

Markov processes in a problem of the stochastic models of hydrological events.

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Markov stochastic models

In hydrological applications, the problem of the definition of a type of the stochastic model of the process under investigation and its parameters estimation is important. The researches carried out during recent decades have shown the description of the runoff fluctuations as the simple Markov chain to be acceptable.

For the description of river runoff fluctuations there are used:

- the solution of Markov equation in the form of the bilinear decomposition on systems of orthogonal functions;
- stochastic differential equations (SDE) in the form Ito or Stratonovich;
- diffusion equations of Fokker-Planck- Kolmogorov (FPK);

The solution of SDE, received in such a way, can be obtained by two methods. The first method named the Langevin one assumes the notation of SDE solution by quadratures and the consideration of this solution as some operator, transforming the input random process into the output one. The other idea will consist in an identification of the written equation of the system with the stochastic differential equation and in the calculation of the FPK equation coefficients with its subsequent solution.

Application (Caspian sea level fluctuations)

The water level fluctuations in inland water bodies (lakes, reservoirs) is the poorly predicted natural phenomenon, nevertheless which is giving in to the description on the basis of stochastic models of hydrometeorological processes. The water balance mechanism of the level fluctuation consists in the following. Water inflow in a reservoir as a river runoff and precipitation, is spent basically for evaporation. The level fluctuations (h) in an inland reservoir can be described with the help of the known equation of water balance

$$\frac{dh}{dt} = v(t) / F(t) - e(t),$$

where $v(t)$ - inflow in a unit of time, $e(t)$ - layer of seen evaporation (evaporation minus precipitation), $F(t)$ - area of a water surface. Using a Markov models for iflow and evaporation it is possible to find a solution of the stochastic equation of water balance and to do the probabilistic forecasts of the water level as an average of a sea level and deviation from this position of the given probability (quantile).