TKE decay. Pino et al., 2006

## $k$ - $\varepsilon$ model

### Wind velocity and potential temperature equations

$$\begin{aligned}\frac{\partial U}{\partial t} - \frac{\partial}{\partial z} K_m \frac{\partial U}{\partial z} &= f(V - V_g) - w_{sub} \frac{\partial U}{\partial z} \\ \frac{\partial V}{\partial t} - \frac{\partial}{\partial z} K_m \frac{\partial V}{\partial z} &= -f(U - U_g) - w_{sub} \frac{\partial V}{\partial z} \\ \frac{\partial \theta}{\partial t} - \frac{\partial}{\partial z} K_h \frac{\partial \theta}{\partial z} &= -w_{sub} \frac{\partial \theta}{\partial z}\end{aligned}$$

, where  $K_m = S_m \frac{E_k^2}{\varepsilon}$  and  $K_h = S_h \frac{E_k^2}{\varepsilon}$  are turbulent coefficients.

### TKE and dissipation rate equations

$$\begin{aligned}\frac{\partial E_k}{\partial t} - \frac{\partial}{\partial z} \frac{K_m}{\sigma_k} \frac{\partial E_k}{\partial z} &= P + B - \varepsilon \\ \frac{\partial \varepsilon}{\partial t} - \frac{\partial}{\partial z} \frac{K_m}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} &= \frac{\varepsilon}{E_k} (C_{1\varepsilon} P - C_{2\varepsilon} \varepsilon + C_{3\varepsilon} B)\end{aligned}$$

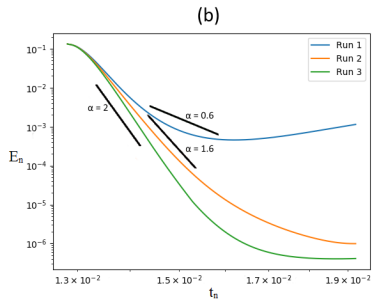
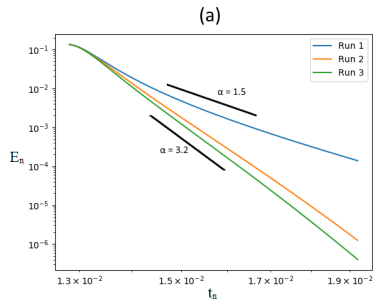
## Experiment setup

- ▶ During the 12-hour run, for the first 6 hours the surface kinematic heat flux is set to be  $F_s = 0.15 \text{ K m s}^{-1}$  to build up a CBL, then it changes abruptly to  $F_s = 0$  (the neutral boundary layer),  $F_s = -0.01 \text{ K m s}^{-1}$  (the SBL) and  $F_s = -0.02 \text{ K m s}^{-1}$  (the strong SBL) in runs 1, 2 and 3, respectively.
- ▶ The geostrophic wind was weakened with setting parameters at  $U_{geo} = 0.01 \text{ m s}^{-1}$  and  $V_{geo} = 0$ , and then introduced with parameters  $U_{geo} = 7.5 \text{ m s}^{-1}$  and  $V_{geo} = 0$ .

At the end of the 6th hour of the run, the thickness of the CBL reaches its maximum height of  $h_{CBL} = 790.9 \text{ m}$ . Deardorff velocity scale (Deardorff, 1970) is  $w_{*0} = (F_b h_{CBL})^{1/3} = 1.517 \text{ m s}^{-1}$ . The surface buoyancy flux  $F_b = g\theta_0^{-1}F_s = 490.5 \text{ m}^2 \text{ s}^{-3}$ , where  $g = 9.81 \text{ m s}^{-2}$  is the gravitational acceleration and  $\theta_0^{-1} = 0.003 \text{ K}^{-1}$  is the air temperature expansion coefficient. The turbulence turnover time scale is  $t_* = h_{CBL}/w_{*0} = 521.3 \text{ s}$ . Thus, the normalized quantities for the decay power law are  $E_n = E/w_{*0}^2$ ,  $t_n = t/t_*$  (El Guernaoui et al., 2019).

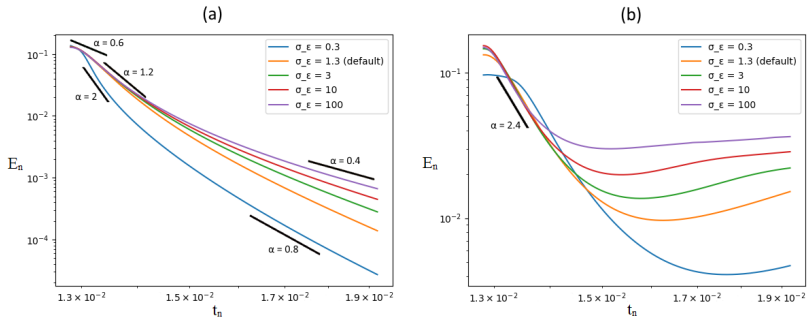


# Experiment results



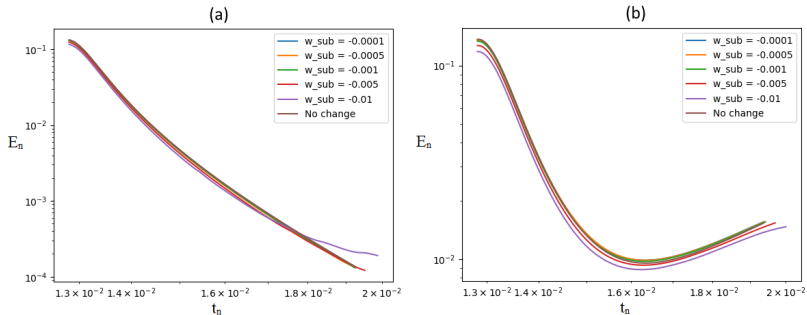
The TKE decay without (a) and with (b) geostrophic wind.

# Influence of the diffusion coefficient $\sigma_\varepsilon$



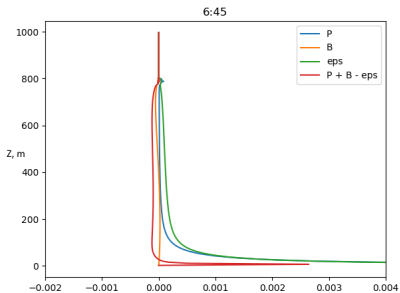
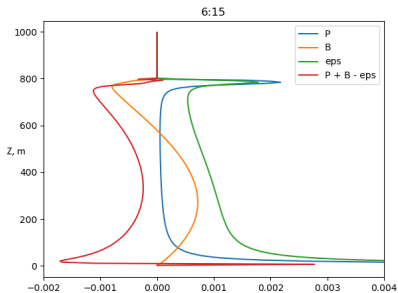
The difference in TKE decay with different values of  $\sigma_\varepsilon$  for Run 1 without (a) and with (b) geostrophic wind.

# Influence of the subsidence rate $w_{sub}$



The difference in TKE decay with different values of  $w_{sub}$  for Run 1 without (a) and with (b) geostrophic wind.

# TKE balance



Graphics of production  $P$ , buoyancy  $B$ , dissipation  $\varepsilon$  and TKE balance  $P + B + (-\varepsilon)$  for Run 1 with geostrophic wind.

## Conclusion

- ▶ It has been shown that there is a significant difference between LES and RANS experiment results when the setups are nearly identical.
- ▶ However, certain parameters can alter the outcome of the experiment when their value is changed (such as  $\sigma_\epsilon$ )
- ▶ Meanwhile, there are parameters, the value of which has little to no influence on the dynamics (such as the subsidence rate  $w_{sub}$ , which has pretty much no effect on the result)
- ▶ As for the influence of the geostrophic wind, the pattern of the simulation results in this study is in agreement with the results by Pino et al. (2006)
- ▶ It is worth investigating further how and why different parameters influence the model's behaviour in RANS experiments, compared to the LES ones, given that it seems to be possible to achieve similar results with carefully chosen values of model constants.

Thank you for your attention!

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